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ENGAGE IN PRECAUTIONARY SAVING?
TWO OBSERVATIONS ON RISK
AND PRECAUTIONARY SAVING**

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Do Non-Prudent Consumers Ever Engage in Precautionary Saving? Two Observations on Risk and Precautionary Saving

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Abstract

Previous authors have asserted that precautionary saving will arise only if consumers are not only risk-averse but also prudent, but in this paper, we first show that when saving occurs in the context of a financial economy featuring at least one other asset, prudence is neither necessary, nor sufficient, to generate precautionary saving, i.e. saving induced only by variance in income. Then, simplifying and elaborating on some results presented in Eeckhoudt and Schlesinger (2008), we address a particular form of precautionary saving, which we name “intertemporal precautionary saving” to distinguish it from purely intertemporal and purely precautionary saving, and show that it will inevitably arise in the case of pure (downside) risk as long as consumers are risk-averse, regardless of whether or not they are prudent, and that prudence will affect only its extent. Thus, our paper challenges the conventional wisdom on precautionary saving in at least two ways.

Journal of Economic Literature classification codes: D11, D14, D15, D81, E21, G51

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1. Introduction

Precautionary saving arising from income risk and various other uncertainties is an important component of household saving. Countless empirical papers have been written about the importance of precautionary saving arising from future income risk, and as the excellent survey by Lugilde, et al. (2019, p. 507) shows, most of these studies find “robust and convincing results as regards the existence of a precautionary motive for saving” although “there is not a consensus on the magnitude of this effect.” For example, Skinner (1988) finds, using data from the Panel Study of Income and Dynamics, that 56% of total household saving in the United States is attributable to precautionary saving arising from income risk, while Dardanoni (1991) finds, using data from the U.K. Family Expenditure Survey, that the comparable figure for the United Kingdom is more than 60% (see Zhou, 2003, for a similar study for Japan). Similarly, Carroll and Samwick (1998) find, using data from the Panel Study of Income Dynamics, that between 32-50% of wealth in their sample of American households is attributable to the extra uncertainty that some consumers face compared to the lowest uncertainty group. A different approach is taken by Horioka and Watanabe (1997), Horioka, et al. (2000), and Horioka and Ventura (2025), who use data from household surveys to analyze the relative importance of various motives for household saving. They find that a substantial proportion of households are saving for precautionary purposes in Japan, Europe, and the United States and furthermore that the quantitative importance of such saving is often quite substantial. Moreover, saving for income risk is presumably an important component of precautionary saving, which also includes saving for illness, accidents, longevity risk, and other unforeseen contingencies.

Thus, it is important to have a profound understanding of the theory of precautionary saving in general and of the impact of income risk on household saving in particular. There have been many theoretical analyses of precautionary saving starting with the seminal paper by Leland (Leland, 1968) more than half a century ago (see Jappelli and Pistaferri, 2017, Chapter 6, and Baiardi, et al., 2020, for surveys of this literature). Leland (1968) was the first to identify the conditions required for income variance to stimulate saving. In particular, he showed that risk aversion alone is not enough for saving to respond to increasing fluctuations of income around an expected future value and that an additional condition is required beyond risk aversion—namely, the convexity of marginal utility or $U'''(.) > 0$ (a positive third derivative of the utility function or “positive prudence,” to use Kimball’s (1990) terminology) (see Sandmo, 1970, for a closely related analysis).

Kimball (1990) extended Leland’s (1968) analysis by defining the coefficient of absolute prudence as minus the third derivative of the utility function divided by its second derivative and conducting an analysis akin to the one developed for risk aversion by Arrow and Pratt, who defined the coefficient of absolute risk aversion as minus the second derivative of the utility function divided by its first derivative (see, for example, Pratt, 1964). In particular, just as Arrow and Pratt showed that the coefficient of absolute risk aversion is a good measure to gauge the extent of risk aversion, Kimball (1990) showed that the coefficient of absolute prudence is a good measure to quantify the intensity of precautionary behavior (or, to use Kimball’s own words, “the propensity to prepare and forearm oneself in the face of uncertainty about future income”). He did so by basically transposing Arrow and Pratt’s analysis to the new context (by replacing total utility with marginal utility) and defining new concepts such as the coefficients of absolute and relative prudence (corresponding to the coefficients of absolute

and relative risk aversion), the “precautionary premium” (corresponding to the “risk premium”), etc.

By contrast, Yaari (1987) proved that, within the anticipated utility framework formulated by Quiggin (1982), a risk-averse consumer will engage in precautionary saving regardless of the sign of the third derivative of its utility function (i.e., regardless of whether or not it is prudent). Moreover, Segal et al. (1988) extended a modification of this result to an n -period model.

In this paper, we first conduct a theoretical analysis of precautionary saving and show that, even in the more standard context of expected utility, prudence is neither necessary, nor sufficient, to generate precautionary saving, when saving occurs in the context of a financial economy, where assets are also traded.

In the second part of the paper, we focus on a “pure risk” concept of income whereby there is only downside risk (i.e., there is only the possibility of a loss of income) (see, for example, Willett, 1901, and Knight, 1921). Note that this type of income risk is quite common in actual practice. For example, in the case of the risk of unemployment, there is the possibility of a loss of income but not the possibility of an increase in income. The same is true of health problems, accidents involving property and vehicles, etc.

The contribution of this paper is twofold. First, we show that, in the context of a financial economy, where individuals can save but can also trade assets, prudence is neither necessary, nor sufficient, to generate precautionary saving. Then, elaborating on the results presented by Eeckhoudt and Schlesinger (2008) for the case of first-order stochastic dominance, and in the very simple framework of a two-state the world economy, we show that a particular form of precautionary saving, which we will call “intertemporal precautionary saving” to distinguish it from purely intertemporal and purely precautionary saving, will inevitably arise in the case of pure (downside) risk as long as consumers are risk-averse, even if they are not prudent. However, we also show that prudence *will* affect the intensive margin (the *amount* of intertemporal precautionary saving that consumers do) even though it will not affect the extensive margin (whether or not consumers engage in intertemporal precautionary saving). Thus, our paper challenges the conventional wisdom on precautionary saving in at least two ways.

2. Precautionary Saving in a Simple Arrow-Debreu Economy without Prudence

The goal of this section is to show, by presenting two simple examples, that if saving is considered, as it should, as part of the financial structure of an economy, (pure) precautionary saving may arise even if consumers are not prudent, and that prudence is not sufficient to generate precautionary saving. Therefore, we turn to the following proposition:

Proposition 1. In an economy with assets, positive prudence is neither a necessary nor a sufficient condition for precautionary saving, i.e. for saving to increase as risk increases.

The proof of this proposition is given by the following two examples.

Example 1 (prudence is not necessary to generate precautionary saving)

Let us consider an exchange economy with two agents, in which one agent has linear preferences, so that she is risk-neutral, while the preferences of the other agent are concave and quadratic. Hence, this agent is risk-averse and displays no prudence (the third derivative of her utility function is equal to zero). The economy extends over two periods, 0 and 1, and risk is represented by two exhaustive and mutually exclusive states of the world, 1 and 2, in the second period. Let us denote by x_0, x_1, x_2 consumption at date 0 and in states 1 and 2 at date 1, respectively.

The economy features one consumption good in period 0, and one consumption good in each of two states of the world in period 1. Agents can transfer consumption across date-event pairs by saving and by using one asset in “zero net supply” (i.e. one for which the sum of portfolio holdings across agents is zero) that pay off in period 1. For simplicity, and without loss of generality, this is an Arrow-Debreu asset, paying one unit of consumption in state 1 only. For simplicity, the time discount factor is set equal to unity, and the probabilities of the two states are equal. The price of the asset is q .

The preferences of the risk-averse agent are represented by the following intertemporal expected utility function:

$$U(x_0, x_1, x_2) = 5x_0 - \frac{1}{2}x_0^2 + \frac{1}{2}\left(5x_1 - \frac{1}{2}x_1^2\right) + \frac{1}{2}\left(5x_2 - \frac{1}{2}x_2^2\right).$$

This agent has an initial endowment of 4 in the first period, and state contingent endowments of $4 + \epsilon$ and $4 - \epsilon$, respectively, in the two states in period 1. This agent's income is therefore affected by a speculative, symmetric risk with zero mean and variance equal to ϵ^2 .

The maximization problem of the risk-averse agent then becomes:

$$\max_{x_0, x_1, x_2} 5x_0 - \frac{1}{2}x_0^2 + \frac{1}{2}\left(5x_1 - \frac{1}{2}x_1^2\right) + \frac{1}{2}\left(5x_2 - \frac{1}{2}x_2^2\right)$$

such that:

$$x_0 = 4 - q_1y_1 - s$$

$$x_1 = 4 + \epsilon + y_1 + s$$

$$x_2 = 4 - \epsilon + s.$$

By substituting these constraints into the utility function, we obtain the equivalent problem:

$$\max_{y_1, y_2} 5(4 - q_1y_1 - s) - \frac{1}{2}(4 - q_1y_1 - s)^2 + \frac{1}{2}\left(5(4 + \epsilon + y_1 + s) - \frac{1}{2}(4 + \epsilon + y_1 + s)^2\right) + \frac{1}{2}\left(5(4 - \epsilon + s) - \frac{1}{2}(4 - \epsilon + s)^2\right)$$

with first-order conditions with respect to asset holdings (y_1, y_2) :

$$-5q_1 + (4 - q_1y_1 - s)q_1 + \frac{1}{2}(5 - 4 - \epsilon - y_1 - s) = 0$$

$$-5q_2 + (4 - q_1y_1 - s) + \frac{1}{2}(5 - 4 - \epsilon - y_1 - s) + \frac{1}{2}(5 - 4 + \epsilon - s) = 0.$$

By solving the problem of the risk-neutral agent, we can immediately show that the equilibrium price of the risky asset is $q_2 = 1/2$. Thus, the presence of the risk-neutral agent allows a simple closure of the model.

By substituting equilibrium asset prices into the first-order conditions and rearranging, we obtain:

$$-\frac{3}{4}y_1 - s - \frac{\epsilon}{2} = 0$$

$$s = -\frac{1}{2}y_1.$$

Solving this system of equations yields:

$$y_1 = -2\epsilon, s = \epsilon.$$

We immediately notice that saving is a linear function of ϵ . This is bona fide precautionary saving, as it is generated by speculative, zero-mean risk in the absence of prudence (zero third derivative).

We immediately notice that the demand for saving, i.e. for the riskless asset, is a linear function of ϵ .

Saving is therefore a function of the (symmetric) shock, and it can be fully characterized as purely precautionary, as the income shock has zero mean.

Lastly, by substituting saving and asset holding in the budget constraint, we obtain a final allocation for this agent equal to (4,4) in the second period--i.e., the risk-neutral agent achieves perfect insurance.

Three clarifying remarks are in order concerning the result shown in this example.

Remark 1. The presence of an Arrow-Debreu asset, instead of a generic risky asset, is immaterial, in the sense that similar results may be obtained by using a more general asset with state contingent returns.

Remark 2. The fact that the saving is implemented by a simple consumption storage technology (self-saving) as opposed to resorting to credit markets is also immaterial. In the Appendix, we present the solution to the problem where saving is implemented by investing in a riskless asset, and the solution is identical to the one obtained above.

Remark 3. Assets are in zero net supply, i.e., they constitute only a means to shift income and consumption across dates and states; therefore, they do not introduce additional risks. The only risk present in our economy is therefore the endowment risk $(+\epsilon, -\epsilon)$.

Example 2 (prudence is not sufficient to generate precautionary saving)

Let us consider an economy with two periods, 0 and 1, and two equally probable states of the world in the second period (denoted for simplicity 1 and 2, as in the previous example).

There are two individuals, with preferences characterized by a linear instantaneous utility function (risk-neutral individual) and a logarithmic instantaneous utility function (risk-averse

and prudent individual). Initial endowments for the risk-averse individual are (10, 4, 4) in period 0 and in the two states at period 1, respectively. Income at period 1 is also affected by a zero mean shock, taking the form $(-\epsilon, +\epsilon)$, with a positive shock ϵ .

Individuals can save an amount s at period 0, and can also buy an Arrow security paying one unit of consumption in state 1 and zero otherwise. The price of this asset is q , and asset holding by the risk-averse individual is y .

The presence of a risk-neutral individual insures that the price of this Arrow security will be equal to the probability of state 1, i.e., $q = 0.5$.

The maximization problem for the risk-averse individual is:

$$\max_{x_0, x_1, x_2} \ln(x_0) + \frac{1}{2} \ln(x_1) + \frac{1}{2} \ln(x_2)$$

such that:

$$x_0 = 10 - s - q_1 y$$

$$x_1 = 4 - \epsilon + s + y$$

$$x_2 = 4 + \epsilon + s.$$

Substituting the constraint in the objective function, we obtain the equivalent problem:

$$\max_{s, y} \ln(10 - s - q_1 y) + \frac{1}{2} \ln(4 - \epsilon + s + y) + \frac{1}{2} \ln(4 + \epsilon + s)$$

with first-order conditions:

$$-\frac{1}{10-s-qy} + \frac{1}{2} \frac{1}{4-\epsilon+s+y} + \frac{1}{2} \frac{1}{4+\epsilon+s+y} = 0$$

$$-\frac{q}{10-s-qy} + \frac{1}{2} \frac{1}{4-\epsilon+s+y} = 0.$$

Substituting $q = \frac{1}{2}$ and solving the resulting system of first-order conditions yields:

$$s = 3 - \epsilon, y = 2\epsilon.$$

Therefore, saving negatively depends on the size of the shock.

Here, it is quite clear that the need to insure against income shocks overcomes the desire to shift consumption to time 1 because of randomness of period 1's endowments, and negative saving is used to hedge against income risk, when one Arrow security is available.

Remarkably, simple calculations show that if we eliminate the Arrow security from our economy, leaving saving as the only instrument to transfer consumption, the standard Leland-Kimball results apply, as the solution of the model, in terms of saving, is:

$$s = \frac{-1 + \sqrt{49 + 2\epsilon^2}}{2}, \text{ which is clearly increasing in risk.}$$

The interpretation of this proposition is that the standard Leland-Kimball result, i.e. that saving is increasing in risk when prudence is positive, applies only to simple economies with saving.

Once we introduce assets in the economy, saving becomes just an additional instrument to obtain the desired allocation of consumption across dates and states of nature, and saving no longer responds monotonically to (speculative) risk. As the relationship between saving and risk is not straightforward in a financial economy, we should not be surprised to see that empirical results about precautionary saving reach such widely diverging conclusions.

3. A Model of Intertemporal Precautionary Saving

In this section, we simplify and elaborate on some results presented in Eeckhoudt and Schlesinger (2008) to address a particular form of precautionary saving, which we name “intertemporal precautionary saving” to distinguish it from purely intertemporal and purely precautionary saving, and show that it will inevitably arise in the case of pure (downside) risk as long as consumers are risk-averse, regardless of whether or not they are prudent.

Eeckhoudt and Schlesinger (2008) provide conditions under which a stochastic increase in n th-degree risk generates more saving using a mathematical result about n th-order stochastic dominance and illustrate some implications of that result for first-order and second-order increases in risk. In what follows, we simplify their analysis in the case of increases in first-order risk by using a stylized model with two states of the world and provide an additional result showing that prudence will affect the amount of intertemporal precautionary saving even though it will not affect whether or not consumers engage in intertemporal precautionary saving.

Consider an intertemporal economy lasting for two periods, 0 and 1, with consumption in period 0 and consumption in period 1 in $c = 1, \dots, C$ possible contingencies (states of the world). In period 0, consumers can save an amount s out of their endowment. For simplicity, initial endowment is w in period 0 and in period 1 for all contingencies. For simplicity, both the time preference factor and the interest factor ($1 + \text{interest rate}$) are set equal to one so that there is no incentive to save for intertemporal motives.

Without loss of generality, pure risk is represented by a loss ϵ in one state of the world, c' , with probability $\pi_{c'}$. The instantaneous utility function is $u(x)$, continuously twice differentiable and strictly concave, so that consumers are risk-averse.

Proposition 2. In the presence of pure (downward) risk, a risk-averse consumer will engage in intertemporal precautionary saving even if she is not prudent.

Proof. The intertemporal maximization problem of a consumer is:

$$\max_s u(w - s) + \sum_{c \neq c'} \pi_c u(w + s) + \pi_{c'} u(w - \epsilon + s)$$

The first-order condition with respect to s is:

$$F(w, \epsilon, s) = -u'(w - s) + \sum_{c \neq c'} \pi_c u'(w + s) + \pi_{c'} u'(w - \epsilon + s) = 0 \quad (1)$$

(The second-order condition holds because of the concavity of $u(x)$.)

It is clear that (1) is solved by $s = 0$ when $\epsilon = 0$ as (1) becomes

$$F(w, 0, 0) = -u'(w) + \sum_{c \neq c'} \pi_c u'(w) + \pi_{c'} u'(w) = -u'(w) + \sum_c \pi_c u'(w) = -u'(w) + u'(w) = 0.$$

By concavity of $u(x)$, when $\epsilon > 0$ $u'(w - \epsilon)$ is greater than $u'(w)$ (how much greater depends on the concavity of $u(x)$) and therefore $F(w, \epsilon, 0) > 0$.

By concavity of $u(x)$, we have that $\frac{\partial F(w, \epsilon, s)}{\partial s} = u''(w - s) + \sum_{c \neq c'} \pi_c u''(w + s) + \pi_{c'} u''(w - \epsilon + s) < 0$ since all of the terms in the summation are negative.

Hence, there will exist a value of $s^* > 0$ such that the first-order condition is satisfied--i.e., $F(w, \epsilon, s^*) = 0$.

Thus, Proposition 2 shows that, facing pure risk, the consumer will engage in intertemporal precautionary saving, i.e., saving induced by a fall in expected income, regardless of whether or not she is prudent. Pure risk has been represented by a shock in just one state of the world, but the proof can easily be generalized to the case in which the shock occurs in multiple states.

The intuition underlying this result is quite straightforward. Consumers allocate consumption across the two periods by equating marginal utility of consumption across periods. Since period 1 features many possible states of the world, marginal utility in period 0 will be equated to average marginal utility in period 1. A fall in the endowment of at least one state of the world in period 1 will increase marginal utility of consumption in that state and consequently increase average marginal utility. To restore equality, consumption in period 0 must decrease (i.e., saving must increase) and consumption in period 1 must increase, on average.

Lastly, expression (1) also shows that $s < \epsilon$ as, if $s = \epsilon$, we would obtain:

$$F(w, \epsilon, \epsilon) = -u'(w - \epsilon) + \sum_{c \neq c'} \pi_c u'(w + \epsilon) + \pi_{c'} u'(w) < 0^1$$

Thus, s would have to decrease to restore equality in light of Proposition 2.

However, prudence *will* contribute toward determining the intensity of intertemporal precautionary saving, as shown by the following:

Proposition 3. The derivative of intertemporal precautionary saving with respect to ϵ , $\frac{ds(\epsilon)}{d\epsilon}$, will depend positively on absolute prudence.

Thus, in the case of pure (downside) risk, prudence *will* affect the intensive margin (the *amount* of intertemporal precautionary saving that consumers do) even though it will not affect the extensive margin (whether or not consumers engage in intertemporal precautionary saving).

Proof. By the implicit function theorem and the continuous differentiability of $F(w, \epsilon, s)$, we know that there exists a continuous function $s(\epsilon)$ in a neighborhood of each solution of the utility maximization problem and that its derivative exists and is given by:

$$\frac{ds}{d\epsilon} = -\frac{\frac{\partial F}{\partial \epsilon}}{\frac{\partial F}{\partial s}}.$$

$\frac{\partial F}{\partial \epsilon} = -\pi_{c'} u''(w - \epsilon + s) \cong -\pi_{c'} (u''(w) - u'''(w)(\epsilon - s))$ by a first-order Taylor expansion around w .

¹ To see this, just consider that in view of concavity of $u(w)$, $\sum_{c \neq c'} \pi_c u'(w + \epsilon) + \pi_{c'} u'(w) < u'(w) < u'(w - \epsilon)$.

Also,

$$\begin{aligned}
\frac{\partial F}{\partial s} &= u''(w-s) + \sum_{c \neq c'} \pi_c u''(w+s) + \pi_{c'} u''(w-\epsilon+s) \\
&\cong u''(w) - u'''(w)s \\
&+ \sum_{c \neq c'} \pi_c (u''(w) + u'''(w)s) + \pi_{c'} (u''(w) + u'''(w)(s-\epsilon)) \\
&= u''(w) - u'''(w)s \\
&+ \sum_{c \neq c'} \pi_c u''(w) + \sum_{c \neq c'} \pi_c u'''(w)s + \pi_{c'} u''(w) + \pi_{c'} u'''(w)s - \pi_{c'} u'''(w)\epsilon \\
&= 2u''(w) - \pi_{c'} u'''(w)\epsilon
\end{aligned}$$

by using first-order Taylor expansions around w and performing some algebraic manipulations.

Their ratio yields:

$$\frac{ds}{d\epsilon} = -\frac{\frac{\partial F}{\partial \epsilon}}{\frac{\partial F}{\partial s}} \cong -\frac{-\pi_{c'}(u''(w)-u'''(w)(\epsilon-s))}{2u''(w)-\pi_{c'}u'''(w)\epsilon} = \frac{\pi_{c'}(u''(w)-u'''(w)(\epsilon-s))}{2u''(w)-\pi_{c'}u'''(w)\epsilon}.$$

Dividing both the numerator and the denominator by $u''(w)$ yields:

$$\frac{ds}{d\epsilon} \cong \frac{\pi_{c'}(1+\rho(w)(\epsilon-s))}{2+\pi_{c'}\rho(w)\epsilon},$$

where $\rho(w)$ is Kimball's (1990) coefficient of absolute prudence evaluated at w .

One can easily show that $\frac{ds}{d\epsilon}$ is increasing in absolute prudence as

$$\frac{d}{d\rho} \left(\frac{ds}{d\epsilon} \right) = \frac{2\pi_{c'}(\epsilon-s) + \pi_{c'}^2 \epsilon}{(2+\pi_{c'}\rho(w)\epsilon)^2} > 0, \text{ as } (\epsilon-s) > 0 \text{ in light of the observation at the end of Proposition 2.}$$

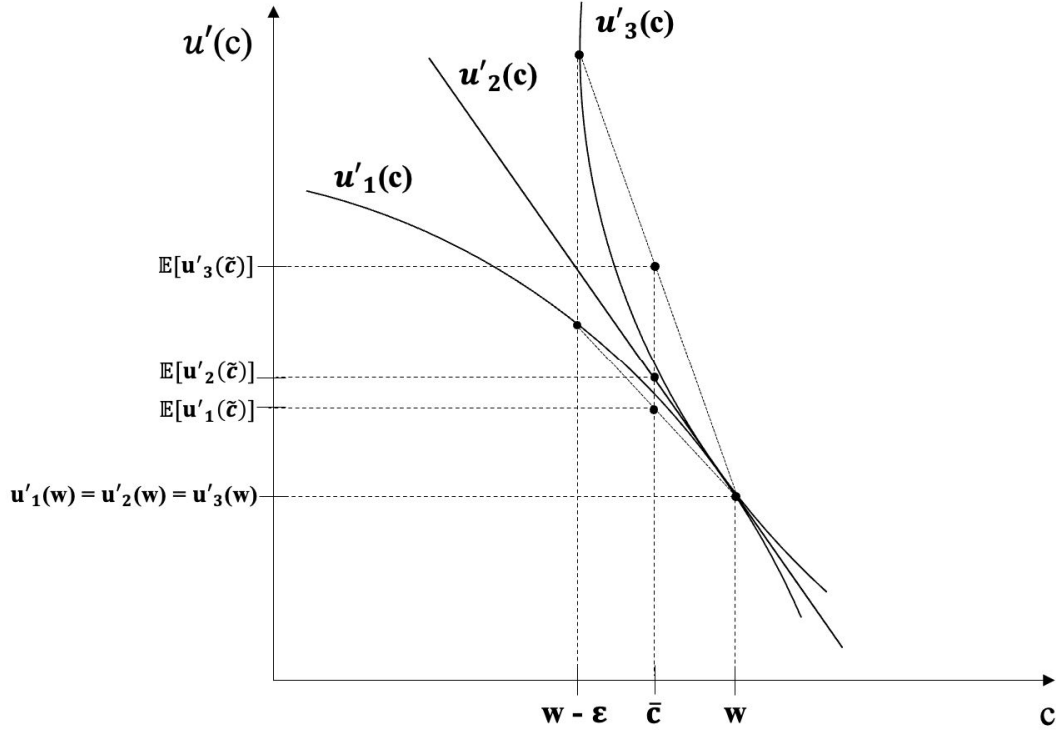
The intuition underlying Proposition 3 is also straightforward. Prudence affects the sensitivity of saving to the shock because the fact that marginal utility is convex (positive prudence) or concave (negative prudence) implies that the marginal utility of consumption in the state affected by the shock will increase more quickly (convex marginal utility, positive prudence) or more slowly (concave marginal utility, negative prudence) than with linear marginal utility (zero prudence).

Figure 1 summarizes the intuition of both propositions. The solid curves represent the marginal utilities of three different utility functions, characterized by varying levels of prudence. At consumption w , which is the endowment of both period 0 and state 1 in period 1, marginal utility is the same for the three functions. As the three curves are tangent at w , the second derivatives are also the same. Thus, absolute risk aversion is equal across utility functions at w . Prudence is not, as the first marginal utility is concave, the second is linear, and the third is convex (corresponding to negative, zero and positive absolute prudence).

The main intuition of Proposition 3 is illustrated by the fact that average marginal utility for the three curves is always higher than marginal utility at w in the face of an expected loss.

Therefore, consumption is optimally shifted from period 0, when marginal utility is $u'(w)$, to period 1, when expected marginal utility is higher.

Figure 1. Risk Aversion, Prudence, and Precautionary Saving



How much higher depends on the curvature of $u'(w)$, which is the main content of Proposition 3. Expected marginal utility is lowest for the concave marginal utility function and highest for the convex one.

4. Saving without Prudence with Pure Risk: An Example

Let us now consider an economy with consumption in period 0 and in period 1 with two equally probable states in period 1, called 1 and 2.

Preferences are represented by an intertemporally separable utility function, where instantaneous utility is quadratic and equal to $u(x) = 100x - x^2$. The third derivative of the utility function is zero, so our consumer does not display prudence, only risk aversion.

Endowments are equal to 20 in all periods and states.

Pure risk materializes as a loss in state 2, represented by a coefficient $k \in [0,1]$ that multiplies the endowment in that state, i.e., $w = 20k$. If $k = 1$, there is no loss in state 2; if $k = 0$, the loss in state 2 is total (zero endowment).

The maximization problem is thus:

$$\max 100x_0 - x_0^2 + \frac{1}{2}(100x_1 - x_1^2) + \frac{1}{2}(100x_2 - x_2^2)$$

such that

$$x_0 = 20 - s$$

$$x_1 = 20 + s$$

$$x_2 = 20k + s$$

Substituting the constraints into the objective function, we obtain the equivalent problem:

$$\max_s 100(20 - s) - (20 - s)^2 + \frac{1}{2}(100(20 + s) - (20 + s)^2) + \frac{1}{2}(100(20k + s) - (20k + s)^2)$$

with the following first-order conditions:

$$-100 + 2(20 - s) + \frac{1}{2}(100 - 2(20 + s)) + \frac{1}{2}(100 - 2(20k + s)) = 0, \text{ i.e.}$$

$$-100 + 40 - 2s + \frac{1}{2}(100 - 40 - 2s + 100 - 40k - 2s) = 0, \text{ i.e.}$$

$$20 - 4s - 20k = 0, \text{ i.e. } 20(1 - k) = 4s, \text{ yielding } s = 5(1 - k).$$

This implies that, in the absence of a loss (i.e., $k = 1$), $s = 0$, whereas when the loss is complete (i.e., $k = 0$), $s = 5$.

Why does risk aversion alone generate precautionary saving in the case of pure risk?

Pure risk decreases consumption in one state of the world, leaving consumption in the other state unaffected and equal to consumption in period 0. This means that the average marginal utility of income in period 1 will certainly be higher than marginal utility in period 0. This will make our consumer willing to shift consumption from period 0 to period 1--i.e., to engage in this weaker form of precautionary saving.

Thus, in the case of pure risk, prudence (the third derivative of the utility function), which describes the rate at which marginal utility increases in the bad state, will affect the amount of saving that is induced by risk aversion. However, in the case of pure risk, intertemporal precautionary saving will arise as long as consumers are risk-averse, regardless of whether or not they are prudent. This also means, of course, that with pure risk, we may have positive saving in the presence of downside risk even if prudence is negative.

5. Concluding Remarks

Previous authors have asserted that precautionary saving will arise only if consumers are not only risk-averse but also prudent, but in this paper, we challenged this conventional wisdom in at least two ways. First, we first considered a more general Arrow-Debreu economy in which saving is fully integrated in the set of assets available to individuals to transfer income and consumption across date-event pairs and to help consumers to achieve perfect insurance, prudence is no longer necessary, or sufficient, to generate pure precautionary saving (i.e., saving generated by risk alone without effects on expected income).

We then conducted a theoretical analysis of precautionary saving in the case of pure (downside) risk and showed that a weaker form of precautionary saving, which we named “intertemporal precautionary saving” to distinguish it from purely intertemporal and purely precautionary saving, will inevitably arise in the case of pure (downside) risk as long as consumers are risk-averse, even if they are not prudent. To do so, we adopted a simplified version of the model in Eeckhoudt and Schlesinger (2008), with only two states of nature, and complemented their result by also showing that prudence *will* affect the intensive margin (the *amount* of intertemporal precautionary saving that consumers do) even though it will not affect the extensive margin (whether or not consumers engage in precautionary saving).

Although these findings may be considered as a simple observation on saving and risk, we note that pure (downside) risk more often motivates consumers’ choices (see, for example, the unemployment, health, and accident examples in the introduction) and that speculative risk (including both downside and upside risk) is more relevant only for financial investors. Thus, the findings of this paper have wide applicability (including in the above examples) and imply that any measure that mitigates pure (downside) risk will always bring about a reduction in precautionary saving.

From a more empirical standpoint, the findings of this paper imply that we should not be surprised by the diverging empirical results concerning the extent of precautionary saving, and that measures of risk aversion (which are much more readily available than measures of prudence) may constitute an extremely relevant control variable in any saving equation.

References

- Baiardi, Donatella; Magnani, Marco; and Menegatti, Mario (2020), “The Theory of Precautionary Saving: An Overview of Recent Developments,” *Review of Economics of the Household*, 18(2): 513-542.
- Carroll, Christopher, and Samwick, Andrew A. (1998), “How Important Is Precautionary Saving,” *Review of Economics and Statistics*, 80(3): 410-419.
- Dardanoni, Valentino (1991), “Precautionary Saving under Income Uncertainty: A Cross-sectional Analysis,” *Applied Economics*, 23: 153-160.
- Eeckhoudt, Louis, and Schlesinger, Harris (2008), “Changes in Risk and the Demand for Saving,” *Journal of Monetary Economics*, 55: 1329-1336.
- Horioka, Charles Yuji, Fujisaki, Hideki, Watanabe, Wako, and Kouno, Takatsugu (2000), “Are Americans More Altruistic than the Japanese? A U.S.-Japan Comparison of Saving and Bequest Motives,” *International Economic Journal*, 14(1): 1-31.
- Horioka, Charles Yuji, and Ventura, Luigi (2025), “Why Do Europeans Save? Micro-Evidence from the Household Finance and Consumption Survey,” *Review of Income and Wealth*, 71(2): e70021
- Horioka, Charles Yuji, and Watanabe, Wako (1997), “Why Do People Save? A Micro-analysis of Motives for Household Saving in Japan,” *Economic Journal*, 107(442): 537-552.

Jappelli, Tullio, and Pistaferri, Luigi (2017), *The Economics of Consumption: Theory and Evidence* (Oxford, England: Oxford University Press).

Kimball, Miles (1990), "Precautionary Saving in the Small and in the Large," *Econometrica*, 58(1): 53-73.

Knight, Frank H. (1921), *Risk, Uncertainty and Profit* (Boston Massachusetts, USA: Hart, Schaffner and Marx; Houghton Mifflin).

Leland, Hayne E. (1968), "Saving and Uncertainty: The Precautionary Demand for Saving," *Quarterly Journal of Economics*, 82(3): 465-473.

Lugilde, Alba; Bande, Roberto; and Riveiro, Dolores (2019), "Precautionary Saving: A Review of the Empirical Literature," *Journal of Economic Surveys*, 33(2): 481-515.

Pratt, John W. (1964), "Risk Aversion in the Small and in the Large," *Econometrica*, 32(1/2): 122-136.

Quiggin, John (1982), "A Theory of Anticipated Utility," *Journal of Economic Behavior and Organization*, 3(4): 323-343.

Sandmo, Agnar (1970), "The Effect of Uncertainty on Saving Decisions," *Review of Economic Studies*, 37(3): 353-360.

Segal, Uzi; Spivak, Avia; and Zeira, Joseph (1988), "Precautionary Saving and Risk Aversion: An Anticipated Utility Approach," *Economics Letters*, 27(3): 223-227.

Skinner, Jonathan (1988), "Risky Income, Life Cycle Consumption, and Precautionary Savings," *Journal of Monetary Economics*, 22(2): 237-255.

Willett, Allan Herbert (1901), *The Economic Theory of Risk and Insurance* (New York, N. Y., USA: Columbia University Press).

Yaari, Menahem E. (1987), "The Dual Theory of Choice under Risk," *Econometrica*, 55(1): 95-115.

Zhou, Yanfei (2003), "Precautionary Saving and Earnings Uncertainty in Japan: A Household-level Analysis," *Journal of the Japanese and International Economies*, 17(2): 192-212.

Appendix: A Model of Saving with a Riskless Asset

Let us restate the model of section 4 substituting the storage technology with a riskless asset that pays one unit of consumption in both states of the world. Buying this asset will thus be equivalent to saving.

The economy features one consumption good in period 0, and one consumption good in each of two states of the world in period 1. Agents can transfer consumption across date-event pairs by trading two assets in “zero net supply” (defined as assets for which the sum across agents of portfolio holdings of these assets is zero) that pay off in period 1. One is an Arrow-Debreu asset, paying one unit of consumption in state 1 only, while the other is a riskless asset, paying one unit of consumption in both states in period 1. Buying this asset is thus equivalent to saving, while selling this asset is equivalent to lending. For simplicity, the time discount factor is set equal to unity, and probabilities are equal. The prices of the two assets are q_1 and q_2 , respectively. Let us denote by x_0, x_1, x_2 consumption at date 0 and in states 1 and 2 at date 1, respectively.

The preferences of the risk-averse agent are represented by the following intertemporal expected utility function:

$$U(x_0, x_1, x_2) = 5x_0 - \frac{1}{2}x_0^2 + \frac{1}{2}\left(5x_1 - \frac{1}{2}x_1^2\right) + \frac{1}{2}\left(5x_2 - \frac{1}{2}x_2^2\right).$$

This agent has an initial endowment of 4 in the first period, and state contingent endowments of $4 + \epsilon$ and $4 - \epsilon$, respectively, in the two states in period 1. This agent’s income is therefore affected by a speculative, symmetric risk with zero mean and variance equal to ϵ^2 .

The maximization problem of this agent is:

$$\max_{x_0, x_1, x_2} 5x_0 - \frac{1}{2}x_0^2 + \frac{1}{2}\left(5x_1 - \frac{1}{2}x_1^2\right) + \frac{1}{2}\left(5x_2 - \frac{1}{2}x_2^2\right)$$

such that:

$$x_0 = 4 - q_1y_1 - q_2y_2$$

$$x_1 = 4 + \epsilon + y_1 + y_2$$

$$x_2 = 4 - \epsilon + y_2.$$

By substituting the constraints into the utility function, we obtain the equivalent problem:

$$\max_{y_1, y_2} 5(4 - q_1y_1 - q_2y_2) - \frac{1}{2}(4 - q_1y_1 - q_2y_2)^2 + \frac{1}{2}\left(5(4 + \epsilon + y_1 + y_2) - \frac{1}{2}(4 + \epsilon + y_1 + y_2)^2\right) + \frac{1}{2}\left(5(4 - \epsilon + y_2) - \frac{1}{2}(4 - \epsilon + y_2)^2\right)$$

with the following first-order conditions with respect to asset holdings (y_1, y_2) :

$$-5q_1 + (4 - q_1y_1 - q_2y_2)q_1 + \frac{1}{2}(5 - 4 - \epsilon - y_1 - y_2) = 0$$

$$-5q_2 + (4 - q_1y_1 - q_2y_2)q_2 + \frac{1}{2}(5 - 4 - \epsilon - y_1 - y_2) + \frac{1}{2}(5 - 4 + \epsilon - y_2) = 0.$$

By solving the problem of the risk-neutral agent, we immediately obtain that the equilibrium asset prices are $q_1 = \frac{1}{2}$ and $q_2 = 1$. Thus, the presence of the risk-neutral agent allows a simple closure of the model.

By substituting equilibrium asset prices into the first-order conditions and rearranging, we obtain:

$$-\frac{3}{4}y_1 - y_2 - \frac{\epsilon}{2} = 0$$

$$y_2 = -\frac{1}{2}y_1.$$

Solving this system of equations yields:

$$y_1 = -2\epsilon, y_2 = \epsilon.$$

We immediately notice that the demand for saving, i.e. for the riskless asset, is a linear function of ϵ .

Saving is therefore a function of the (symmetric) shock, and it can be fully characterized as purely precautionary, as the income shock has zero mean.

Lastly, by substituting asset holdings in the budget constraint, we obtain a final allocation for this agent of (4,4) in the second period--i.e., the risk-averse agent achieves perfect insurance.

The reason why this is so is that, in this stylized Arrow-Debreu economy, saving is carried out by buying a risk-free asset, which crucially contributes to expanding the insurance opportunities available to the agents and, consequently, the opportunities to transfer consumption across dates and contingencies. In this example, markets are complete, and they are so thanks to the presence of the riskless asset (saving) in addition to a financial asset. Risk aversion, in and by itself, may thus well generate precautionary saving, even in the absence of prudence. The only situation in which the effect on saving highlighted in our example would not arise is if markets were complete to begin with, even in the absence of a riskless asset, a circumstance that we can safely rule out in actual practice.

Lastly, let us observe that the presence of an Arrow-Debreu asset, instead of a generic risky asset, is immaterial, in the sense that similar results may be obtained by using a more general asset with state contingent returns.