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TRANSITION TO GREEN INDUSTRY AND RECYCLING IN A HETEROGENEOUS-INDUSTRY AND ENDOGENOUS GROWTH MODEL

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Transition to green industry and recycling in a heterogeneous-industry and endogenous growth model*

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Abstract

This study incorporates two heterogeneous industries into an endogenous growth model within the framework of a circular economy. In the model, industries are classified as either brown or green, and each can transition between states through R&D activities related to innovation and greening. Greening R&D is conducted exclusively by firms in the brown industry and enables the transition to the green industry. We analyze the effects of subsidies for greening R&D and show that such subsidies increases labor allocation to both innovation and greening R&D. As a result, the model yields win-win outcome: economic growth is promoted not only by productivity-driven growth acceleration but also by a decline in the share of brown industries that rely on exhaustible resources, which mitigates the negative impact of resource depletion on growth. These findings suggest that advancing a circular economy can be compatible with sustained economic growth.

Keywords: Endogenous growth · Circular economy · Recycling · Heterogeneous-industry model

JEL Classification Numbers: O31 · O44 · Q53

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1 Introduction

1.1 Introduction

A "circular economy" (CE) is an economic and social system that should be aimed at in the future, and various efforts are being made mainly in Japan, Germany, China, and other countries. The CE is an economic activity that aims to change the economic system of mass production, mass consumption, and mass disposal, and to use scarce resources and energy sustainability;. According to Heshmati (2017), several reasons why the circular economy should be promoted: to address environmental issues; to address the lack of demand for resources and energy associated with rapid economic growth; and to enhance national security by promoting alternative energy resources and making the use of materials more efficient. Circular economy initiatives are not limited to those initiated by the government, but are increasingly being undertaken voluntarily by firms. TOYOTA states that by 2030 it will use at least 30 percent recycled materials in the production of its vehicles (TOYOTA, 2025). Apple inc. procures 24 percent of its materials in its products from recycled or renewable sources (Apple inc., 2025), and this trend is expected to continue in the future.

This study constructs an R&D-based growth model that accounts for industry heterogeneity and examines the long-run economic impact of recycling and a fraction of industry. There are two types of intermediate goods-producing industries in the economy: one produces with inputs of exhaustible resources (brown industry) and the other produces with inputs of recycled goods (green industry). The final good is produced by using a unit continuum of intermediate goods. Recycled goods are made from waste stock consumed by households. Each industry can transition between states as a result of R&D activities related to innovation and greening. Innovation, which enhances productivity, occurs in both industries. In contrast, only firms in brown industries engage in greening R&D, which facilitates the transition from brown to green industries. Regardless of their states, once R&D activities for innovation succeed, productivity improvements make the industry a brown one. We follow Chu et al.(2023) and Maeda et al. (2025) for the model framework where the industries switch their states. In addition, we conduct a policy analysis of a subsidy for greening R&D with numerical examples.

We show that the existence of the unique equilibrium labor allocation on the balanced growth path. In addition, a subsidy for the greening R&D increases the both R&D labor, thereby stimulating economic growth. The growth rate in this model consists of the rate of technological progress and the rate of raw material reduction, which depends on the share of brown industries. An increase in the labor that engages in innovation increases the productivity, and an increase in the labor that engages in greening decreases a share of brown industries. This economic grows due to these two effects by the subsidy. Moreover, the numerical analysis reveals that the promotion of economic growth is largely driven the decline of brown industries, which the slowdown in growth caused by resource depletion. This finding suggests that promoting a circular economy can be compatible with sustaining economic growth.

1.2 Related literature

In line with the growing international momentum for recycling, there has been an increasing number of studies in economics analyzing recycling and the circular economy in recent years. The literature that has constructed the dynamic models of recycling includes Hoel (1978), Di Vita (2001, 2004), Pittel et al. (2010), Akimoto and Futagami (2018), Lafforgue and Rouge (2019), Rosendahl and Rubiano (2019), and Zhou and Smulders (2021). Hoel (1978) derives the optimal path assuming resources and recycled goods are perfect substitutes. Di Vita (2001, 2004) shows many positive effects of waste recycling on the economy. Pittel et al. (2010) compare the social planner economy with the decentralized economy under the material balance and analyze the market failures caused by not considering the waste market. Akimoto and Futagami (2019) and Lafforgue and Rouge (2019) discuss the transition from a linear economy to a circular economy. Akimoto and Futagami (2018), using the Ramsey model, obtain the Environmental Kuznets Curve along the optimum path. Lafforgue and Rouge (2019) assume that, initially, recycled resources are not as productive as non-renewable resources, and that, by investing in R&D, productivity will increase, and recycled resources will also be used in production, thus achieving a circular economy. Rosendahl and Rubiano (2019) focus on the Lithium market and examine to what extent recycling improves resource scarcity. Several papers have reported the positive impacts of recycling on the economy, Zhou and Smulders (2021) state that an increase in recycling rates will result in economic losses if innovations are strongly resource-saving, and argue that we should be cautious once and for all about the introduction of recycling. These studies listed above do not discuss industrial heterogeneity, and in this respect, this study has a contribution to make regarding the connection between recycling and economic growth.

The remainder of the paper is organized as follows: Section 2 describes the setting of the model. Section 3 characterizes the balanced growth path. Section 4 conducts a policy analysis. Section 5 concludes this paper.

2 The model

We introduce a circular economy into an R&D-based growth model. There exist two industries, in the economy, green and brown. Both industries produce intermediate goods, the production process in the brown uses exhaustible raw materials, and in the green industry uses recycled goods which are produced by converting the waste stock through the input of labor. Each industry can transition between states as a result of R&D activities related to innovation and greening. Innovation, which enhances productivity, occurs in both industries. In contrast, only firms in brown industry engage in greening R&D, which facilitates the transition to from brown to green industry. The representative household maximizes the lifetime utility and supplies labor inelastically. The following section derives the growth rate on the balanced growth path.

2.1 Final good

Final goods y_t are produced by competitive firms, which use a unit of continuum of differentiated intermediate goods:

$$y_t = \exp\left(\int_0^1 \ln x_t(i) di\right). \tag{1}$$

$x_t(i)$ denotes intermediate good $i \in [0, 1]$, and (1) gives the conditional demand function for $x_t(i)$ is

$$x_t(i) = \frac{y_t}{p_t(i)},$$

where $p_t(i)$ is the price of $x_t(i)$.

2.2 Intermediate goods

There is a unit continuum of industries, which is indexed by $i \in [0, 1]$ producing intermediate goods. There exist two types of industry, green and brown, We define the set of green industries as Λ, and brown as Θ. A fraction of green (brown) industries is denoted by θ_t $(1 - \theta_t)$.

2.2.1 Brown industry

If industry $i \in \Theta$ is classified as a brown industry, its production process uses exhaustible raw materials such as metals:

$$x_t(i) = q^{n_t(i)} m_t(i), \tag{2}$$

where $q > 1$ is the parameter of the step size of productivity improvement, $n_t(i)$ is the number of quality improvements in industry i at time t, $m_t(i)$ is the amount of raw material used in industry i. The marginal cost of the leader in a brown industry i is $p_{m,t}/q^{n_t(i)}$ where $p_{m,t}$ is the material price. The government regulates the monopolistic price, which cannot be greater than $\mu > 1$. Then, the leader chooses the price for maximizing profit such as

$$p_t(i) \leq \mu \frac{p_{m,t}}{q^{n_t(i)}} \Rightarrow p_t(i) = \mu \frac{p_{m,t}}{q^{n_t(i)}}.$$

In this case, the payment for a material in brown industry i is

$$p_{m,t} m_t(i) = \frac{p_t(i) x_t(i)}{\mu} = \frac{y_t}{\mu}, \tag{3}$$

and the monopolistic profit in the brown industry is

$$\pi_t^m(i) = p_t(i)x_t(i) - p_{m,t}m_t(i) = \frac{\mu-1}{\mu}p_t(i)x_t(i) = \frac{\mu-1}{\mu}y_t. \tag{4}$$

2.2.2 Resource extracting firm

A competitive price-taking resource extraction firm supplies raw materials for brown industries. Resource extraction costs are zero. The resource extraction firm maximizes the net present value of extraction profits subject to the resource stock such as $\dot{S}_t = -m_t$, where $m_t = \int_\Theta m_t(i)di$. Now, we can derive the Hotelling rule such as[1]

$$\frac{\dot{p}_{m,t}}{p_{m,t}} = r_t. \tag{5}$$

where r_t is an interest rate. As raw material use increases, leading to higher marginal costs for intermediate goods production in the brown sector. In response, firms invest in R&D either to innovate and improve productivity or shift toward the green industry.

2.2.3 Green industry

If industry $i \in \Lambda$ is classified as a green industry, its production process uses recycled goods $z_t(i)$. These recycled goods are produced by converting the waste stock through the input of labor. Then, the production function in the green industries is as follows:

$$\begin{aligned} x_t(i) &= z_t(i), \\ z_t(i) &= \eta q^{n_t(i)} l_{z,t}(i) \end{aligned} \tag{6}$$

where η is the exogenous productivity parameter, and $l_{z,t}(i)$ is the amount of labor in the green industry i. The marginal cost of the leader is $w_t/\eta q^{n_t(i)}$ where w_t is the wage rate. As in the brown industry, the

[1] Through the paper, a dot notation denotes differentiation with respect to time.

leader in green chooses the price under the government regulation such as

$$p_t(i) \leq \mu \frac{w_t}{\eta q^{n_t(i)}} \Rightarrow p_t(i) = \mu \frac{w_t}{\eta q^{n_t(i)}}.$$

Then, the payment for recycled goods in a green industry is

$$w_t z_t(i) = \frac{1}{\mu} p_t(i) x_t(i) = \frac{1}{\mu} y_t, \tag{7}$$

and the monopolistic profit in the green industry is

$$\pi_t^z(i) = \frac{\mu - 1}{\mu} p_t(i) x_t(i) = \frac{\mu - 1}{\mu} y_t. \tag{8}$$

The waste stock is assumed to accumulate as a fraction ($\beta \in (0,1)$) of consumed final goods and to decrease by the amount regenerated by the green industry. Additionally, an initial waste stock W_0 is present at $t = 0$. The dynamics of the waste stock is given by:

$$\dot{W}_t = -\int_{\Lambda} z_t(i) di + \beta y_t. \tag{9}$$

2.3 R&D sector

In our model, all industries hire labor $l_{r,t}(i)$ for innovation to improve the productivity, and if industry i is the brown, industry i hires labor $l_{g,t}(i)$ for greening which means that transformation from brown industry to green industry. To ensure that each R&D sector has incentives to engage in R&D activities, we assume a certain range of parameters.

For R&D sectors in brown industries to engage in greening, we assume that greening reduces the marginal cost of producing intermediate goods. This condition is given by:

$$\frac{\mu p_{m,t}}{q^{n_t(i)}} > \frac{\mu w_t}{\eta q^{n_t(i)}}.$$

Similarly, for R&D sectors in green industries to engage in innovation, we assume that the step size of the productivity improvement must be large enough so that the marginal cost of intermediate goods production using raw materials is lower. This condition is given by:

$$\frac{\mu w_t}{\eta q^{n_t(i)}} > \frac{\mu p_{m,t}}{q^{n_t(i)+1}}.$$

From these conditions, we obtain the following parameter range:

$$1 < \frac{\eta p_{m,t}}{w_t} < q. \tag{10}$$

The second term in the above equation reflects that, along the balanced growth path, the growth rate of the material price exceeds that of the wage. Therefore, this study restricts the analysis to the range of parameters in which this condition holds.

(4) and (8) show that the profit is symmetric such as $\pi_t^m(i) = \pi_t^m$, $\pi_t^z(i) = \pi_t^z$, and $\pi_t^m = \pi_t^z = \pi_t = \frac{\mu-1}{\mu} y_t$. Therefore, the value of each industry is also symmetric, $v_t^m(i) = v_t^m$, $v_t^z(i) = v_t^z$. The no-arbitrage condition of v_t^m becomes:

$$r_t v_t^m = \pi_t^m + \dot{v}_t^m - (\lambda_t + \alpha_t) v_t^m, \tag{11}$$

where λ_t is the arrival rate of innovation, and α_t is the arrival rate of greening. The no-arbitrage condition of v_t^z also becomes:

$$r_t v_t^z = \pi_t^z + \dot{v}_t^z - \lambda_t v_t^z. \tag{12}$$

Given the wage rate of R&D for innovation, R&D sector in industry i hires labor $l_{r,t}(i)$ for performing innovation, which spills over to all industries. Suppose that the arrival rate of innovation in industry i is given by:

$$\lambda_t(i) = \phi_t l_{r,t}(i), \tag{13}$$

where $\phi_t \equiv \phi l_{r,t}^{\varepsilon-1}, \varphi > 0, \varepsilon \in (0,1)$. The aggregate arrival rate at time t is given by $\lambda_t = \varphi l_{r,t}^{\varepsilon}$, which captures that R&D is decreasing return to scale in aggregate level. In a symmetric equilibrium, the free-entry condition of innovation is given by:

$$v_t^m \lambda_t = w_t l_{r,t} \Leftrightarrow \phi v_t^m = w_t l_{r,t}^{1-\varepsilon}. \tag{14}$$

Similarly, the R&D sector in brown industry also hires the labor $l_{g,t}$ and performs R&D for greening. The arrival rate of greening in industry $i \in \Theta$ is given by:

$$\alpha_t(i) = \psi_t l_{g,t}(i), \tag{15}$$

where $\psi_t \equiv \psi(1-\theta_t)$, which captures that the larger fraction of brown industries $(1-\theta_t)$ makes greening easier to complete. The aggregate arrival rate becomes $\alpha_t = \psi l_{g,t}$, where $l_{g,t} = (1-\theta_t) l_{g,t}(i)$. In a symmetric equilibrium, the free-entry condition becomes:

$$\alpha_t v_t^z = w_t \frac{l_{g,t}}{1-\theta_t} \Leftrightarrow \psi v_t^z = \frac{w_t}{1-\theta_t}. \tag{16}$$

Through the above R&D activities, the dynamics of the green industry share (θ_t) is given by:

$$\dot{\theta}_t = \alpha_t(1-\theta_t) - \lambda_t \theta_t. \tag{17}$$

2.4 Households

The representative household supplies one unit of labor inelastically. Labor market clearing condition is given by

$$l_{z,t} + l_{r,t} + l_{g,t} = 1. \tag{18}$$

The household's utility is derived consumption c_t at time t, and the instantaneous utility function form is $\ln c_t$. The household holds the equity of resource extraction firm and intermediate goods firm, and maximizes the lifetime utility as follows:

$$\int_0^\infty e^{-\rho t} \ln c_t dt, \tag{19}$$

s.t. $\dot{a}_t = r_t a_t + p_{m,t} m_t + w_t - c_t$,
where a_t is a total asset, $\rho > 0$ is a discount rate. From a dynamic optimization problem for households, the Euler equation is given by:

$$\frac{\dot{c}_t}{c_t} = r_t - \rho. \tag{20}$$

2.5 Aggregate economy

We define aggregate technology Q_t as

$$Q_t \equiv \exp\left(\int_0^1 n_t(i) di \ln q\right) = \exp\left(\int_0^t \lambda_s ds \ln q\right). \tag{21}$$

Taking the log of (21) and differentiating it with respect to time gives the growth rate of technology:

$$g_Q = \frac{\dot{Q}_t}{Q_t} = \lambda_t \ln q. \tag{22}$$

In the symmetric equilibrium, each input in intermediate firms is symmetric. Therefore, the aggregate amount of each input is $m_t = (1-\theta_t)m_t(i)$, and $z_t = \theta_t z_t(i)$. Substituting (2), (6), and these aggregate amount of each input into (1) yields the aggregate production function:

$$y_t = Q_t \left(\frac{\eta z_t}{\theta_t}\right)^{\theta_t} \left(\frac{m_t}{1-\theta_t}\right)^{1-\theta_t}. \tag{23}$$

In equilibrium, (3) and (7) can be rewritten as the aggregate form as follows:

$$\frac{p_{m,t} m_t}{y_t} = \frac{1-\theta_t}{\mu}, \tag{24}$$

$$\frac{w_t l_{z,t}}{y_t} = \frac{\theta_t}{\mu}. \tag{25}$$

2.6 Market equilibrium

The final goods market equilibrium is given by

$$y_t = c_t.$$

The labor market equilibrium is given by

$$\int_0^1 l_{r,t}(i)di + \int_\Theta l_{g,t}(i)di + \int_\Lambda l_{z,t}(i)di = 1.$$

The raw material market equilibrium is given by

$$\int_\Theta m_t(i)di = m_t.$$

The value if R&D is equal to the value of household's asset such as

$$\int_\Lambda v_t^z(i)di + \int_\Theta v_t^m(i)di = a_t.$$

3 Balanced growth path

In this section, we derive a balanced growth path (BGP) and show its existence. Let us define the BGP as follows.

Definition 1. A balanced growth path is characterized by the following features.

1. All of c_t, y_t, π_t, v_t^m, v_t^z, and w_t grow at a constant rate g, i.e.,

$$\frac{\dot{c}_t}{c_t} = \frac{\dot{y}_t}{y_t} = \frac{\dot{\pi}_t}{\pi_t} = \frac{\dot{v}_t^m}{v_t^m} = \frac{\dot{v}_t^z}{v_t^z} = \frac{\dot{w}_t}{w_t} = g.$$

2. l_z, l_r, l_g, and θ are constant over time.

Substituting the Euler equation (20) into the no-arbitrage condition of v_t^m (11) and rearranging for v_t^m yield

$$v_t^m = \frac{\pi_t^m}{\rho + \lambda + \alpha} = \frac{\mu - 1}{\mu} \frac{y_t}{\rho + \phi l_r^{\varepsilon} + \psi l_g}, \tag{26}$$

where $g = r - \rho$, $\pi_t^m = \frac{\mu-1}{\mu} y_t$, $\lambda = \phi l_r^{\varepsilon}$, and $\alpha = \psi l_g$. Substituting (26) into the free-entry condition of innovation (14) yields

$$\phi \frac{\mu - 1}{\mu} \frac{y_t}{w_t} = l_r^{1-\varepsilon} (\rho + \phi l_r^{\varepsilon} + \psi l_g). \tag{27}$$

Similarly, substituting (20) into (12) yields

$$v_t^z = \frac{\pi_t^z}{\rho + \varphi l_r^{\varepsilon}} = \frac{\mu - 1}{\mu} \frac{y_t}{\rho + \varphi l_r^{\varepsilon}}. \tag{28}$$

Substituting (28) into the free-entry condition of greening (16) yields

$$\psi \frac{\mu - 1}{\mu} \frac{y_t}{w_t} = \frac{\psi l_g + \phi l_r^{\varepsilon}}{\phi l_r^{\varepsilon}} (\rho + \phi l_r^{\varepsilon}). \tag{29}$$

where $\theta = \alpha/(\alpha+\lambda) = \psi l_g/(\psi l_g + \phi l_r^{\epsilon})$. Combining (25), the labor market clearing condition (18) yields

$$\frac{y_t}{w_t} = \frac{\mu(\psi l_g + \phi l_r^{\varepsilon})}{\psi l_g}(1 - l_r - l_g). \tag{30}$$

By substituting (30) into (27) and (29), we obtain the equilibrium conditions on the BGP as follows:

$$(\mu - 1)(1 - l_r - l_g) = \frac{\rho + \phi l_r^{\varepsilon}}{\phi l_r^{\varepsilon}} l_g, \tag{31}$$

$$\phi(\mu - 1)(1 - l_r - l_g)\frac{\psi l_g + \phi l_r^{\varepsilon}}{\psi l_g} = l_r^{1-\varepsilon}(\rho + \phi l_r^{\varepsilon} + \psi l_g). \tag{32}$$

Finally, by rearranging (31) for l_g and substituting (32) into (31), we obtain the following proposition:

Proposition 1. Let us assume that $\rho < \phi < \psi < \phi + \rho$. Then, there uniquely exists a BGP in which the following two equations are satisfied:

$$l_g = (\mu - 1)\frac{\phi l_r^{\varepsilon}(1 - l_r)}{\rho + \mu\phi l_r^{\varepsilon}} \equiv h(l_r) \tag{33}$$

$$l_g = \frac{(\rho + \phi l_r^{\varepsilon})(\psi l_r - \phi)}{\psi(\rho + \phi l_r^{\varepsilon} - \psi l_r)} \equiv H(l_r). \tag{34}$$

Proof. See Appendix A. ∎

The assumption that $\rho < \phi < \psi < \phi + \rho$ implies that the productivity of greening R&D is higher, but the two R&D productivity do not differ substantially, and that they are sufficiently larger than the discount rate. Figure 1 illustrates an equilibrium labor allocation on the BGP. The equilibrium values of l_r^* and l_g^* are determined at the intersection of $h(l_r)$ and $H(l_r)$. The length of the vertical line from l_r^* is equal to $l_g^* + l_z^*$. Since the labor allocation in equilibrium is thus determined, the arrival rates of the two types of R&D are also determined, which in turn simultaneously determines the share of green industries and growth rate.

With the equilibrium labor allocation along the BGP derived, the growth rate on the model can be obtained and is formally states in the following proposition.

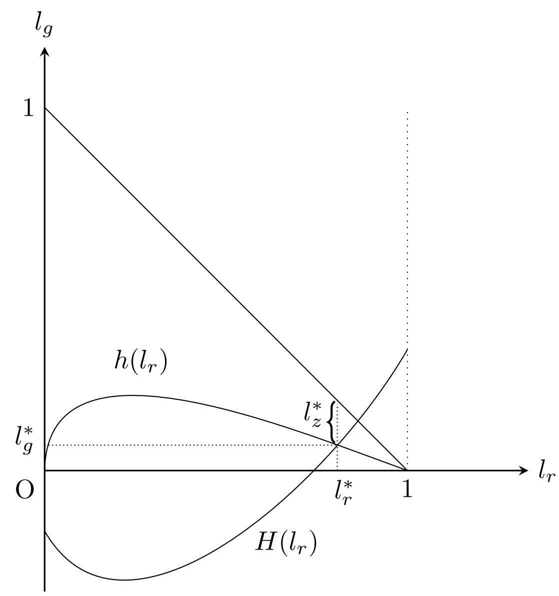


Figure 1: Equilibrium labor allocation on the BGP

Proposition 2. The growth rate on the BGP is obtainedas follows:

$$g = \underbrace{g_Q}_{\text{technological progress}} + \underbrace{(1-\theta)g_m}_{\text{raw material reduction}} = \lambda \ln q - \frac{\rho\lambda}{\alpha+\lambda} = \phi(l_r^*)^{\varepsilon} \ln q - \frac{\rho\phi(l_r^*)^{\varepsilon}}{\psi l_g^* + \phi(l_r^*)^{\varepsilon}}.$$

In this economy, growth is promoted by both technological progress and a reduction in the size of the brown industry.

Proof. Taking time derivative of (23), yields $g = g_Q + (1-\theta)g_m$ where $g_m = \frac{\dot{m}_t}{m_t} = -\rho$ from (24). By substituting $\lambda = \phi l_r^{1-\varepsilon}$, $\alpha = \psi l_g$, and $\theta = \alpha/(\alpha+\lambda)$ into $g = g_Q + (1-\theta)g_m$, we can obtain the above equation. ■

Note that the speed of raw material reduction depends solely on the discount rate. This result is due to the logarithmic form of utility function. The growth rate on the BGP consists of the rate of technological progress and the product of the rate of raw material reduction and the share of brown industries. Even in this model, we can confirm the effect noted in growth models that incorporate the constraint of exhaustible resources—such as discussed in Jones and Vollrath (2013), Chapter 10—where the growth rate declines in accordance with the depletion rate of resources. However, under our model setting, this effect can be mitigated by an increase in green industries (i.e., a decrease in brown industry), suggesting a positive relationship between the promotion of a circular economy and economic growth.

4 Policy analysis

This section investigates the effect of greening subsidies on the BGP as a part of the policy analysis.

4.1 Effect of subsidies for greening

Rearranging the free-entry condition of greening (16) as follows:

$$\alpha_t v_t^z = (1-s)w_t \frac{l_{g,t}}{1-\theta_t} \Leftrightarrow \psi v_t^z = (1-s)\frac{w_t}{1-\theta_t}, \tag{35}$$

where $s < 1$ is the greening R&D subsidy. The government's budget constraint comprises: the lump-sum tax from the household $\mathcal{T}_t$, the subsidy to the greening R&D $sw_t l_{g,t}$. Therefore, the balanced-budget condition is:

$$sw_t l_{g,t} = \mathcal{T}_t. \tag{36}$$

As Section 3, we derive the BGP considering the subsidy. (33) is rewritten by:

$$l_g^{\tau} = (\mu-1)\frac{\phi l_r^{\varepsilon}(1-l_r)}{(1-s)\rho + (\mu-s)\phi l_r^{\varepsilon}}. \tag{37}$$

The results of the subsidy policy analysis are summarized in the following proposition:

Proposition 3. An increase in the greening R&D subsidy ($s \uparrow$) increases both R&D labor ($l_r^*, l_g^* \uparrow$) and decreases the production labor ($l_z^* \downarrow$).

Proof. It can be verified that an increase in the subsidy s shifts only equation (37) upward, implying that both l_g and l_r increase. ■

This proposition shows that an increase in the greening subsidy leads to an increase in labor allocation to both R&D sectors. As a result of the subsidy increase, the cost of greening R&D decreases, which leads to a greater allocation of labor to the greening sector. In addition, industries that have transitioned to the green have incentives for innovations that lower the marginal cost of production, which increases the demand for innovation and, consequently, the need for more labor. Therefore, subsidies for greening raise the demand for boty types of R&D. Figure 2 illustrates how the equilibrium labor allocation changes in response to the greening R&D subsidy. The impacts of the greening subsidy on some economic states remain ambiguous. Therefore, a numerical analysis is conducted to further investigate these effects in the next subsection.

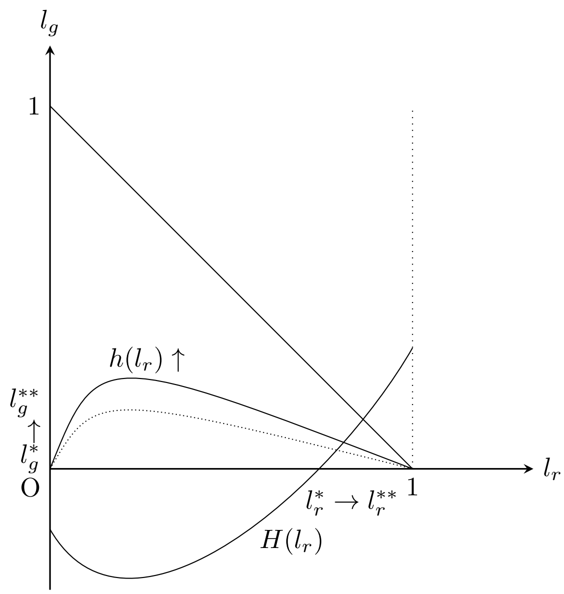


Figure 2: Equilibrium labor allocation on the BGP with the subsidy

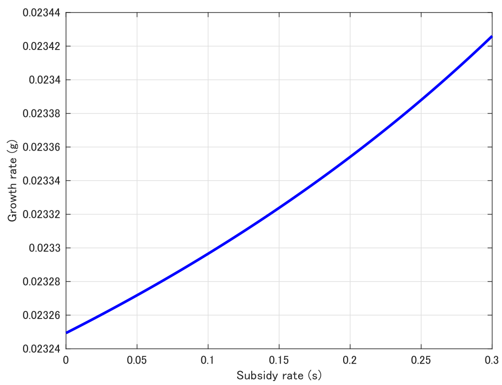
4.2 Numerical analysis

In this subsection, we examine, through numerical analysis, the effects on economic growth rate and fraction of green industries on the BGP. The parameters ρ, μ, and ε follow the settings adopted in Chu et al. (2023). The discount rate ρ is set to 0.05. The markup rate μ is assumed to to be 1.10, following Laitner and Stolyarov (2004). The parameter ε, which represents the diminishing returns to innovation, is set to 0.5 as in Jones and Williams (2000). The step size of innovation q is set to 1.2, following Impullitti (2010). The target value for the growth rate follows Maeda et al. (2025) and is set to the average US growth rate over the period 2010-2023, $g = 0.023$. The productivity parameters of each R&D sector, ϕ and ψ, are calibrated using the US growth rate so that the equilibrium value of labor allocation exists in this model. The level of greening subsidy s is varied from from 0 to 0.3. As for the exogenous production parameter η, we set its value to 0.6 to satisfy (10). These parameter values are shown in Table 1.

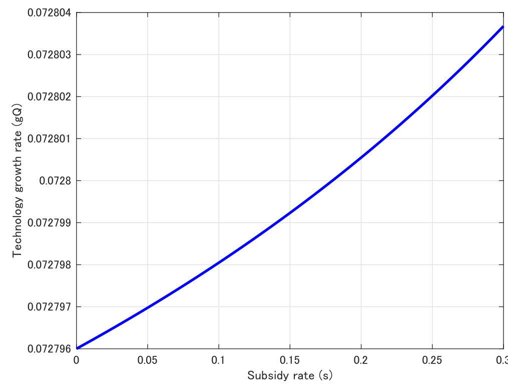
ρ	μ	q	ε	ϕ	ψ	η
0.05	1.10	1.20	0.50	0.42	0.465	0.6

Table 1: Parameter values in the numerical analysis

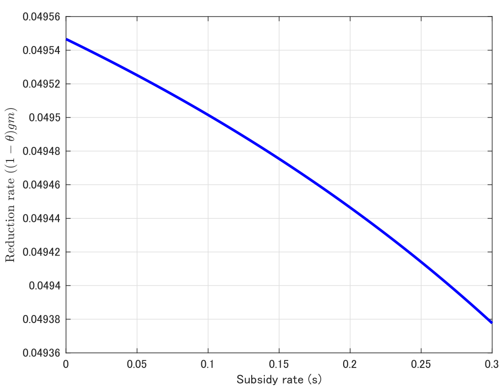
Figure 3 illustrates effects of the greening subsidy rate on the steady-state values values when the level of subsidy is varied from 0 to 0.3. Figure 3a shows the aggregate growth rate increases by 0.019%. Figure 3b indicates a 0.0024% rise in the rate of technological progress, while Figure 3c shows a 0.0169% decline in the growth rate of raw material use. These results suggest the promotion of economic growth through the subsidy policy in this model can be attributed to the expansion of green industry. Figure 3d shows the fraction of green industry increases by 0.338%. In the numerical results based on the model of this study, the amount of labor engaged in innovation at equilibrium is sufficiently close to one, and thus the effect of policy on the technology growth rate is limited. This result is likely attributable to the diminishing marginal returns in innovation.



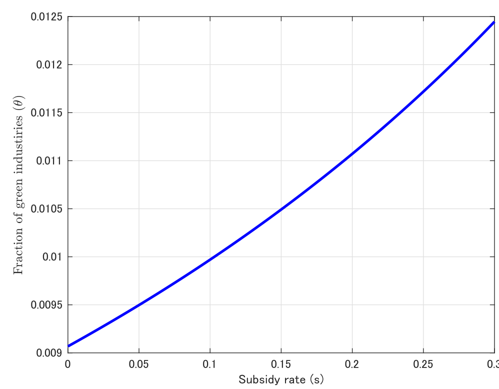
(a) Effect of s on g



(b) Effect of s on g_Q



(c) Effect of s on $(1-\theta)g_m$



(d) Effect of s on θ

Figure 3: Effects of the greening subsidy rate on the steady-state values: growth rate, technology growth rat e, reduction rate, fraction of green industries

5 Concluding remarks

In this paper, we constructed a circular economy model considering industry transition. There are two types of industries that use different goods in the production of intermediate goods. We consider a green industry that produces using of recycled goods and brown industry that relies on exhaustible raw materials. A fraction of consumed final good accumulates as waste stock, which is reduced through reproduction by the green industry. The relative shares of these two industries are endogenously determined by the level of investment in R&D, which drives innovation and promotes industry greening. In addition, we conduct a policy analysis of a subsidy for greening R&D.

We show that the existence of the unique equilibrium labor allocation on the BGP. Moreover, the subsidy increases the both R&D labor. As a result, economic growth is promoted not only by productivity-driven growth acceleration but also by a decline in the share of brown industries that rely on exhaustible resources, which mitigates the negative impact of resource depletion on growth. The numerical analysis reveals that the promotion of economic growth is largely driven the decline of brown industries. These findings suggest that promoting a circular economy can be compatible with sustaining economic growth.

Although this study does not incorporate a framework in which recycled goods are produced through the direct conversion of a portion of completed final or intermediate goods, we consider this a promising extension that could provide a more realistic representation of the circular economy. Moreover, the analysis in this study is limited to the case in which the price of raw materials has not yet fully increased, and both raw materials and recycled goods are used in production. As a possible extension of this study, one could analyze a long-run transition from a linear economy to a circular economy—starting with the exclusive use of raw materials in the early stage of the economy, followed by the combined use of both inputs, and eventually shifting to production using only recycled goods. We leave these for a future research.

Appendix

A Proof of Proposition 1

Let us assume that $\rho < \phi < \psi < \phi + \rho$. First, differentiating $h(l_r)$ gives

$$h'(l_r) = \frac{(\mu-1)\phi}{l_r^{1-\varepsilon}(\rho+\mu\phi l_r^{\varepsilon})^2}[\rho\varepsilon - \rho(1+\varepsilon)l_r - \mu\phi l_r^{1+\varepsilon}],$$
$$h''(l_r) = \frac{(\mu-1)\phi}{l_r^{2(1-\varepsilon)}(\rho+\mu\phi l_r^{\varepsilon})^4}[-\rho\mu\phi\varepsilon(1-\varepsilon)l_r - \varepsilon(1+\varepsilon)\rho^2 l_r^{1-\varepsilon} - \rho^2\varepsilon(1-\varepsilon)l_r^{-\varepsilon} - 2\rho\mu\phi\varepsilon^2 - \rho\mu\phi\varepsilon(1-\varepsilon)] < 0.$$

We can verify that $h(0) = h(1) = 0$, $h(l_r) \geq 0$, $\lim_{l_r \to 0} h'(l_r) = \infty$, and $h'(1) < 0$. Therefore, $h(l_r)$ exhibits an inverted U-shaped for all $l_r \in [0, 1]$. Next, differentiating $H(l_r)$ gives

$$H'(l_r) = \frac{1}{\psi^2(\rho+\phi l_r^{\varepsilon} - \psi l_r)^2}[\{\phi\varepsilon l_r^{1-\varepsilon}(\psi l_r - \phi) + \psi(\rho+\phi l_r^{\varepsilon})\}(\rho+\phi l_r^{\varepsilon} - \psi l_r) - (\rho+\phi l_r^{\varepsilon})(\psi l_r - \rho)(\phi\varepsilon l_r^{\varepsilon-1} - \psi)]. \tag{A1}$$

Similarly, we can verify that $H(0) = -\phi/\psi < 0$ and $H(1) = \dfrac{(\rho+\phi)(\psi-\phi)}{\psi(\rho+\phi-\psi)} > 0$. Additively, we show that $H(l_r)$ is increasing monotonically when $H(l_r) \geq 0$ holds.

$$\begin{aligned} \rho + \phi l_r^{\varepsilon} - \psi l_r &\geq \rho + \phi l_r - \psi l_r \\ &= \rho + \phi - \phi(1-l_r) - \psi l_r \\ &> \psi - \phi(1-l_r) - \psi l_r \\ &= (\psi - \phi)(1-l_r) \geq 0 \end{aligned}$$

We can see that the denominator of $H(l_r)$ takes a positive value for $l_r \in [0, 1]$. Therefore, $H(l_r) \geq 0$ when $\phi/\psi \leq l_r \leq 1$ holds. For the second term in the brackets in (A1),

$$\begin{aligned} \phi\varepsilon l_r^{\varepsilon-1} - \psi &= (\phi\varepsilon - \psi l_r^{1-\varepsilon})/l_r^{1-\varepsilon} \\ &< (\phi - \psi l_r^{1-\varepsilon})/l_r^{1-\varepsilon} \\ &\leq (\phi - \psi l_r)/l_r^{1-\varepsilon} < 0. \end{aligned}$$

Therefore, $H'(l_r) > 0$ for $\phi/\psi \leq l_r \leq 1$, i.e., $H(l_r)$ is increasing when $H(l_r) \geq 0$.Thus, we can show that $h(l_r)$ and $H(l_r)$ intersect at only one point.

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