

**TRANSITION TO GREEN INDUSTRY  
AND RECYCLING IN A HETEROGENEOUS-  
INDUSTRY AND ENDOGENOUS  
GROWTH MODEL**

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# Transition to green industry and recycling in a heterogeneous-industry and endogenous growth model \*

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## Abstract

This study introduces two heterogeneous industries into an endogenous growth model in a circular economy. In our model, there are two types of industries, brown industries using exhaustible resources for production, and green industries using recycled goods which are reproduced from the used final good by a recycling firm. Each industry switches the state as a result of R&D activities for innovation and greening. Innovation improves the level of productivity and occurs in both industries. In contrast, only firms in brown industries invest in R&D activities for greening, which transfers the brown industries toward the green industries. This paper examines the effect of recycling and the share of green industries on the growth rate. We show that an increase in the recycling rate does not have a negative effect on the economy, and improves the welfare of households.

**Keywords:** Endogenous growth · Circular economy · Recycling · Heterogeneous-industry model

**JEL Classification Numbers:** O31 · O44 · Q53

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# 1 Introduction

## 1.1 Introduction

A “circular economy” (CE) is an economic and social system that should be aimed at in the future, and various efforts are being made mainly in Japan, Germany, China, and other countries. The CE is an economic activity that aims to change the economic system of mass production, mass consumption, and mass disposal, and to use scarce resources and energy sustainability;. According to Heshmati (2017), several reasons why the circular economy should be promoted: to address environmental issues; to address the lack of demand for resources and energy associated with rapid economic growth; and to enhance national security by promoting alternative energy resources and making the use of materials more efficient. Circular economy initiatives are not limited to those initiated by the government, but are increasingly being undertaken voluntarily by firms. TOYOTA states that by 2030 it will use at least 30 percent recycled materials in the production of its vehicles (TOYOTA, 2025). Apple inc. procures 24 percent of its materials in its products from recycled or renewable sources (Apple inc., 2025), and this trend is expected to continue in the future.

This study constructs an R&D-based growth model that accounts for industry heterogeneity and examines the long-run economic impact of recycling and a fraction of industry. There are two types of intermediate goods-producing industries in the economy: one produces with inputs of exhaustible resources (brown industry) and the other produces with inputs of recycled goods (green industry). The final good is produced by using a unit continuum of intermediate goods. Recycled goods are made from final goods consumed by households and reproduced by recycling firms. Each industry switches the state as a result of R&D activities for innovation and greening. Innovation improves the level of productivity and occurs in both industries. In contrast, only firms in brown industries invest in R&D activities for greening, which

transfers the brown industries toward the green industries. Regardless of their states, once R&D activities for innovation succeed, productivity improvements make the industry a brown one. We follow Chu et al.(2023) for the model framework where the industries switch their states.

We show that an increase in the recycling rate improves the welfare of households, and the growth rate is independent of the recycling rate. Therefore, we conclude that the recycling rate does not have a negative effect on the economy. In our model, the recycling rate only affects aggregate production, not on the clearing condition of the labor market. Because of this, the R&D activities are also independent of the recycling rate. However, since there is a difference between the allocation of labor in R&D, which maximizes the growth rate, and the allocation of labor determined by equilibrium, it is suggested that policy enforcement here would make the economy more efficient and achieve the optimal industry fraction.

## 1.2 Related literature

In line with the growing international momentum for recycling, there has been an increasing number of studies in economics analyzing recycling and the circular economy in recent years. The literature that has constructed the dynamic models of recycling includes Hoel (1978), Di Vita (2001, 2004), Pittel et al. (2010), Akimoto and Futagami (2018), Lafforgue and Rouge (2019), Rosendahl and Rubiano (2019), and Zhou and Smulders (2021). Hoel (1978) derives the optimal path assuming resources and recycled goods are perfect substitutes. Di Vita (2001, 2004) shows many positive effects of waste recycling on the economy. Pittel et al. (2010) compare the social planner economy with the decentralized economy under the material balance and analyze the market failures caused by not considering the waste market. Akimoto and Futagami (2019) and Lafforgue and Rouge (2019) discuss the transition from a linear economy to a circular economy. Akimoto and Futagami (2018), using the Ramsey model, obtain the Environmental Kuznets Curve along the optimum path. Lafforgue and Rouge (2019) assume that, initially, recycled

resources are not as productive as non-renewable resources, and that, by investing in R&D, productivity will increase, and recycled resources will also be used in production, thus achieving a circular economy. Rosendahl and Rubiano (2019) focus on the Lithium market and examine to what extent recycling improves resource scarcity. Several papers have reported the positive impacts of recycling on the economy, Zhou and Smulders (2021) state that an increase in recycling rates will result in economic losses if innovations are strongly resource-saving, and argue that we should be cautious once and for all about the introduction of recycling. These studies listed above do not discuss industrial heterogeneity, and in this respect, this study has a contribution to make regarding the connection between recycling and economic growth.

The remainder of the paper is organized as follows: Section 2 describes the setting of the model. Section 3 characterizes the balanced growth path. Section 4 concludes this paper.

## 2 The model

We introduce a circular economy into an R&D-based growth model. There exist two industries, in the economy, green and brown. Both industries produce intermediate goods, the production process in the green industry uses recycled goods, and in the brown uses raw materials and labor. Here, we assume that the industries are more labor-intensive than the green ones. We will derive the growth rate on the balanced growth path and the fraction between green and brown.

## 2.1 Final good

Final goods  $y_t$  are produced by competitive firms, which use a unit of continuum of differentiated intermediate goods:

$$y_t = \exp \left( \int_0^1 \ln x_t(i) di \right). \quad (1)$$

$x_t(i)$  denotes intermediate good  $i \in [0, 1]$ , and (1) gives the conditional demand function for  $x_t(i)$  is

$$x_t(i) = \frac{y_t}{p_t(i)}, \quad (2)$$

where  $p_t(i)$  is the price of  $x_t(i)$ .

## 2.2 Intermediate goods

There is a unit continuum of industries, which is indexed by  $i \in [0, 1]$  producing intermediate goods. There exist two types of industry, green and brown, We define the set of green industries as  $\Lambda$ , and brown as  $\Theta$ . A fraction of green (brown) industries is denoted by  $\theta_t$  ( $1 - \theta_t$ ).

### 2.2.1 Brown industry

If an industry is brown industry ( $i \in \Theta$ ), then the production process uses the raw material:

$$x_t(i) = q^{n_t(i)} m_t(i), \quad (3)$$

where  $q > 1$  is the parameter of the step size of productivity improvement,  $n_t(i)$  is the number of quality improvements in industry  $i$  at time  $t$ ,  $m_t(i)$  is the amount of raw material used in industry  $i$ . The marginal

cost of the leader in a brown industry  $i$  is  $p_{m,t}/q^{n_t(i)}$  where  $p_{m,t}$  is the material price. The government regulates the monopolistic price, which cannot be greater than  $\mu > 1$ . Then, the leader chooses the price for maximizing profit such as

$$p_t(i) \leq \mu \frac{p_{m,t}}{q^{n_t(i)}} \Rightarrow p_t(i) = \mu \frac{p_{m,t}}{q^{n_t(i)}}. \quad (4)$$

In this case, the payment for a material in brown industry  $i$  is

$$p_{m,t}m_t(i) = \frac{p_t(i)x_t(i)}{\mu} = \frac{y_t}{\mu}, \quad (5)$$

and the monopolistic profit in the brown industry is

$$\pi_t^m(i) = p_t(i)x_t(i) - p_{m,t}m_t(i) = \frac{\mu - 1}{\mu} p_t(i)x_t(i) = \frac{\mu - 1}{\mu} y_t. \quad (6)$$

### 2.2.2 Resource extracting firm

A competitive price-taking resource extraction firm supplies raw materials for brown industries. Resource extraction costs are zero. The resource extraction firm maximizes the net present value of extraction profits subject to the resource stock such as  $\dot{S}_t = -m_t$ , where  $m_t = \int_{\Theta} m_t(i)di$ . Now, we can derive the Hotelling rule such as<sup>1</sup>

$$\frac{\dot{p}_{m,t}}{p_{m,t}} = r_t. \quad (7)$$

where  $r_t$  is an interest rate.

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<sup>1</sup>Through the paper, a dot notation denotes differentiation with respect to time.

### 2.2.3 Green industry

If an industry is green  $i \in \Lambda$ , then the production process uses recycled goods, and the production function is

$$x_t(i) = Aq^{n_t(i)}z_t(i), \quad (8)$$

where  $A$  is the exogenous productivity parameter, and  $z_t(i)$  is the amount of recycled goods in industry  $i$ . The marginal cost of the leader in a green industry  $i$  is  $p_{z,t}/Aq^{n_t(i)}$  where  $p_{z,t}$  is the price of recycled goods. As in the brown industry, the leader in green chooses the price under the government regulation such as

$$p_t(i) \leq \mu \frac{p_{z,t}}{Aq^{n_t(i)}} \Rightarrow p_t(i) = \mu \frac{p_{z,t}}{Aq^{n_t(i)}}. \quad (9)$$

Then, the payment for recycled goods in a green industry is

$$p_{z,t}z_t(i) = \frac{1}{\mu}p_t(i)x_t(i) = \frac{1}{\mu}y_t, \quad (10)$$

and the monopolistic profit in the green industry is

$$\pi_t^z(i) = \frac{\mu-1}{\mu}p_t(i)x_t(i) = \frac{\mu-1}{\mu}y_t. \quad (11)$$

### 2.2.4 Recycling firm

A competitive recycling firm reproduces a fraction  $\beta$  of the used final good as recycled goods  $z_t$ . The recycling goods market is subject to perfect competition and free-entry. The production function of



recycled goods is given by

$$z_t = \beta y_t, \quad (12)$$

where  $z_t = \int_{\Lambda} z_t(i) di$ . The recycling firm sets the price of recycled goods  $p_{z,t}$  to equal the marginal cost of reproduction that makes zero profit. Akimoto and Futagami (2018) and Zhou and Smulders (2021) also represent recycling activities in a similar way.<sup>2</sup>

### 2.3 R&D sector

In our model, all industry hires labor  $l_{r,t}(i)$  for innovation to improve the productivity, and if industry  $i$  is the brown, industry  $i$  hires labor  $l_{g,t}(i)$  for greening which means that transformation from brown industry to green industry. (6) and (11) show that the profit is symmetric such as  $\pi_t^m(i) = \pi_t^m$ ,  $\pi_t^z(i) = \pi_t^z$ . Therefore, the value of each industry is also symmetric,  $v_t^m(i) = v_t^m$ ,  $v_t^z(i) = v_t^z$ . The no-arbitrage condition of  $v_t^m$  is

$$r_t v_t^m = \pi_t^m + \dot{v}_t^m - (\lambda_t + \alpha_t) v_t^m, \quad (13)$$

where  $\lambda_t$  is the arrival rate of innovation, and  $\alpha_t$  is the arrival rate of greening. The no-arbitrage condition of  $v_t^z$  also becomes

$$r_t v_t^z = \pi_t^z + \dot{v}_t^z - \lambda_t v_t^z. \quad (14)$$

Given the wage rate of R&D for innovation  $w_{r,t}$ , R&D sector in industry  $i$  hires labor  $l_{r,t}(i)$  for

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<sup>2</sup>Other related literature of recycling considers a flow and stock of waste and derive the dynamics of the price of recycled goods under that constraint; Di Vita (2001), Pittel et al. (2010), Lafforgue and Rouge (2019). Under this setting, the growth rate of a recycled goods price is equal to an interest rate where a recycling cost are zero.

performing innovation, which spills over to all industries. Suppose that the arrival rate of innovation in industry  $i$  is given by

$$\lambda_t(i) = \varphi_t l_{r,t}(i), \quad (15)$$

where  $\varphi_t \equiv \varphi l_{r,t}^{\varepsilon-1}$ ,  $\varphi > 0, \varepsilon \in (0, 1)$ . The aggregate arrival rate at time  $t$  is given by  $\lambda_t = \varphi l_{r,t}^\varepsilon$ , which captures that R&D is decreasing return to scale in aggregate level. In a symmetric equilibrium, the free-entry condition of R&D is

$$v_t^m \lambda_t = w_{r,t} l_{r,t} \Leftrightarrow \varphi v_t^m = w_{r,t} l_{r,t}^{1-\varepsilon}. \quad (16)$$

Given the wage rate  $w_{g,t}$ , the R&D sector in brown industry also hires the labor  $l_{g,t}$  and performs R&D for greening. As well as innovation, the arrival rate of greening in industry  $i \in \Theta$  is given by

$$\alpha_t(i) = \phi_t l_{g,t}(i), \quad (17)$$

where  $\phi_t \equiv \phi(1 - \theta_t) l_{g,t}^{\varepsilon-1}$ , which captures that R&D for greening is decreasing return to scale as well as innovation, and in addition, the larger fraction of brown industries  $(1 - \theta_t)$  makes greening easier to complete. The aggregate arrival rate becomes  $\alpha_t = \phi l_{g,t}^\varepsilon$ , where  $l_{g,t} = (1 - \theta_t) l_{g,t}(i)$ . In a symmetric equilibrium, the free-entry condition becomes

$$\alpha_t v_t^z = w_{g,t} \frac{l_{g,t}}{1 - \theta_t} \Leftrightarrow \phi v_t^z = w_{g,t} \frac{l_{g,t}^{1-\varepsilon}}{1 - \theta_t}. \quad (18)$$

## 2.4 Households

The representative household supplies one unit of labor inelastically. Labor market clearing condition is given by

$$l_{r,t} + l_{g,t} = 1. \quad (19)$$

The household's utility is derived consumption  $c_t$  at time  $t$ , and the instantaneous utility function form is  $\ln c_t$ . The household holds the equity of resource extraction firm and intermediate goods firm, and maximizes the lifetime utility as follows:

$$\int_0^\infty e^{-\rho t} \ln c_t dt, \quad (20)$$

$$\text{s.t. } \dot{a}_t = r_t a_t + p_{m,t} m_t + w_{r,t} l_{r,t} + w_{g,t} l_{g,t} - c_t,$$

where  $a_t$  is a total asset,  $\rho > 0$  is a discount rate. From a dynamic optimization problem for households, the Euler equation is given by

$$\frac{\dot{c}_t}{c_t} = r_t - \rho. \quad (21)$$

## 2.5 Aggregate economy

We define aggregate technology  $Q_t$  as

$$Q_t \equiv \exp \left( \int_0^1 n_t(i) di \ln q \right) = \exp \left( \int_0^t \lambda_s ds \ln q \right). \quad (22)$$

Taking the log of (22) and differentiating it with respect to time gives the growth rate of technology:

$$g_Q = \frac{\dot{Q}_t}{Q_t} = \lambda_t \ln q. \quad (23)$$

In the symmetric equilibrium, each input in intermediate firms is symmetric. Therefore, the aggregate amount of each input is  $m_t = (1 - \theta_t)m_t(i)$ , and  $z_t = \theta_t z_t(i)$ . Substituting (3), (8), and these aggregate amount of each input into (1) yields the aggregate production function:

$$y_t = Q_t \left( \frac{Az_t}{\theta_t} \right)^{\theta_t} \left( \frac{m_t}{1 - \theta_t} \right)^{1 - \theta_t} \quad (24)$$

$$\Leftrightarrow y_t = Q_t^{1/(1 - \theta_t)} \left( \frac{A\beta}{\theta_t} \right)^{\theta_t/(1 - \theta_t)} \frac{m_t}{1 - \theta_t} \quad (25)$$

where we use (12) in the second equation. From (27), and (10), each input share of income is given by

$$\frac{p_{z,t}z_t}{y_t} = \frac{\theta_t}{\mu} \Leftrightarrow \beta p_{z,t} = \frac{\theta_t}{\mu}, \quad (26)$$

$$\frac{p_{m,t}m_t}{y_t} = \frac{1 - \theta_t}{\mu}. \quad (27)$$

## 2.6 Market equilibrium

Each market equilibrium in our model is given by:

- the market-clearing condition for the final good holds such that  $y_t = c_t$ ;
- the market-clearing condition for the labor holds such that  $\int_0^1 l_{r,t}(i)di + \int_{\Theta} l_{g,t}(i)di = 1$ ;
- the market-clearing condition for the raw material holds such that  $\int_{\Theta} m_t(i)di = m_t$ ;

- the market-clearing condition for the recycled goods holds such that  $\int_{\Lambda} z_t(i)di = z_t$ ; and
- the value of R&D is equal to the value of household's asset such that  $\int_{\Lambda} v_t^z(i)di + \int_{\Theta} v_t^m(i)di = a_t$ .

### 3 Balanced growth path

On the balanced growth path, the fraction of industry  $\theta_t$  and the amount of labor  $l_r, l_g$  are constant over time, and the growth rates of consumption, production, profit, and value of intermediate firms are equal  $g = g_c = g_y = g_{\pi} = g_v$ . Substituting (21),  $\lambda = \varphi l_r^{\varepsilon}$ , and  $\alpha = \phi l_g^{\varepsilon}$  into (13) yields

$$v_t^m = \frac{\pi_t^m}{\rho + \varphi l_r^{\varepsilon} + \phi l_g^{\varepsilon}}. \quad (28)$$

As well substituting (21) into (14) yields

$$v_t^z = \frac{\pi_t^z}{\rho + \varphi l_r^{\varepsilon}}. \quad (29)$$

Substituting (28) into the free-entry condition of innovation (16) yields

$$\frac{\varphi \pi_t^m}{\rho + \varphi l_r^{\varepsilon} + \phi l_g^{\varepsilon}} = w_{r,t} l_r^{1-\varepsilon}. \quad (30)$$

Substituting (29) into the free-entry condition of innovation (18) yields

$$\frac{\phi \pi_t^z}{\rho + \varphi l_r^{\varepsilon}} = w_{g,t} \frac{l_g^{1-\varepsilon}}{1-\theta}. \quad (31)$$

Combining (30), (31),  $\pi_t^m = \pi_t^z = \pi_t = \frac{\mu-1}{\mu}y_t$  and  $w_{r,t} = w_{g,t}$  yields

$$\frac{\varphi}{\phi} \frac{\rho + \varphi l_r^\varepsilon}{(1-\theta)(\rho + \varphi l_r^\varepsilon + \phi l_g^\varepsilon)} = \frac{l_r^{1-\varepsilon}}{l_g^{1-\varepsilon}} \quad (32)$$

which can be rearranged as follows:

$$\frac{\varphi}{\phi} + \left( \frac{1-l_r}{l_r} \right)^\varepsilon = \left( \frac{l_r}{1-l_r} \right)^{1-\varepsilon} + \left( \frac{l_r}{1-l_r} \right)^{1-2\varepsilon} \frac{\phi}{\varphi + \rho/l_r^\varepsilon}, \quad (33)$$

where  $\theta = \alpha/(\alpha + \lambda) = \frac{\phi l_g^\varepsilon}{\phi l_g^\varepsilon + \varphi l_r^\varepsilon}$  and  $l_g = 1 - l_r$  are used. For  $\varepsilon \leq 1/2$ , RHS of (33) is monotonically increasing with  $l_r$ , which results in existing the equilibrium labor for innovation  $l_r^* \in (0, 1)$ .

Here, we derive the growth rate on the balanced growth path. Taking log of (27) and differentiating with respect to time yields the material decreasing rate:

$$g_m \equiv \frac{\dot{m}_t}{m_t} = - \left( \frac{\dot{p}_{m,t}}{p_{m,t}} - g \right) = -(r - g) = -\rho, \quad (34)$$

where the Hotelling rule (7) and the Euler equation(21) hold. Next, from (23) and (25), the growth rate of the aggregate production is given by

$$g = \frac{1}{1-\theta}g_Q + g_m = \frac{\lambda \ln q}{1-\theta} - \rho = (\phi(1-l_r)^\varepsilon + \varphi l_r^\varepsilon) \ln q - \rho. \quad (35)$$

The welfare of a representative household is given by

$$U_0 = \int_0^\infty e^{-\rho t} \ln c_0 e^{gt} dt = \frac{1}{\rho} \ln c_0 + \frac{g}{\rho^2}. \quad (36)$$

The raw material input at time 0  $m_0$  is given by  $\rho S_0$ . According to (25), we obtain the initial

consumption  $c_0(=y_o)$  as follows:

$$c_0 = y_0 = \frac{\rho S_0 Q_0^{1/(1-\theta)}}{1-\theta} \left( \frac{A\beta}{\theta} \right)^{\frac{\theta}{1-\theta}}. \quad (37)$$

We can see that an increase in the recycling rate  $\beta$  results in improving the household's welfare. The following proposition summarizes the result of our model.

**Proposition 1.** *Along the balanced growth path, the recycling rate does not affect the fraction of industries, the growth rate, and the decreasing rate of raw material, and an increase in the recycling rate has a positive effect on the welfare of a household.*

The reason why the recycling rate does not affect the fraction of industries is as follows. The fraction of industries in equilibrium is determined by the proportion of arrival rates of each R&D, and the only economic variable that depends on this is labor input. Since labor input is not used to produce intermediate goods, it does not affect the rate of raw material consumption over time. In terms of the income share of each intermediate good input, the fraction of the brown industry remains constant because the rate of decline in the consumption of raw materials is offset by the rate of increase in material price and aggregate output. For these reasons, it does not affect the recycling rate, the fraction of industries, or the distribution of labor, and therefore does not affect the growth rate.

However, this does not mean that recycling has no effect on the economy; it can increase production at any time, and an economy with a high recycling rate can improve its welfare value compared to an economy with a low recycling rate.

## 4 Concluding remarks

In this paper, we constructed a circular economy model with heterogeneous industries. There are two types of industries that use different goods in the production of intermediate goods. We consider a green industry that produces making use of recycled goods and a brown industry that uses exhaustible raw materials. The final good is reproduced to the recycled goods by the recycling firm. The fraction of these two industries is endogenously determined by the level of investment in R&D that performs innovation and that causes industry greening.

We show that recycling improves the welfare of households, but it does not affect other economic variables, for example, the growth rate, the raw material depletion rate, and the fraction of green industry. Therefore, we state that recycling does not have a negative effect on the economy.

This study did not consider R&D which would increase the recycling rate, and extensions to the model still need to be considered. For example, a setting where the recycling rate is involved in determining the labor allocation should be considered. Furthermore, policy analyses such as subsidies for R&D investment or taxes on raw material extraction would be useful. We examine these extensions and policies for future research.



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