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IS THE 2 PERCENT INFLATION TARGET SUFFICIENT? THE WEALTH PREFERENCE APPROACH

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Is the 2 percent inflation target sufficient? The wealth preference approach*

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Abstract

We construct a macroeconomic model based on household wealth preferences to identify the theoretical conditions under which full-employment and/or stagnation steady states hold. The theoretical conditions also specify the minimum level of inflation target that shifts the economy from stagnation to full employment. Applying these conditions to Japanese and US data, we assess whether both economies have experienced stagnation in recent decades. Our findings suggest that both steady states are feasible in Japan, while only the full-employment steady state holds in the US. If Japan were to transition to full employment solely through monetary expansion, the inflation target would need to be 5% or higher, with an immediate and significant price increase unavoidable. Moreover, even if a 5 percent inflation target had been implemented in the late 1990s, it would have led to a welfare loss owing to the substantial reduction in the real value of financial assets caused by the initial price surge and subsequent inflation, which outweigh welfare gains from consumption.

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1 Introduction

Since the mid-1990s, the Japanese economy has experienced persistently low inflation. To address this, the Bank of Japan implemented a series of large monetary easings and adopted inflation targeting over the past two decades, though inflation remained subdued. In contrast, the US economy recovered from the deep recession caused by the Global Financial Crisis in the late 2000s. While both countries adopted aggressive monetary stimulus with a 2 percent inflation target, their effectiveness differed considerably.

To explore the reasons behind this difference, we develop a neoclassical growth model incorporating insatiable household wealth preferences and downward nominal wage rigidity (DNWR), which sets a lower bound for inflation. This model can describe dynamic equilibrium paths with full employment or stagnation within a unified framework.

The model gives two steady states in an economy: (a) a full-employment steady state, where wealth preferences are not satiated and DNWR does not bind, and (b) a stagnation steady state, where wealth preferences are satiated and DNWR binds. We identify the theoretical conditions for the validity of these steady states, emphasizing the importance of parameters such as the nominal money growth rate, wealth preferences, and the lower bound of nominal wage inflation.

The theoretical conditions also reveal the minimum inflation target required to shift an economy from stagnation to full employment. This makes the stagnation steady state incompatible with the transversality condition and only the full-employment steady state feasible.

We calibrate the model with data from Japan and the US to assess the feasibility of either or both steady states in each country. The results show that both full-employment and stagnation steady states coexist in Japan, while only the full-employment steady state holds in the US. For Japan to escape stagnation, the required inflation target is at least 5%, significantly higher than the current target. Furthermore, late implementation of this policy worsens welfare. We find that it would have led to a welfare loss if a 5 percent inflation target had been implemented in the late 2010s. This holds true even if it had been implemented in the late 1990s. It is primarily because raising the target to 5% causes an immediate price surge and subsequent higher inflation. These price increases substantially reduce the real value of financial assets and increase the cost of holding assets, outweighing the welfare gains from increased consumption.

The insatiable wealth preference plays a critical role in explaining persistent stagnation in our model. This assumption posits that the marginal utility of holding assets remains positive over time. With insatiable wealth preference, increasing real financial assets continues to provide positive marginal utility, while the marginal utility of consumption diminishes to zero, halting consumption growth. As capital and production capacities expand, consumption remains insufficient to achieve full employment, leading to unemployment and persistent demand shortages. In this context, DNWR prevents a deflationary spiral and shrinking consumption. Without DNWR, falling prices would intensify, making the holding of financial assets more appealing than consumption, which could trigger an unsustainable deflationary spiral.

When both wealth preference and DNWR are present, monetary policy becomes crucial in determining whether the economy can reach one of two steady states. A full-employment steady state exists when the cost of holding assets exceeds their minimum benefit at full employment. If holding assets is too costly, households will consume sufficiently to ensure full employment. Conversely, a stagnation steady state occurs under two conditions: (i) the cost of holding assets in stagnation is less than the minimum benefit of holding assets at full employment, and (ii) the transversality condition holds. Condition (ii) is satisfied only if the cost of holding assets in stagnation exceeds the nominal money growth rate. Monetary policy affects these steady states by controlling nominal money growth, which in turn influences the cost of holding assets and the validity of the transversality condition. The government can shift the economy from stagnation to full employment through monetary expansion, specifically by increasing the nominal money growth rate to meet the required inflation target, thereby violating the transversality condition that holds in stagnation.

The remainder of this paper is structured as follows: Section 2 introduces the model, Section 3 characterizes the two steady states and the conditions for their validity, Section 4 outlines the model calibration, Section 5 presents the simulation results, and Section 6 concludes.

Related literature

Our work builds on the literature addressing inflation targeting under stagnation, including studies by Krugman, Dominquez and Rogoff (1998), Williams (2009), Bernanke, Kiley and Roberts (2019), Andrade et al. (2019, 2021) and Budianto, Nakata and Schmidt (2023). These studies, using the New Keynesian framework, explore optimal inflation targeting and monetary policy in the context of short-run stagnation, particularly when the nominal interest rate hits the zero lower bound (ZLB).[1] In contrast, our study focuses on monetary policy in the context of persistent, demand-driven secular stagnation. Billi, Galí and Nakov (2024) explore optimal inflation targeting and monetary policy under secular stagnation caused by a persistently binding zero lower bound, whereas we examine the inflation target and monetary expansion rate required to shift the economy from secular demand stagnation to full employment.

Another body of literature investigates the relevance of the 2 percent inflation target for advanced countries.[2] Coibion, Gorodnichenko and Wieland (2012) argue that a higher inflation target would increase inflation-related costs, such as greater price dispersion and inflation volatility, which could outweigh the benefits. In contrast, Blanchard, Dell'Ariccia and Mauro (2010), Ball et al. (2016), and Blanco (2021) support a higher inflation target than 2% in the US, highlighting its role in reducing the risk of hitting the effective lower bound of nominal interest rates. De Michelis and Iacoviello (2016) contend that raising the inflation target is less effective unless it is perceived as credible in a very low-inflation environment.

While these studies focus on the inflation target from the perspective of monetary policy availability, our study examines the target needed to shift the economy from a stagnation path to a full-employment path. We also assess the welfare effects of this state-shifting inflation target. Our analysis suggests that delaying implementation of this policy measure may worsen the welfare. It is because the later implementation leads to a larger initial jump in prices, resulting in a larger loss in the real value of assets.

Our research also contributes to the growing body of literature on secular stagnation that assumes the utility of wealth, with marginal utility remaining strictly positive.[3] Ono (1994, 2001) first derived this property from an insatiable wealth preference for money and demonstrated that it led to secular demand stagnation and expanding asset bubbles. This concept has since been

[1] Another strand of research examines the optimal inflation target in the absence of demand shortages. Notable studies in this area include Oikawa and Ueda (2018), Miyakawa, Oikawa and Ueda (2022), and Adam, Pfäuti and Reinelt (2024).

[2] For discussions on the 2 percent inflation target from central bank practitioners, see Amano, Carter and Schembri (2020) and Wells (2024).

[3] A vast body of literature exists on secular stagnation without wealth preferences. Early discussions on the topic can be found in Eichengreen (2015), Summers (2015), and Gordon (2015). More recent contributions, such as Eggertsson, Mehrotra and Robbins (2019), use an overlapping generations framework to analyze secular stagnation.

applied in studies on secular stagnation, with Illing, Ono and Schlegl (2018) and Inagaki, Ono and Tsuruga (2023) analyzing Japan's Lost Decades using this framework. The empirical evidence for insatiable wealth preferences was confirmed by Ono, Ogawa and Yoshida (2004) and Akesaka, Mikami and Ono (2024). Besides insatiable wealth preferences, other factors contribute to this property. Murota and Ono (2011), Ono and Yamada (2018), Michaillat and Saez (2021, 2022) and Cuba-Borda and Singh (2024) explore the role of status preferences regarding asset holdings, while Michau (2018) and Hashimoto, Ono and Schlegl (2023) examine preferences for wealth excluding government liabilities (i.e., money and bonds). Both factors maintain a positive marginal utility of wealth, contributing to secular demand stagnation.

In this analysis, the DNWR assumption is also important for the feasibility of a steady state with secular demand stagnation. Schmitt-Grohé and Uribe (2017) and Benigno and Fornaro (2017) incorporate a DNWR with an effective lower bound in their New Keynesian models, where it is key to generating stagnation. In our model, the DNWR prevents a deflationary spiral, enabling the economy to reach a stagnation steady state.

Overview of Japan's monetary policy since the 1990s

Since the mid-1990s, the Japanese economy has experienced stagnant consumption and rapid money growth in a low-inflation environment. As shown in Figure 1(a), Japan entered a low-inflation period by the late 1990s. Facing the risk of prolonged deflation, the Bank of Japan initiated a series of large-scale monetary expansions in the early 2000s, along with other unconventional policy measures. Quantitative easing (QE) was implemented in 2001 and ended in 2006, followed by comprehensive monetary easing (CME) in 2010 and quantitative and qualitative easing (QQE) in 2013. These asset purchase programs raised the money supply from approximately 600 trillion yen to over 1,000 trillion yen between 2000 and 2019, as depicted in Figure 1(b).

[Figure 1 about here.]

In addition to the asset purchase programs, the Bank of Japan introduced the "Price Stability Target" in January 2013 which was set to 2 percent. This came several years after the introduction of an implicit 1 percent target in the 2006 announcement of "An Understanding of Medium- to Long-Term Price Stability." The target was further clarified in two subsequent statements: "Clarification of the Understanding of Medium- to Long-Term Price Stability" in December 2009 and "Price Stability Goal in the Medium to Long Term" in February 2012.[4,5]

2 Model

This section outlines the model setup, integrating households' wealth preference and DNWR into the standard neoclassical growth model. It also presents the conditions under which full-employment and stagnation paths are valid.

2.1 Setup

Households

There is a mass one of identical infinitely lived households, each with a labor endowment normalized to unity. These households accumulate real capital, denoted as K_t, which evolves according to

$$\dot{K}_t = i_t - \delta K_t, \tag{1}$$

where i represents real investment and δ is the rate of capital depreciation. Households rent capital to firms at the rental price R_t^K. They also hold financial wealth A_t, consisting of nominal bonds B_t (with nominal rate of return R_t) and nominal money M_t (with nominal rate of return zero). Thus, $A_t = B_t + M_t$, which, in real terms, is expressed as

$$a_t = b_t + m_t, \tag{2}$$

where $a_t = A_t/P_t$, $b_t = B_t/P_t$, and $m_t = M_t/P_t$, with P_t representing the price of goods. Households receive labor income $W_t n_t^s$ from supplying labor n_t^s to firms, capital rent $R_t^K K_t$ from firms, returns $R_t B_t$ from bonds, and government transfers T_t. They spend on consumption C_t and investment I_t.

[4] In the statement "The Introduction of a New Framework for the Conduct of Monetary Policy" on March 9, 2006, the Bank of Japan noted that "an approximate range between zero and two percent was generally consistent with the distribution of each Board member's understanding of medium- to long-term price stability. The median figures of the target proposed by most Board members fell on both sides of one percent."

[5] Although the Bank of Japan did not officially declare an inflation target before the 2006 announcement, recent studies suggest that the implicit target may have been much lower than 2 percent. For instance, Hayashi and Koeda (2019) estimate the implicit target inflation rate from 1992 to 2012 and find it to be around 0.5%.

The flow budget equation is

$$\dot{A}_t = W_t n_t^s + R_t^K K_t + R_t B_t + T_t - C_t - I_t,$$

which, in real terms, becomes

$$\dot{a} = w_t n_t^s + r_t^K K_t + r_t b_t + \tau_t - \pi_t m_t - c_t - i_t, \tag{3}$$

where $\pi_t (= \dot{P}_t / P_t)$ is the rate of price inflation, $r_t^K = R_t^K - \pi_t$ is the real capital rent, $r_t = R_t - \pi_t$ is the real return from bond holdings, and τ_t is T_t / P_t.

Households' lifetime utility is expressed as

$$\max \int_0^\infty e^{-\rho t} \left[u(c_t) + v(m_t) + \beta(a_t) \right] dt, \tag{4}$$

where $u(c_t)$ is the utility of consumption, $v(m_t)$ is the utility of money for transaction motive, and $\beta(a_t)$ is the utility of asset holdings representing wealth preference. These functions satisfy the following conditions:

$$\begin{gathered} u'(c_t) > 0,\ u''(c_t) < 0; \\ v'(m_t) > 0,\ v''(m_t) < 0 \text{ for } m_t < \bar{m};\ \ v'(m_t) = 0 \text{ for } m_t \geq \bar{m}; \\ \beta'(a_t) > 0,\ \beta''(a_t) \leq 0,\ \beta'(\infty) = \bar{\beta} > 0. \end{gathered} \tag{5}$$

Note that $v(m)$ becomes satiated when m reaches $\bar{m}$, while $\beta(a)$ is insatiable, with $\beta'(a)$ approaching a strictly positive constant $\bar{\beta}$ as a increases.[6]

Households maximize their lifetime utility (4) subject to the capital accumulation equation (1), the flow budget equation (3), and the stock budget constraint (2). From this, the following

[6] A growing body of research on secular stagnation explores household preference for holding liquid assets to explain weak aggregate demand (Kumhof, Rancière and Winant (2015); Hansen and İmrohoroğlu (2016, 2023), and Mian, Straub and Sufi (2021)).

first-order conditions are obtained:

$$\frac{v'(m_t)}{u'(c_t)} = r_t + \pi_t (= R_t), \tag{6}$$

$$\sigma \frac{\dot{c}_t}{c_t} = r_t - \rho + \frac{\beta'(a_t)}{u'(c_t)}, \tag{7}$$

$$r_t + \frac{\beta'(a_t)}{u'(c_t)} = r_t^K - \delta, \tag{8}$$

where σ represents the elasticity of the marginal utility of consumption. The transversality condition is

$$\lim_{t \to \infty} e^{-\rho t} u'(c_t) a_t = 0. \tag{9}$$

Equation (6) represents the standard money demand function. Equation (7) is the Euler equation incorporating wealth preference $\beta(a_t)$. It implies that the marginal benefit of present consumption, $\rho + \sigma \dot{c}_t / c_t$, equals the marginal benefit of bond holding, $r_t + \beta'(a_t)/u'(c_t)$, which combines the real rate of return, r_t, and the marginal utility of bond holding in terms of real consumption, $\beta'(a_t)/u'(c_t)$. This indicates that households with wealth preference have a weaker incentive to consume compared to those without such a preference.[7] Equation (8) shows that the marginal benefit of bond holding equals the marginal benefit of capital, which is the marginal productivity of capital, r_t^K, minus the rate of capital depreciation, δ.

Firms

Firms rent real capital K_t and hire labor n_t^d from households to produce real output y_t. Their production function exhibits constant returns to scale with respect to capital K_t and labor n_t^d:

$$y_t = F(K_t, n_t^d) = f(k_t) n_t^d, \tag{10}$$

$$\text{where } k_t = \frac{K_t}{n_t^d}, \; f' > 0, \; f'' < 0.$$

[7] Note that (7) reduces to the standard Euler equation ($\sigma \dot{c}_t / c_t = r_t - \rho$) when wealth preference is absent ($\beta(a_t) = 0$).

They maximize profits represented as follows:

$$F(K_t, n_t^d) - r_t^K K_t - wn_t^d, \tag{11}$$

and satisfy the following conditions:

$$\begin{aligned} r_t^K &= f'(k_t), & (12)\\ (\frac{W_t}{P_t} =)\, w_t &= f(k_t) - f'(k_t)k_t. & (13) \end{aligned}$$

Unemployment may exist. Thus, the labor supply n_t^s and labor demand n_t^d satisfy

$$n_t^s = \min(1, n_t^d). \tag{14}$$

Government

The government consolidates the treasury and central bank; hence, its budget constraint is

$$\dot{M}_t^s + \dot{B}_t^s = R_t B_t^s + T_t,$$

where M^s is the nominal money supply and B^s represents the government bonds supplied to households. In real terms, this simplifies to

$$\dot{m}_t^s + \dot{b}_t^s = r_t b_t^s + \tau_t - \pi_t m_t^s. \tag{15}$$

The real money supply m_t^s evolves according to

$$\frac{\dot{m}_t^s}{m_t^s} = \mu_t - \pi_t, \tag{16}$$

where μ_t is the monetary expansion rate controlled by the central bank.

2.2 Markets

The money and bond markets

The equilibrium conditions in the money and bond markets are

$$m_t = m_t^s,$$

$$b_t (= a_t - m_t) = b_t^s = \bar{b}, \tag{17}$$

where the government's bond supply to households b_t^s is assumed to be constant at $\bar{b}$, for simplicity.

The labor market

Assuming DNWR, as discussed in Schmitt-Grohé and Uribe (2016), the inflation rate of nominal wages W_t cannot fall below γ:

$$\frac{\dot{W}_t}{W_t} = \pi_t^W \geq \gamma. \tag{18}$$

The wage inflation rate π_t^W adjusts to achieve full employment ($n_t^d = 1$) as long as it exceeds γ. If the wage inflation rate required for full employment is below γ, then π_t^W is fixed at γ, resulting in involuntary unemployment ($n_t^d < 1$). This can be expressed as a complementary slackness condition in the labor market:

$$\left(\pi_t^W - \gamma\right)\left(1 - n_t^d\right) = 0. \tag{19}$$

The goods market

Goods prices adjust perfectly to ensure that supply, given by (10), always equals demand, which consists of consumption c and investment i. Thus, we have

$$c_t + i_t = f(k_t) n_t^d, \tag{20}$$

where i satisfies (1).

2.3 Dynamics

We describe the dynamics of the economy using market equilibrium conditions. From (1) and (20), the dynamics of K_t is given by

$$\dot{K}_t = \left(\frac{f(k_t)}{k_t} - \delta \right) K_t - c_t, \tag{21}$$

$$\text{where } K_t = n_t^d k_t.$$

Noting that $a_t = m_t + \bar{b}$ from (17), we find $\beta(a_t) = \beta(m_t + \bar{b})$. Let $\psi(m_t)$ be defined as

$$\psi(m_t) \equiv v(m_t) + \beta(m_t + \bar{b}), \tag{22}$$

$$\psi'(m_t) > 0,\ \psi''(m_t) \leq 0,\ \psi'(\infty) = \bar{\beta} > 0.$$

From (6), (8), (12), the time derivative of (13), and (22), we obtain the following expression for goods price inflation π_t:

$$\pi_t = \frac{\psi'(m_t)}{u'(c_t)} - (f'(k_t) - \delta) = \pi_t^W - \alpha(k_t)\frac{\dot{k}_t}{k_t}, \tag{23}$$

$$\text{where } \alpha(k_t) = -\frac{f''(k_t)k_t^2}{f(k_t) - f'(k_t)k_t} (= \alpha \text{ for } f(k_t) = \Omega k_t^\alpha,\ \alpha \text{ and } \Omega \text{ are constant}).$$

Using (19), (21), and (23), we derive the dynamics of k_t under full employment and unemployment conditions as follows:

Region A (full employment): $\gamma < \frac{\psi'(m_t)}{u'(c_t)} - [f'(k_t) - \delta] + \alpha(k_t) \left[\frac{f(k_t)}{k_t} - \delta - \frac{c_t}{k_t} \right] \left(= \pi_t^W \right),$

$$\dot{k}_t = f(k_t) - \delta k_t - c_t, \tag{24}$$

$$K = k (\Longleftrightarrow n_t^d = 1),$$

Region B (unemployment): $\frac{\psi'(m_t)}{u'(c_t)} - [f'(k_t) - \delta] + \alpha(k_t) \left[\frac{f(k_t)}{k_t} - \delta - \frac{c_t}{k_t} \right] \leq \gamma \left(= \pi_t^W \right),$

$$\alpha(k_t)\frac{\dot{k}_t}{k_t} = f'(k_t) - \delta + \gamma - \frac{\psi'(m)}{u'(c)}, \tag{25}$$

$$K < k (\Longleftrightarrow n_t^d < 1).$$

The boundary between the two regions is given by the following hyperplane in terms of (k, c, m):

$$\mathit{\Gamma}(k, c, m) = 0: \ \frac{\psi'(m)}{u'(c)} - [f'(k) - \delta] + \alpha(k)\left[\frac{f(k)}{k} - \delta - \frac{c}{k}\right] = \gamma. \tag{26}$$

The dynamics of consumption c_t and real money m_t are the same in both Regions A and B. The dynamics of consumption c_t is obtained from (7) and (8) as

$$\sigma\frac{\dot{c}_t}{c_t} = f'(k_t) - \delta - \rho. \tag{27}$$

The dynamics of real money m_t is described by (16) because the money market equilibrium is $m_t = m_t^s$. Substituting π_t from (23) into this equation yields

$$\frac{\dot{m}_t}{m_t}(= \mu_t - \pi_t) = \mu_t + f'(k_t) - \delta - \frac{\psi'(m_t)}{u'(c_t)}. \tag{28}$$

Thus, the autonomous dynamics of k, c, and m are summarized as follows:

Lemma 1. If k, c, and m are in Region A, (24), (27), and (28) describe the autonomous dynamics of k, c, and m, along which full employment prevails. These dynamics are the same as those in standard neoclassical growth models. If k, c, and m are in Region B, (25), (27), and (28) describe the autonomous dynamics of k, c, and m, along which unemployment prevails.

3 Two steady states and monetary policy

In this section, we describe two steady states—the full-employment steady state and the stagnation steady state—and the conditions under which they hold. We then demonstrate how monetary policy shifts the economy from stagnation to full employment by invalidating the stagnation steady state. Before addressing each steady state, we first outline the steady-state conditions common to both. Whether in full employment or stagnation, (27) holds. Thus, in both steady states, applying (12), (13) and (27) with $\dot{c}/c_t = 0$, we derive

$$f'(k^*) = \delta + \rho, \tag{29}$$

$$w^* = f(k^*) - f'(k^*)k^*, \tag{30}$$

where the superscript $*$ denotes the steady state, applicable regardless of whether full employment or stagnation prevails.

Full-employment steady state

In the full-employment steady state, (24) and (28) hold, as shown in Lemma 1, and (29) is valid. Therefore, the inflation rate, real consumption, and money (π^f, c^f, m^f) satisfy

$$c^f = f(k^*) - \delta k^*, \tag{31}$$

$$\pi^f = \mu = \frac{\psi'(m^f)}{u'(c^f)} - \rho, \tag{32}$$

where the superscript f represents the full-employment steady state. The nominal wage inflation has a lower bound, γ (i.e., $\pi^W \geq \gamma$), and $w(= W/P)$ remains at w^*, as given by (30), implying $\pi = \pi^W$. Hence, the government must set the monetary expansion rate μ greater than γ :

$$\pi^f = \mu > \gamma, \tag{33}$$

to support m^f.

From the properties of $\psi'(m)$ given in (22), we derive the conditions for the solution of m^f given in (32) to exist, which also determine whether the full-employment steady state can exist. These conditions are

$$\begin{aligned} &\text{(i) } \frac{\bar{\beta}}{u'(c^f)} < \rho + \mu : \text{the full-employment steady state exists,} \\ &\text{(ii) Otherwise : the full-employment steady state does not exist,} \end{aligned} \tag{34}$$

where the left-hand side of the inequality represents the lower bound for the preference to hold assets at the full-employment level of consumption, and the right-hand side represents the cost of holding assets. In case (i), the preference for holding assets is lower than the cost; thus, households increase consumption enough to reach full employment. In case (ii), the preference is higher than the cost, and households do not increase consumption sufficiently to achieve full employment. In other words, $\rho - \bar{\beta}/u'(c^f)$ is the upper bound of the real interest rate in the full-employment steady state (the natural interest rate), $r^n = \rho - \beta'(m^f + \bar{b})/u'(c^f)$, while $-\mu$ represents the lower bound of the feasible real interest rate, since $R(= r + \pi)$ cannot fall below zero. Thus, condition (34) implies that the full-employment steady state is infeasible if and only if the upper bound of r^n is lower than the real interest rate when the nominal interest rate hits the zero lower bound.

The stagnation steady state

In the stagnation steady state, (25) and (28) hold, with $\dot{k}_t = 0$, as established in Lemma 1. Using (22), (23) and (27) with $\dot{c}_t = 0$, we derive the following conditions for π, m and c:

$$\pi^{ss} = \gamma\,(< \mu), \tag{35}$$

$$\frac{\dot{m}_t}{m_t} = \mu - \gamma > 0, \tag{36}$$

$$\rho + \gamma = \frac{\bar{\beta}}{u'(c^{ss})}, \tag{37}$$

where the superscript ss denotes the stagnation steady state. The consumption level, c^{ss}, must be lower than c^f; otherwise, real consumption would reach the full-employment level, c^f. Since $u''(c) < 0$, the following condition must hold for stagnation to be feasible:

$$\frac{\bar{\beta}}{u'(c^f)} > \rho + \gamma \left(= \frac{\bar{\beta}}{u'(c^{ss})} \right), \tag{38}$$

In this state, m_t continues to expand, as shown in (36), leading $\psi'(m_t)$ to $\bar{\beta}$. Consequently, the nominal interest rate, $R(= v'(m)/u'(c))$, becomes zero, because the utility of money, based on the transaction motive, $v(m)$, is satiated for $m \geq \bar{m}$, as assumed in (5). Although m_t diverges to infinity while c_t remains constant at c^{ss}, the transversality condition in (9) holds if and only if

$$\text{TVC} : \frac{\dot{a}_t}{a_t} - \rho = \frac{\dot{m}_t}{m_t} - \rho = \mu - (\gamma + \rho) < 0. \tag{39}$$

For the stagnation steady state to be feasible, both the existence condition (38) and the TVC

(39) must be satisfied; hence,

$$\text{(i) } \frac{\bar{\beta}}{u'(c^f)} > \rho + \gamma > \mu : \text{the stagnation steady state exists,}$$

$$\text{(ii) Otherwise : the stagnation steady state does not exist.} \tag{40}$$

The conditions for the two steady states to hold

From (34) and (40), we derive the conditions under which the two steady states can hold, summarized as follows:

Lemma 2. One or both steady states are feasible under the following conditions:

(A) (i) of (34) and (i) of (40): Both the full-employment and the stagnation steady states are feasible.

(B) (i) of (34) and (ii) of (40): Only the full-employment steady state is feasible.

(C) (ii) of (34) and (i) of (40): Only the stagnation steady state is feasible.

(D) (ii) of (34) and (ii) of (40): Neither the full-employment nor the stagnation steady state is feasible.

Given that $v'(m) = 0$ for $m \geq \overline{m}$, as stated in (5), and that $R = v'(m)/u'(c)$ from (6), we have

$$R > 0 \quad \text{if } m < \bar{m},$$

$$R = 0 \quad \text{if } m \geq \bar{m}.$$

m^f given by (32) can be either larger or smaller than $\bar{m}$. Therefore, if case (i) of (34) is valid and $m^f < \bar{m}$, full employment is achieved with $R > 0$. If case (i) of (34) is valid and $m^f \geq \bar{m}$, full employment is still achieved, but $R = 0$. In case (i) of (40), the stagnation steady state is valid, where m continues to expand, and $R = 0$. Thus, facing the ZLB of R is not necessarily indicative of stagnation; R can be zero even when full employment is present.

The dynamic path described by Lemma 1 leads to either the full-employment steady state (given by (31) and (32)) or the stagnation steady state (defined by (35), (36) and (37)) depending on the case in Lemma 2. As noted in Lemma 1, the dynamics in Region A follow the standard neoclassical growth model, meaning the path to the full-employment steady state is unique. The saddle stability around the stagnation steady state is demonstrated in the Appendix, confirming a unique path toward the stagnation steady state in Region B. Looking backward in time, the path passes a unique point at the border between Regions A and B. In Region A, the path leading to this point is selected as the dynamic equilibrium path, along which full employment is maintained.

The phase diagram for case (A) of Lemma 2 is shown in Figure 2. In this diagram, the red line always remains in Region A, following the equilibrium path of the standard monetary growth model, where full employment holds, and the full-employment steady state is eventually reached. The black line starts in Region A, where full employment prevails until the point $(\hat{c}, \hat{k}, \hat{m})$ is reached, satisfying (26):

$$\Gamma(\hat{c}, \hat{k}, \hat{m}) = 0.$$

After reaching this point, the path enters Region B, where unemployment appears and asymptotically approaches the stagnation steady state. The dashed lines indicate the projections of each path on the (k, c) and (m, c) planes.

[Figure 2 about here.]

We rewrite Lemma 2 to clarify the conditions under which the monetary expansion rate μ supports each steady state, as summarized in the following proposition.

Proposition 1. The conditions for the monetary expansion rate μ to support the full-employment and stagnation steady states are the following:

(A) If $\bar{\beta}/u'(c^f) \leq \rho + \gamma$, only the full-employment steady state holds for any $\mu \geq \gamma$.

(B) If $\bar{\beta}/u'(c^f) - \rho \leq \rho + \gamma < \bar{\beta}/u'(c^f)$, the validity of the two steady states depends on μ:

1. If $\mu < \bar{\beta}/u'(c^f) - \rho$, only the stagnation steady state holds.
2. If $\bar{\beta}/u'(c^f) - \rho \leq \mu < \rho + \gamma$, both full-employment and stagnation steady states are valid.
3. If $\rho + \gamma \leq \mu$, only the full-employment steady state holds.

(C) If $\rho + \gamma < \bar{\beta}/u'(c^f) - \rho$, the validity of the two steady states depends on μ:

1. If $\mu < \rho + \gamma$, only the stagnation steady state holds.
2. If $\rho + \gamma \leq \mu < \bar{\beta}/u'(c^f) - \rho$, no steady state exists.

3. If $\bar{\beta}/u'(c^f) - \rho \leq \mu$, only the full-employment steady state holds.

Proposition 1 demonstrates that the monetary expansion rate is crucial in determining the economy's regime. In case (A), the economy always follows the full-employment path. In cases (B) and (C), the economy may follow either the full-employment or stagnation path, depending on μ.

In case (B)-2, both steady states are feasible. If the full-employment path is chosen, the economy maintains full employment; if the stagnation path is chosen, the economy enters secular demand stagnation. In the latter case, to shift the economy from stagnation to full employment, the government must increase μ sufficiently to satisfy case (B)-3, invalidating the transversality condition along the stagnation path. Without this, transitioning to the full-employment path becomes almost impossible, as households behave rationally along the stagnation path. Raising the target inflation through an insufficient rise in μ can affect neither the inflation rate $\pi^{ss} = \gamma$ nor consumption. The government might be able to stimulate consumption by directly raising the nominal wage inflation rate γ, though this approach is politically challenging.

4 Calibration

We calibrate the model using the steady-state conditions presented in Section 3 to examine the feasibility of the two steady states and determine the necessary inflation target (the nominal money growth rate in the full-employment steady state) to transition the economy from stagnation to full employment. To achieve this, we specify the utility and production functions.

The unit period is set to one year. We assume the following utility functions for consumption, money, and assets:

$$u(c_t) = \log(c_t), \tag{41}$$

$$v(m_t) = \begin{cases} \nu \left(\log \frac{m_t}{\bar{m}} - \frac{m_t}{\bar{m}}\right) & \text{if } m_t \leq \bar{m}, \\ -\nu & \text{if } m_t > \bar{m}, \end{cases} \tag{42}$$

$$\beta(m_t + \bar{b}) = \bar{\beta} \times (m_t + \bar{b}). \tag{43}$$

Thus, the function $\psi(m_t)$ in (22) satisfies

$$\psi'(m_t) = \begin{cases} \nu\left(\frac{1}{m_t} - \frac{1}{\bar{m}}\right) + \bar{\beta} & \text{if } m_t \leq \bar{m}, \\ \bar{\beta} & \text{if } m_t > \bar{m}. \end{cases} \tag{44}$$

The production function (10) follows the Cobb-Douglas form, with a capital share α and productivity A:

$$f(k_t) = Ak_t^{\alpha}. \tag{45}$$

Given that the steady-state k satisfies (29), this production function leads to the following output in the full-employment steady state, which is normalized to 1:

$$f(k^*) = A\left(\frac{\rho+\delta}{A\alpha}\right)^{\frac{\alpha}{\alpha-1}} = 1. \tag{46}$$

From (31) and (46), we have

$$A = \left(\frac{\rho+\delta}{\alpha}\right)^{\alpha}, \tag{47}$$

$$k^* = \frac{\alpha}{\rho+\delta}, \tag{48}$$

$$c^f = \frac{\rho+(1-\alpha)\delta}{\rho+\delta}. \tag{49}$$

By substituting the value of c^f given by (49), the utility functions in (41) and (43), and $\dot{c}/c = 0$ into (7), we obtain the real interest rate in the full-employment steady state r^f:

$$r^f = \rho - \bar{\beta}\left[\frac{\rho+(1-\alpha)\delta}{\rho+\delta}\right]. \tag{50}$$

The results of the calibration are summarized in Table 1.

[Table 1 about here.]

We set $\rho = 0.04$, following recent studies on secular stagnation in the Japanese economy, including Hansen and İmrohoroğlu (2016), Hansen and İmrohoroğlu (2023), and Michau (2024). The capital share and capital depreciation rate are set to typical values assumed in the literature: $\alpha = 0.4$ and $\delta = 0.09$, respectively.

We calibrate the values of $\bar{\beta}$, $\bar{m}$, and ν sequentially, under the assumption that the Japanese economy was at full employment in the period 1981–1991 and at stagnation in the period 1999–2019. We first compute $\bar{\beta}$ by setting r^f and substituting r^f, α, δ, and ρ into (50). r^f is set to -0.007 (-0.7%), which is based on the average real interest rate and per-capita GDP growth rate in the period 1981–1991, which are 3.1% and 3.8%, respectively.[8] Substituting these values into (50) and rearranging the results yields $\bar{\beta} = 0.065$.

We then calibrate the threshold of real money ($\bar{m}$), which represents the real money value when the nominal interest rate first reaches zero, to its 1999 level—the year in which Japan's zero interest rate policy began. To do this, we compute the autonomous dynamics of c, m, and k in Region B, as given by (25), (27), and (28), respectively, and the dynamics of K as described by (21). The lower bound of the nominal wage inflation rate γ, which is needed to compute these dynamics, is set to 0.002, based on the average rate of Japanese nominal wage inflation in the period 1999–2019.[9] Solving these dynamics with the 1999 consumption-money ratio (0.388) and the unemployment rate (0.08) yields $\bar{m} = 1.625$.[10]

We set the scale parameter ν of the utility of money for transactions using the money demand function. Applying (41) and (42) to (6) yields

$$(R^f =) r^f + \pi^f = \nu \left(\frac{c^f}{m^f} - \frac{c^f}{\bar{m}} \right). \tag{51}$$

The calibration of ν requires the values of π^f, c^f, and m^f, as well as those of r^f and $\bar{m}$ that have been already calibrated. We set π^f to 0.02, based on the average consumer price inflation rate in the period 1981–1991. We obtain $c^f = 0.723$ from (49), and set c^f/m^f to 0.491, the average ratio of household consumption to M2 in the period 1981–1991. This results in $m^f = c^f/(c^f/m^f) = 1.473$. Substituting these values together with r^f and $\bar{m}$ into (51) yields $\nu = 0.282$.

The calibration results suggest that the Japanese economy is in case (B)-2 of Proposition 1 in Section 3. Specifically, the value of $\rho + \gamma = 0.042$ lies between $\bar{\beta}/u'(c^f) - \rho = 0.007$ and

[8] The real interest rate is calculated as the 1-year Japanese bond yield minus the long-term expected inflation rate of 3%. According to Chart 9 in Uchida (2024), the expected inflation rate for the 6-10 years ahead in the early 1990s ranged from 2% to 3%. Since our model assumes no economic growth, r^f is detrended by the average real per-capita GDP growth rate in the period 1981–1991.

[9] We exclude the year 2009 from the calculation to eliminate the impact of the Global Financial Crisis. The value is not detrended because the average real per-capita GDP growth rate in the period 1999–2019 is around zero.

[10] We set the 1999 unemployment rate at 8%($n = 0.92$), based on Chart 18 in Hara et al. (2006).

$\bar{\beta}/u'(c^f) = 0.047$, satisfying the condition for case (B). Thus, the nominal money growth rate determines the validity of the two steady states. In the period 1999–2019, the average growth rate of M2 (per capita) is 2.6%, which falls between $\bar{\beta}/u'(c^f) - \rho$ and $\rho + \gamma$. This confirms that case (B)-2 is applicable to the Japanese economy.[11] In other words, both the full-employment and stagnation steady states are valid in Japan. While Proposition 1 does not specify which steady state is valid in case (B)-2, the experience of Japan's lost decades strongly suggests that the economy has been stagnant since the late 1990s.

As Proposition 1 indicates, increasing the rate of monetary expansion to satisfy $\mu \geq \rho + \gamma$ can shift the economy from case (B)-2 to case (B)-3 by validating only the full-employment steady state, and move the economy to the full-employment path. Given that the inflation rate in the full employment steady state π^f equals $\mu(\geq \rho + \gamma)$, the inflation target required to escape stagnation is equal to or higher than $\rho + \gamma = 0.042$ (4.2%). This target is substantially higher than the current target.

For comparison, we examine whether the US economy was along the stagnation path after the Global Financial Crisis. We calibrate the model under the assumption that the US was fully employed in the period 1986-2006. Using the same values for ρ, α and δ as in Japan (0.04, 0.4, and 0.09, respectively), we set the real interest rate in the full-employment steady state to $r^f = 0.0309$ with the GDP growth rate of 0.0204, yielding the detrended real interest rate of 0.0105. The resulting value of $\bar{\beta}$ is 0.0409.[12] This value of $\bar{\beta}$ is obviously lower than that of Japan (0.065), suggesting that US households have weaker wealth preferences than Japanese households. The remaining parameters are calibrated as follows: γ is set to 0.0125, reflecting the average personal consumption expenditure inflation rate in the period 2009–2015. Using the values of $\bar{\beta}$ and c^f, the latter of which is obtained by substituting the values of ρ, α and δ into (49), we find $\bar{\beta}/u'(c^f) = 0.0295$, which is lower than $\rho + \gamma = 0.055$. Thus, the US economy is compatible with case (A) in Proposition 1. The calibration results imply that the US economy should have been on the full-employment path since the Global Financial Crisis. This finding aligns with recent studies suggesting that the deep recession in the US caused by the Global Financial Crisis was not due to

[11] We set $\mu^{ss} = 0.026$, based on the money growth rate in the period 1999–2019.

[12] We use a long-term real interest rate series from the Cleveland Fed and real GDP growth rate from the FRED by the St. Louis Fed.

a lack of demand.[13]

5 Model simulations

In Section 4, we find that at least 4.2% inflation was necessary to shift the Japanese economy from stagnation to full employment. This section numerically examines the impact of setting the inflation target at 0.05(>0.042), with monetary expansion to support it, on consumption and welfare.[14] Two scenarios are considered: (i) the policy starting in 1999, during Japan's financial crisis, marked by a chain of failures of major financial institutions, and (ii) the policy starting in 2019, just before the COVID-19 pandemic.

The solid and dashed lines in Figure 3 depict the simulation results for scenarios (i) and (ii), respectively. In the first period, the central bank announces a 5 percent inflation target and raises nominal money growth to meet the target. These policy changes increase the cost of holding financial assets, leading households to reduce asset holdings and increase consumption. As a result, consumption c and prices P rise immediately, and the increase in prices simultaneously causes real money m to decrease, as shown in Figures 3(a)-(c). The nominal money growth rate μ increases by about 4 percentage points, as illustrated in Figure 3(d). Delaying the policy change intensifies the initial responses. In scenario (i), P and m change by approximately 50% and -35%, respectively (1.5 and 0.7 times). In scenario (ii), the changes are larger, at about 140% and -60% (2.4 and 0.4 times).

[Figure 3 about here.]

Figure 4 presents the total welfare effects of the policy change and their breakdown across the two scenarios. Notably, both scenarios experience welfare losses, with the loss in 2019 being greater than in 1999. As Figure 4(b) shows, increased consumption (blue) improves welfare, but decreased real money, affecting both transaction (red) and wealth preference (yellow), leads to welfare losses. This results in overall negative effects in both scenarios, suggesting that the later the policy is implemented, the greater the welfare loss.

[13] Jones (2023) reports that the aging of the US population has been a major factor in the slow recovery from the Great Recession by lowering interest rates.

[14] While the new target can be any value greater than or equal to 4.2%, central banks typically set inflation targets in multiples of one percent. We begin the two scenarios when the calculated consumption-money ratio on the stagnation path is closest to the actual ratio in the respective years.

[Figure 4 about here.]

The following points are noted:

1) An increase in the inflation target and the corresponding monetary expansion necessary to move the Japanese economy from stagnation to full employment should have been implemented before 1999. This finding suggests that effective policy intervention may be too late after the onset of unemployment. If implemented, households would face at least 5% inflation.

2) Welfare declines more as real wealth increases because the policy reduces real wealth more. Thus, in the presence of wealth inequality, the richer tend to prefer secular stagnation, while the poorer favor policy shifts.

3) When production capacity is underutilized, fiscal expansion employing idle labor always benefits the economy for two key reasons. First, it creates public services. Second, importantly, using idle labor incurs no opportunity costs. Therefore, Japan should prioritize fiscal expansions over monetary ones to better utilize idle labor.

6 Conclusion

We constructed a macroeconomic model based on household wealth preferences to identify the conditions of preference, production and policy parameters for either or both of the full-employment steady state and the stagnation steady state to be valid. We then tested these conditions using data from Japan and the US. The results indicate that the parameters of Japan satisfy the conditions under which both steady states are feasible, with the stagnation path chosen by chance. In contrast, the parameters of the US satisfy the conditions under which only the full-employment path is possible. Additionally, we determined the minimum monetary expansion rate and inflation target required to transition the Japanese economy from stagnation to full employment. The effects of this policy on consumption, prices, real financial assets, and welfare were also discussed.

Our quantitative analysis shows that Japan's inflation target must be at least 5%. To meet this target, the nominal money growth rate must rise along the path, as illustrated in Figure 3(d). Notably, implementing this policy results in an immediate and significant price increase. Simulation results indicate that prices would have risen by 50% (1.5 times) if the policy were implemented in 1999 and by 140% (2.4 times) if implemented in 2019. Such a sharp price increase vastly reduces the real value of household financial assets, deteriorating economic welfare. Although the policy succeeds in moving the economy out of stagnation, this can lead to an overall welfare loss. The simulation results show that it would have been the case even if a 5 percent inflation target had been implemented in 1999. These findings suggest that inflation target measures aimed at escaping stagnation may not be optimal during the Lost Decades.

Our analysis has one important caveat: we assume central banks can perfectly control the monetary expansion rate. However, in practice, central banks can only directly control the monetary base, not M2, which includes currency in circulation and deposits. In reality, increasing M2 as planned is difficult. In Japan, the monetary base has grown significantly faster than M2, particularly since the Global Financial Crisis. From 2009 to 2019, the monetary base grew at an annual rate of 17%, while M2 increased by about 3% annually. Perfect control of the money supply requires the government to adopt forceful measures, such as helicopter drops of money. This involves the fiscal authority issuing bonds to distribute money directly to households as subsidies, with the central bank purchasing these bonds by creating base money.

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Appendix: Local stability

First, we examine the stability around the full-employment steady state, given by (29), (31), and (32). Equations (32) and (33) establish the condition:

$$\rho + \gamma < \frac{\psi'(m^f)}{u'(c^f)} = \rho + \mu,$$

which implies that the steady state lies in Region A. From Lemma 1, the autonomous dynamics are as follows:

$$\dot{k}_t = f(k_t) - \delta k_t - c_t, \;\; K = k,$$

$$\sigma \frac{\dot{c}_t}{c_t} = f'(k_t) - \delta - \rho,$$

$$\frac{\dot{m}_t}{m_t} = \mu_t + f'(k_t) - \delta - \frac{\psi'(m_t)}{u'(c_t)}.$$

These equations reflect the standard dynamics of a monetary economy, indicating that the steady state is saddle-stable.

On the stagnation path, according to Lemma 1, the dynamics are

$$\alpha(k_t)\frac{\dot{k}_t}{k_t} = f'(k_t) - \delta + \gamma - \frac{\bar{\beta}}{u'(c)},$$

$$\sigma \frac{\dot{c}_t}{c_t} = f'(k_t) - \delta - \rho,$$

$$\frac{\dot{m}_t^v}{m_t^v} = -\mu_t - f'(k_t) + \delta + \frac{\bar{\beta}}{u'(c_t)}, \;\; \text{where } m_t^v \equiv 1/m_t,$$

$$\dot{K}_t = \left(\frac{f(k_t)}{k_t} - \delta\right) K_t - c_t.$$

Around the stagnation steady state, defined by (29), (36), and (37), we have

$$(k, c, m_t^v, K) = \left(k^*, c^{ss}, 0, \frac{k^* c^{ss}}{f(k^*) - \delta k^*}\right).$$

The corresponding characteristic equation is

$$\begin{vmatrix} \frac{f''k^*}{\alpha} - \lambda & -\frac{k^*\sigma\bar{\beta}}{\alpha u'(c^{ss})c^{ss}} & 0 & 0 \\ \frac{f''c^{ss}}{\sigma} & -\lambda & 0 & 0 \\ 0 & 0 & (\gamma-\mu)-\lambda & 0 \\ -\frac{(f(k^*)-f'(k^*)k^*)K^*}{k^{*2}} & -1 & 0 & \frac{f(k^*)-\delta k^*}{k^*} - \lambda \end{vmatrix} = 0,$$

which simplifies to

$$\left(\lambda^2 - \frac{f''k^*}{\alpha}\lambda + \frac{f''k^*\bar{\beta}}{\alpha u'(c^{ss})}\right)\left[\frac{f(k^*)-\delta k^*}{k^*} - \lambda\right] = 0.$$

Since $f'' < 0$ and $\gamma - \mu < 0$ from (33), λ has two negative and two positive solutions. Thus, the system is saddle-stable, with K and m being unjumpable, while c and $k\,(= K/n)$ are jumpable.

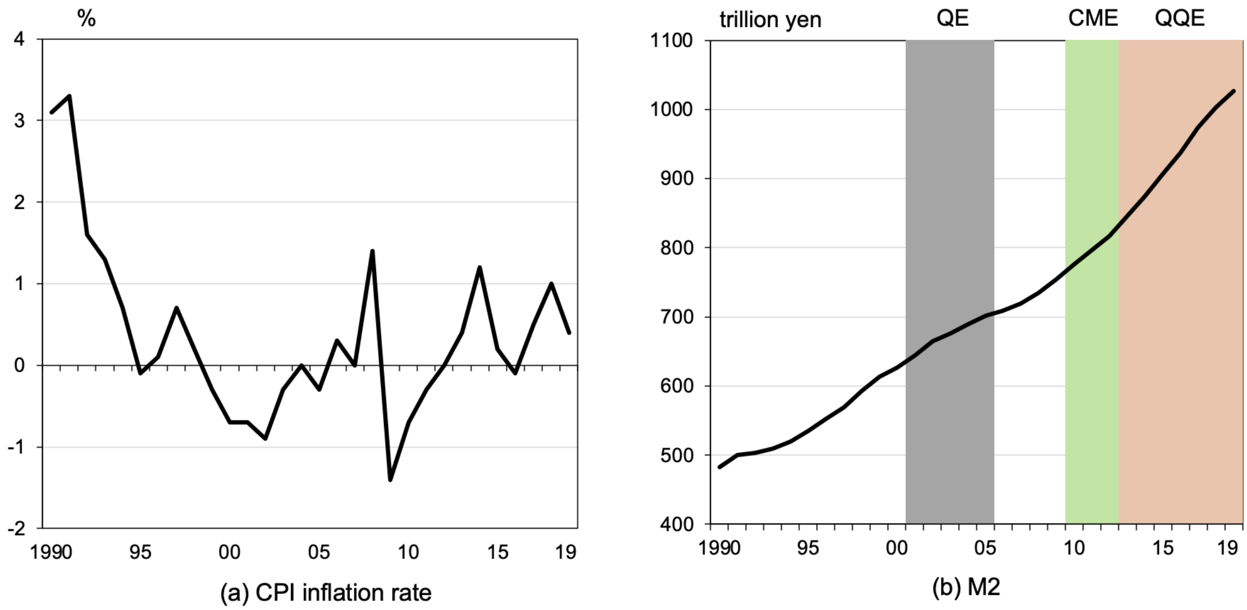

Figure 1: Japan's inflation and money stock from 1990 to 2019

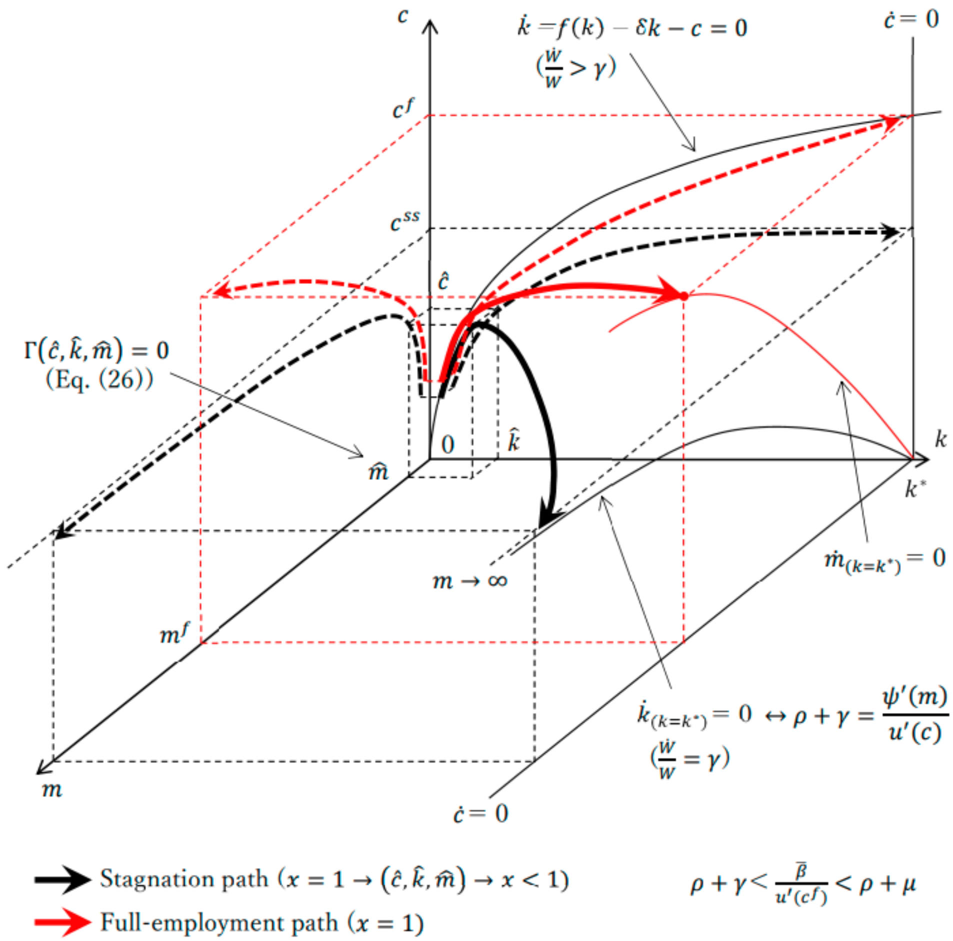

Figure 2: Phase Diagram

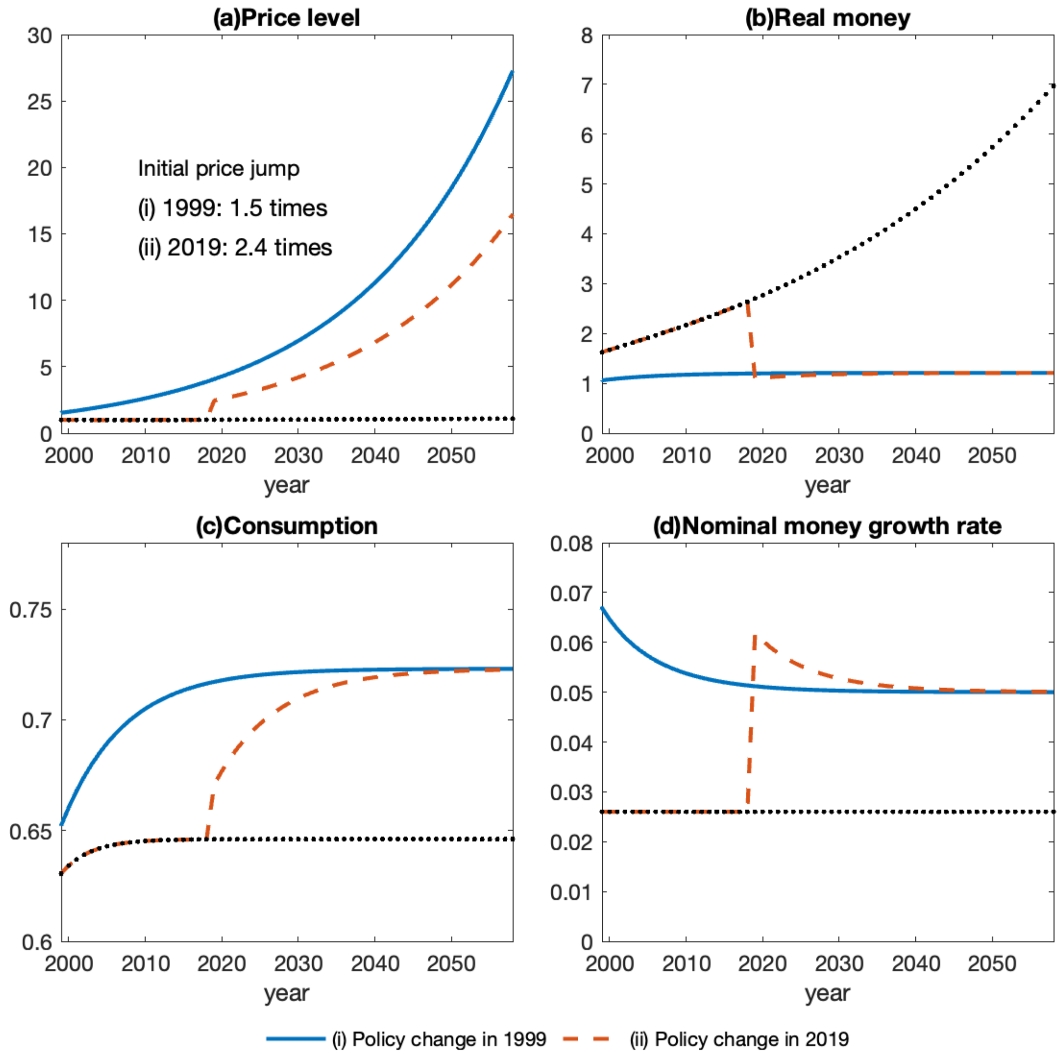

Figure 3: Responses to monetary policy changes

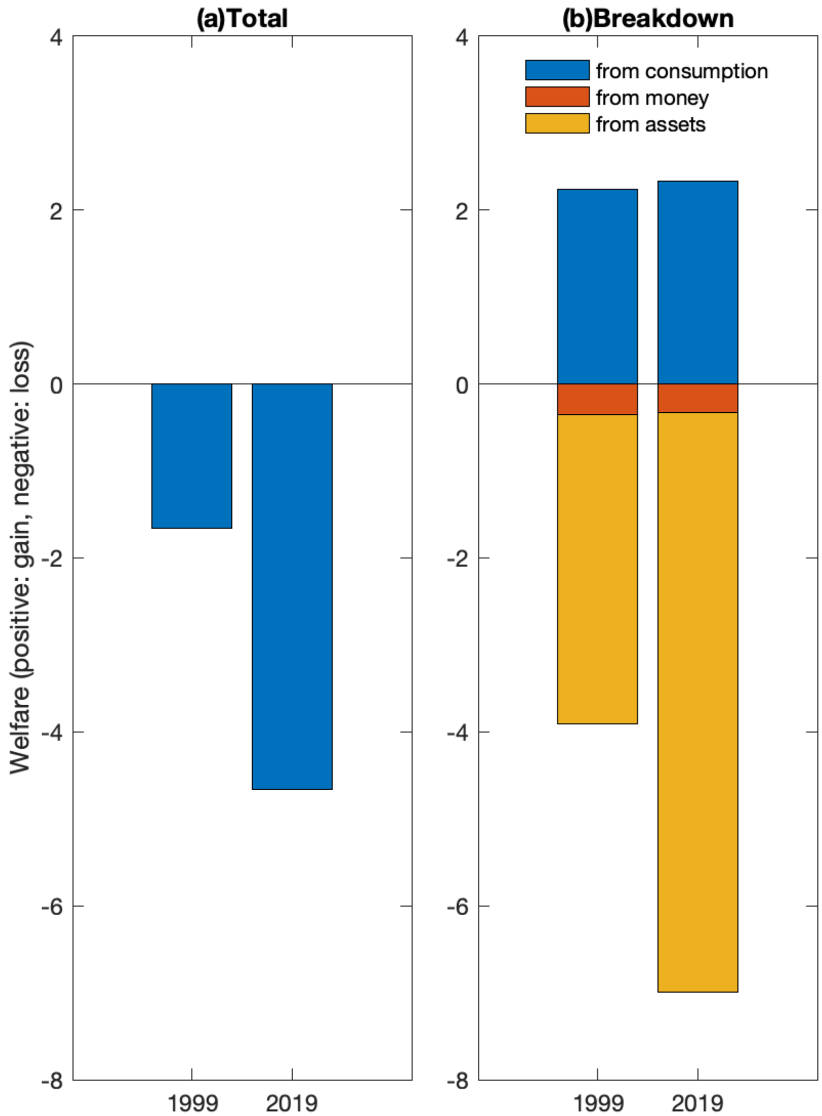

Figure 4: Welfare in two policy scenarios

Parameter		Value	Reference or moment
Subjective discount rate	ρ	0.04	Standard
Capital share	α	0.40	Standard
Capital depreciation	δ	0.09	Standard
Lower bound on wage growth	γ	0.002	Nominal wage inflation rate over 1999-2019
Marginal utility from assets	$\bar{\beta}$	0.065	(50) with Japanese data over 1981-1991
Threshold of real money in utility	$\bar{m}$	1.625	Matching consumption-money ratio in 1999
Scale parameter of money utility	ν	0.282	(51)

Table 1: Parameter calibration for the Japanese economy