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ASSIGNING A TREATMENT UNDER HIDDEN SPILLOVERS

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Abstract

This paper studies the assignment of a treatment by a social planner when the valuations of the treatment are interdependent across individuals in the population. Specifically, an individual's valuation of the treatment is influenced by the treatment status of some group of individuals and is positive if and only if any member of the group is treated. The identities of those who have positive spillovers on an individual is his private type, and the social planner assigns the treatment based on their reported types aiming to maximize the number of treated individuals less subsidies. We study the property of an assignment mechanism that offers a subsidy to a single individual and provides the treatment to everyone over whom this individual has positive spillovers either directly or indirectly. We use the percolation theorem of McDiarmid (1981) to show that the number of treated individuals under such a mechanism is independent of the reciprocal property of the spillover relationship, and is asymptotically optimal when the population grows large.

Key words: reciprocity, networks, divide and conquer, private information, local externalities.
JEL Codes: D42, D47, D62, D82, L12

1 Introduction

Economic treatments such as enrolment in public schools, programs for healthy lifestyles, vaccination, occupational training, *etc.,* all intend to improve social welfare through the maximal participation of individuals. How to assign a treatment to individuals most effectively is a central theme in economic policy making, and

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the literature on the subject recognizes the significant impact of spillovers on the inducement of participation in various treatments. For example, Dahl et al. (2014) study the impact on a father's decision to take a paternity leave of the presence of his peers who have taken such a leave: The presence of those who have taken such leaves provides reassurance to other fathers concerning employers' response to such an action. As another example, an individual may find participation in a quit-smoking program not worth the opportunity cost if his peers don't participate, but may receive the treatment otherwise either because of the reduced cost from the shared experience or the increased benefit from mutual monitoring. As yet another example, a firm may find it too costly to relocate to a newly developed industrial park if it implies physical separation from its input supplier, but may relocate if it is accompanied by the supplier. In many of these settings, however, the social planner does not directly observe the spillover relationships among the agents and must rely on their self reports. In this paper, we formulate a stylized model of privately observed spillover relationships among economic agents, and study what kind of mechanism can effectively induce participation in a treatment. As recognized in the literature (Nakajima, 2007), one important consideration in formulating spillovers across individuals is their reciprocity. In some instances, spillovers are "peer effects" where they are reciprocal and two ways. In some other instances, occurrence of spillovers is independent so that the presence of a spillover in one direction provides no information on the presence of a spillover in the other direction. Yet in other applications, spillovers are "role model effects" where they are non-reciprocal and one way. One focus of the present study is to examine how such a reciprocity property of the spillover network interacts with the working of the mechanism.

Formally, each agent in our model has the set of other agents who have positive spillovers over him as his private type. Their valuation of the treatment is negative if none of them receives the treatment, but is positive otherwise. The social planner solicits information from the agents about their private types and then proposes the set of treated agents along with the subsidy profile. The agents then play the *participation game* where they accept or reject the proposal. We require this two-stage mechanism to be strategy-proof in the first stage so that it is weakly dominant for each agent to report their types truthfully, and dominance solvable in the second stage so that accepting the proposal is an iteratively dominant action for every agent. Importantly, dominance solvability addresses the problem of coordination failures which is typical in games with externalities. In other words, an agent finds it optimal to be treated if those who have positive spillovers over him also choose to be treated, but optimal not to be treated if none of them chooses to be treated.[1] We

[1]Such equilibrium multiplicity is often referred to as the chicken-end-egg problem (Caillaud &

refer to a mechanism satisfying strategy-proofness in the first stage and dominance solvability in the second stage as *uniquely enforceable* (UE).

In the complete information benchmark where the social planner observes the spillover network among the agents, the treatment can be assigned based on an analogue of a divide-and-conquer strategy with subsidies to some set of agents. Specifically, it offers a subsidy just sufficient to induce some set of agents to receive the treatment (even with no spillovers) and then assigns the treatment with no subsidy to all those agents over whom the first set of subsidized agents have spillovers. It then assigns the treatment to the third set of agents over whom the second set of agents have spillovers. Proceeding in the same manner, it assigns the treatment to all those agents over whom the first set of subsidized agents have either direct or indirect spillovers. In the optimum, the set of subsidized agents is minimized whereas the set of treated agents with no subsidy is maximized.

Under incomplete information, we present a class of UE assignment mechanisms based on the same type of divide-and-conquer strategy as described above. These mechanisms, called *single-seed mechanisms,* differ from the first-best mechanism described above in that only a single agent i_1 is offered a positive subsidy and that the choice of the seed agent is *not* contingent on the agents' (reported) type profile. Just as the first-best mechanism, this mechanism identifies (now based on the reported type profile) the set I_2 of agents over whom i_1 has positive spillovers, then identifies the set I_3 of agents over whom the members of I_2 have positive spillovers, and so on. The subsidy to the seed agent i_1 is just sufficient to induce him to receive the treatment, whereas no subsidy is offered to the agents in $I_2, I_3, \ldots,$. We can readily show that this mechanism satisfies the requirements of UE.

The performance of the single-seed mechanism generally depends on the properties of the spillover-network. For example, the single-seed mechanism performs poorly if each agent on average has few spillover relations. Surprisingly, however, we can show that the probability distribution over the sets of treated agents under the single-seed mechanism is *independent* of the reciprocity property of the spillover network. This conclusion, which is due to the percolation theorem (McDiarmid, 1981), implies payoff equivalence of the single-seed mechanism in terms of the spillover reciprocity. In comparison with the first-best under complete information, incomplete information typically implies a lower payoff of the social planner. We can show, however, that the single-seed mechanism is asymptotically optimal as the population grows large. In particular, as the population grows large, the probability that every agent is treated under the single-seed mechanism approaches one,

Jullien, 2003) and is a central concern in the analysis of network externalities (Dybvig & Spatt, 1983).

implying that the social planner can treat everyone with a single subsidy. When the spillover-network is perfectly reciprocal, this result holds since it is identified with the standard Erdős-Rényi random graph, and as such, the network is connected with probability close to one when the number of agents becomes large. The percolation theorem then shows that this result holds for any spillover network with imperfect reciprocity.

The contributions of this paper can be summarized as follows: First, we model treatment assignment in the presence of spillovers. While the model ignores heterogeneity across individuals through their covariates, the simplified framework highlights how direct and indirect spillovers help induce participation in treatments.

Second, we consider a problem of network externalities when they are local and only privately observable. This is a major departure from the standard assumption in the literature which assumes that the externalities are either global and depends on the size of adoption, or local but publicly observable.

Third, the paper introduces the percolation theorem in probability theory to show that the performance of some class of mechanism is independent of the reciprocity property of the spillover network. It also allows us to obtain asymptotic properties of the mechanism based on the standard conclusion from the Erdős-Rényi random networks.

Fourth, the paper shows the validity of an analogue of the divide-and-conquer strategy in the presence of incomplete information. Specifically, unlike the standard divide-and-conquer strategy, the treatment strategy considered in this paper takes advantage of not only the direct externalities among individuals, but also the indirect externalities among them.

The paper is organized as follows. We discuss the related literature in Section 2. Section 3 formulates a model and defines an assignment mechanism. Section 4 analyzes the first-best under complete information and compares it with the second-best under incomplete information. The definition and properties of the single-seed mechanism are presented in Section 5. We conclude with a discussion in Section 6.

2 Related Literature

The present paper is related to a few strands of the literature.

First, it belongs to the literature on local network externalities that express externalities by networks of agents. The primary focus of the literature however is on the monopoly sale of a good to the agents. Among them, Candogan et al. (2012) characterize the relationship between the location of a buyer in the network and the price he faces under imperfect and perfect price discrimination, and Bloch and

Quérou (2013) examine the optimality of price discrimination when each buyer is privately informed about the stand-alone valuation of the monopolist's good. Chen et al. (2018, 2020) formulate models of price competition between sellers of goods with local network externalities. Incomplete information about the valuation of the good provided by the monopolist is also studied in Aoyagi (2013), which assumes the value of the good to each agent is the product of the agent's private type and the size of adoption. Closer to the present paper, some recent papers introduce incomplete information concerning the relationship among agents and study its impact on the optimal strategies of a monopolist. Fainmesser and Galeotti (2016, 2020) and Zhang and Chen (2020) study optimal pricing when agents are privately informed about the number of their neighnors. Kanoria and Saban (2021) and Basu et al. (2024) both formulate relation-specific preferences over others as in the present paper.

How peer-effects impact the value of a group to its member is discussed in a model of group formation (e.g., Board, 2009; Sarkisian & Yamashita, 2024; Veiga, 2013). These papers specify the value of a group as a function of the qualities of its members. Similar assumptions can also be found in matching theory (Alcalde & Revilla, 2004; Cechlárová & Romero-Medina, 2001; Dimitrov et al., 2006; Rodríguez-Álvarez, 2009).

The present paper is also related to the literature on network interventions (Galeotti et al., 2020; Sun et al., 2023). Under the assumption that the agent network is publicly observable, the papers in the literature measure the effects of treating some agents through the change of their payoff functions or their connections with others.

As mentioned in the Introduction, the social planner uses a subsidy scheme analogous to the divide-and-conquer strategy with the aim of eliminating coordination failures among agents. Bernstein and Winter (2012) present comprehensive analysis of the optimal divide-and-conquer pricing strategy in a monopolistic setting when the externalities are heterogeneous and agents differ in their popularity to others. Aoyagi (2018) examines the extent to which the divide-and-conquer pricing strategy influences the equilibrium price configuration in a competitive setting.

Treatment spillovers are empirically documented in such areas as health-related behavior including vaccination and smoking, educational achievement in various contexts, and microfinance (e.g. Dahl et al., 2014; Ferracci et al., 2014; Kitagawa & Wang, 2026). An observation most closely related to the present paper is made by Dahl et al. (2014), who show that treatment participation snowballs: The exogenously treated individuals first induce participation by those around them, who in turn induce participation around them and so on. This is exactly the treatment participation pattern induced by the mechanism studied in the present paper.

3 Model

3.1 Valuation of the treatment

A social planner assigns a treatment to the population consisting of the set $I = \{1, \ldots, n\}$ of ex ante identical agents ($n \geq 2$). Each agent i has a private *type* $\theta_i = (\theta_i^j)_{j \neq i} \in \Theta_i \equiv \{0,1\}^{n-1}$. As described below, $\theta_i^j = 1$ implies the influence of agent j's treatment status on agent i's valuation of the treatment. Denote by $\Theta = \prod_i \Theta_i$ the set of type profiles $\theta = (\theta_i)_{i \in I}$ of all agents. The joint probability distribution of θ is described below. Let $t_i \in \{0,1\}$ denote the treatment status of agent i: $t_i = 1$ if i is treated and $t_i = 0$ otherwise. An *assignment* $t = (t_i)_{i \in I} \in \{0,1\}^I$ is the profile of treatment status of all agents. Let $G(t) = \{j \in I : t_j = 1\}$ denote the set of treated agents in the population under assignment t, and $G_i(t, \theta_i) = G(t) \cap \{j : \theta_i^j = 1\}$ the set of treated agents under assignment t whose have spillovers over agent i's valuation of the treatment. We assume that agent i's valuation of the treatment when his type is θ_i is given by

$$u_i(t, \theta_i) = v^{|G_i(t,\theta_i)|},$$

where $v^0, v^1, \ldots, v^{n-1} \in \mathbf{R}$ satisfies

$$v^0 < 0 < \min_{n \geq 1} v^n.$$

In other words, i's valuation is strictly negative if no agent j with $\theta_i^j = 1$ is treated, and strictly positive if at least one such agent is treated.

3.2 Assignment mechanism

An *assignment mechanism* Γ is an indirect mechanism over two stages described as follows: In stage 1, it announces an *assignment rule* $f : \Theta \to T$ which determines an assignment as a function of θ, and a subsidy rule $y = (y_i)_{i \in I} : \Theta \to \mathbf{R}_+^I$ which determines subsidies to all the agents also as a function of θ. It then solicits from every agent i a report on his type θ_i and determines the assignment and subsidies $((f(\theta), y(\theta))$. In stage 2, the mechanism publicly discloses $((f(\theta), y(\theta))$ and θ, and has the agents play a *participation game* in which they simultaneously choose between accept and reject. The treatment is provided to agent i if and only if he accepts. When an agent choose to reject, he takes the outside option whose value is normalized to zero. Let $A_i = \{0,1\}$ denote the set of actions available to agent $i \in I$ in the participation game, where $a_i = 1$ represents accept and $a_i = 0$ represents reject. If agent i is not assigned the treatment ($t_i = 0$), then his choice a_i only determines whether or not he accepts the transfer y_i. The decision of any such agent is hence irrelevant for the other agents. Assignment $t^a = (t_i^a)_{i \in I}$ following the action profile a and the proposal (f, y) is given by

$$t^a = t \wedge a,$$

where $\wedge$ is the component-wise minimum of the two vectors t and a. Accordingly, given (f, y) and θ, agent i's payoff from the action profile a is given by

$$U_i(a \mid \theta, f, y) = \begin{cases} u_i(f(\theta) \wedge a, \theta_i) + y_i & \text{if } a_i = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Let $a^* = (1, \ldots, 1)$ denote the action profile corresponding to acceptance by all the agents. A mechanism Γ is *strategy-proof* (SP) if truthful reporting is a (weakly) dominant strategy for every agent provided that a^* is played in the participation game: For every θ_i, $\theta_i' \in \Theta_i$, $\theta_{-i} \in \Theta_{-i}$, and $i \in I$,

$$\begin{aligned} & U_i(a^* \mid \theta_i, \theta_{-i}, f, y) \geq U_i(a^* \mid \theta_i', \theta_{-i}, f, y) \\ \Leftrightarrow \quad & u_i(f(\theta_i, \theta_{-i}), \theta_i) + y_i(\theta_i, \theta_{-i}) \geq u_i(f(\theta_i', \theta_{-i}), \theta_i) + y_i(\theta_i', \theta_{-i}). \end{aligned}$$

Γ is *uniquely acceptable* (UA) if for every $\theta \in \Theta$, the participation game $(I, A, (U_i(a \mid \theta_i, f(\theta), y(\theta)))_{i \in I})$ is dominance solvable with a^* surviving the iterative elimination of strictly dominated strategies. The requirement of UA addresses equilibrium multiplicity under adoption externalities and replaces the standard participation (individual rationality) constraint. Γ is *uniquely enforceable* (UE) if it is SP and UA.[2] By SP, no unilateral deviation that involves misreporting in stage 1 and acceptance in stage 2 is profitable. No unilateral deviation that involves rejection in stage 2 is profitable either since any such deviation yields zero, whereas truthful reporting and acceptance yield at least zero by UA. It follows that if the mechanism is UE, then agents will report their types truthfully as a weakly dominant reporting strategy, and then accept the assignment proposal by the social planner as an iteratively strictly dominant participation decision. Since SP and UA are both properties of the assignment and transfer rules, we represent an assignment mechanism Γ by (f, y) with some abuse of notation.

Given the assignment t and transfer profile y, the social planner's payoff $\pi(t, y)$ is given by

$$\pi(t, y) = \sum_{i \in I} (V t_i - y_i),$$

where $V > 0$ is a constant representing the marginal benefit of treating one additional agent. It follows that the social planner wants to maximize the number of

[2]Note however that UE does not imply the uniqueness of a PBE in the two-stage game, making our requirement different from that for unique (full) implementation.

treated agents while minimizing the sum of subsidies. Note that a subsidy may be offered to an agent who is not treated and that no transfer is made from the agents to the social planner. We further assume that

$$V + v^0 < 0, \quad \text{and} \quad 2V + v^0 > 0.$$

The first inequality implies that the marginal benefit to the social planner of treating one additional agent is less than the disutility incurred by any agent from receiving the treatment in isolation. On the other hand, the second inequality implies that the social planner's payoff is positive when he can treat two agents one with a subsidy equal to $-v^0$ and the other with no subsidy. When $\Gamma = (f, y)$ is UE, the social planner's ex post payoff given the type profile θ equals $\pi\big(f(\theta), y(\theta)\big)$, and his expected payoff equals

$$\Pi(\Gamma) = E_\theta[\pi\big(f(\theta), y(\theta)\big)].$$

3.3 Type distribution and the spillover network

As mentioned above, agent i's valuation is influenced by the treatment status of agent j if and only if $\theta_i^j = 1$. We assume that θ_i is identically distributed across agents, and that the distribution of the type profile θ satisfies

$$\Pr(\theta) = \prod_{\{(i,j):\, i \neq j\}} \Pr(\theta_i^j, \theta_j^i) \quad \text{for every } \theta \in \Theta.$$

In other words, the distribution of the type components regarding the pairwise relationship is independent across all different pairs, but correlation is possible in the distribution of (θ_i^j, θ_j^i). The latter assumption captures different possibilities regarding the relationship between any pair of agents. The relationship may be mutual friendship whereby the action of one agent influences the other, or a one-way relationship whereby the action of one agent influences the other, but not the other way around. Formally, we suppose that there exist constants $p \in (0, 1)$ and $\rho \geq \max\{0,\, 2 - \frac{1}{p}\}$ such that the joint distribution of θ_i^j and θ_j^i for $i \neq j$ is given in Table 1.

Hence, we have $p = \Pr(\theta_i^j = 1)$ for any $i \neq j$, and ρ represents the degree of reciprocity between any pair of agents: When $\rho = 1$, $\theta_i^j = 1 \Leftrightarrow \theta_j^i = 1$, and when $p \leq \frac{1}{2}$ and $\rho = 0$, $\theta_i^j = 1 \Rightarrow \theta_j^i = 0$. Now define the binary random variable X_{ij} by

$$X_{ij} = \begin{cases} \theta_i^j & \text{if } i \neq j, \\ 0 & \text{if } i = j. \end{cases}$$

Table 1: Joint distribution of θ_i^j and θ_j^i

		θ_j^i		
		0	1	
θ_i^j	0	$1-p(2-\rho)$	$(1-\rho)p$	$1-p$
	1	$(1-\rho)p$	ρp	p
		$1-p$	p	1

We identify $X = (X_{ij})_{i,j \in I}$ as a directed graph with the node set I and the link set $\{ij : X_{ij} = 1\}$. In X, agent i is *strongly connected* to agent j, denoted $i \rightsquigarrow_\theta j$, if there is a directed path from i to j: There exists a sequence of agents $i_0, \ldots, i_K$ ($K \geq 1$) in I such that $i_0 = i$, $i_K = j$, and for every i_k and i_{k+1} ($k = 0, \ldots, K-1$), there is a directed link $i_k \to i_{k+1}$ ($\Leftrightarrow X_{i_k i_{k+1}}(\theta) = 1$). We write $\rightsquigarrow$ instead of $\rightsquigarrow_\theta$ when θ is evident. A subset $H \subset I$ of agents is *strongly connected* if $i \rightsquigarrow j$ for every pair (i, j) of agents in H. i is *connected* to j if there is a path from i to j when the direction of each link is ignored.

4 First- and second-best with a finite population

4.1 First-best under complete information

We begin our analysis with the benchmark case where the social planner has complete information about the agents' type profile θ, or equivalently, the underlying network $X(\theta)$. In this case, we only require the mechanism Γ to be UA. Define $\Pi^*(\theta)$ to be the supremum of the social planner's payoff from such mechanisms:

$$\Pi^*(\theta) = \sup \left\{ \pi\big(f(\theta), y(\theta)\big) : \Gamma \text{ is UA} \right\},$$

and $\Pi^* = E_\theta[\Pi^*(\theta)]$.

For any set $F \subset I$ of agents and $i \notin F$, we write $i \rightsquigarrow_\theta F$ if $i \rightsquigarrow_\theta j$ for some $j \in F$. Define $(F^*, Y^*) \equiv (F^*(\theta), Y^*(\theta))$ to be a solution to the following maximization problem:

$$\max_{(F,Y)} |Y|V + |F|(V + v^0) \quad \text{subject to} \quad \begin{cases} F, Y \subset I, \\ Y = \{j \notin F : j \rightsquigarrow_\theta F\}. \end{cases} \tag{1}$$

(1) has a solution since I is finite, and gives the maximal payoff that the social planner can achieve from any UA mechanism as seen in Proposition 1 below. The

intuition is as follows: The objective function corresponds to the social planner's payoff when it treats the agents in Y with no subsidy, but the treats the agents in F with a subsidy equal to $-v^0 > 0$. Since each member of Y is strongly connected to some agent in F, they are willing to be treated with no subsidy if the agents in F are also treated. The social planner then maximizes its payoff by taking Y as large as possible (since $V > 0$) and F as small as possible (since $V + v^0 < 0$).

We next show that for any $\varepsilon > 0$, there exists a UA mechanism Γ such that $\Pi(\theta \mid \Gamma) = |Y^*(\theta)|V + |F^*(\theta)|(V + v^0) - \varepsilon$ for the solution $(F^*(\theta), Y^*(\theta))$ to (1). Let (f, y) be defined by

$$t_i(\theta) = \begin{cases} 1 & \text{if } i \in F^*(\theta) \cup Y^*(\theta), \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

and

$$y_i(\theta) = \begin{cases} \frac{\varepsilon}{n} & \text{if } i \notin F^*(\theta), \\ -v^0 + \frac{\varepsilon}{n} & \text{if } i \in F^*(\theta), \end{cases} \tag{3}$$

In other words, agents in $F^*(\theta)$ are offered a subsidy slightly above $-v^0$, whereas other agents are offered a small subsidy.[3]

Proposition 1. *Suppose that the social planner has complete information about θ. Then Γ defined in (2) and (3) is UA for any $\varepsilon > 0$, and satisfies $\Pi(\theta \mid \Gamma) = \Pi^*(\theta) - \varepsilon$ for every θ. It follows that*

$$\Pi^*(\theta) = |Y^*(\theta)|V + |F^*(\theta)|(V + v^0).$$

Note that the maximized payoff is strictly positive as long as $|Y^*| \geq 1$ since $2V + v^0 > 0$. When the network $X(\theta)$ is as described in Figure 1, for example, (1) has the solution $F^* = \{5, 6\}$ and $Y^* = \{1, 2, 4, 7, 8, 9, 10\}$, and its value equals $\Pi^* = 7V + 2(V + v^0) = 9V + 2v^0$. Note that agent 3 is not treated since doing so requires a subsidy in excess of V but does not contribute to the expansion of Y. In the participation game, acceptance is a strictly dominant action for every agent in F^*, and is an iteratively strictly dominant action for agents in Y^*. The number of iterations required for each agent in Y^* to find out that acceptance is an optimal action equals the length of the shortest directed path that connects him with agents in F^*.

[3] As specified, agents who are offered no treatment also receive a small subsidy. This is to ensure that the participation game is dominance solvable, but is irrelevant for the decisions of agents who are assigned the treatment. Alternatively, we may suppose that the participation game is played only by those agents who are assigned the treatment.

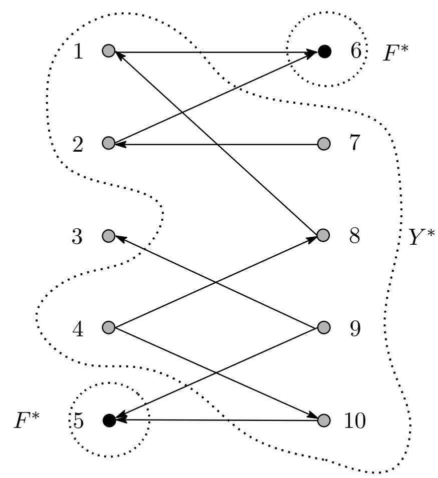

Figure 1: Optimal mechanism under complete information

The figure depicts a two sided-market but the construction is the same in a one-sided market.

4.2 Second-best under incomplete information

We now return to the incomplete information environment in which the realization of the types is private information of the agents. Specifically, we show that the social planner's payoff from the optimal mechanism Γ under incomplete information is bounded away from the first-best level described above for a fixed population n.

Proposition 2. *Suppose that $\rho \in (0,1)$ and that $v^2 \leq \cdots \leq v^{n-1}$. There exists $\kappa \equiv \kappa(n,\rho) > 0$ such that if the social planner's expected payoff under any UE mechanism Γ is bounded away from the first-best level by κ: $\Pi(\Gamma) \leq \Pi^* - \kappa$.*

The intuition is simple: Suppose that the true type profile $\theta = (\theta_i, \theta_{-i})$ is such that every agent is strongly connected and consider any mechanism Γ under incomplete information. Take any agent i who is treated with no subsidy under Γ. If agent i misreports his type as $\hat{\theta}_i$ such that $\hat{\theta}_i^j = 0$ for every $j \neq i$, the set of treated agents under Γ must be smaller than that under the true θ: If it were the same, agent i would still be treated and enjoy the same externality benefit as under θ while he now receives a subsidy at least as large as $-v^0$. Since the first-best mechanism has every agent treated under $(\hat{\theta}_i, \theta_{-i})$ by providing a subsidy to agent i, this implies that the treatment size under Γ is smaller than the first-best, implying a lower payoff to the social planner.

5 Single-seed mechanism

5.1 Definition

This section introduces a class of UE mechanisms under incomplete information. Specifically, a *single-seed mechanism* offers the treatment to a fixed agent i_1 along with all agents who are strongly connected to i_1. It offers a subsidy slightly above $-v^0$ to i_1, and a small subsidy to all other treated agents. Formally, the single-seed mechanism $\Gamma^s \equiv \Gamma^s(i_1)$ based on $i_1 \in I$ has the assignment rule f and the transfer rule y such that

$$f_i(\theta) = \begin{cases} 1 & \text{if } i \in \{i_1\} \cup \{j : j \rightsquigarrow_\theta i_1\}, \\ 0 & \text{otherwise,} \end{cases}$$

and for $\varepsilon > 0$,

$$y_i(\theta) = \begin{cases} -v^0 + \frac{\varepsilon}{n} & \text{if } i = i_1, \\ \frac{\varepsilon}{n} & \text{otherwise.} \end{cases}$$

Figure 2 illustrates a single-seed mechanism based on $i_1 = 6$. Note that the choice of agent i_1 is not contingent on the realization of θ. Upon learning the type profile, it is tempting for the social planner to designate the most "influential" agent as i_1 (i.e., agent i such that $\sum_j \theta_j^i$ is the maximum). With unobservable types, however, choice of i_1 according to their influence based on the agents' reported types creates an incentive problem. For example, if agents i and j both have the highest influence according to θ, then it can create room for profitable misreporting by i: Instead of reporting his true type θ_i for which $\theta_i^j = 1$, i can report $\hat{\theta}_i$ such that $\hat{\theta}_i^j = 0$ so that $\sum_k \hat{\theta}_k^i = \sum_k \theta_k^i > \sum_k \hat{\theta}_k^j$. Such misreporting reduces j's influence by one, while maintaining i's own influence, making i himself uniquely the most influential agent under $(\hat{\theta}_i, \theta_{-i})$.

5.2 Properties

Under the single-seed mechanism, it is clear that the seed agent i_1 has no incentive to misreport his type since it does not change the outcome in any way. The single-seed mechanism has the following important property: For any agent assigned the treatment, he cannot change the set of treated agents by misreporting his type as long as it keeps his treatment status. With this property, if agent j is strongly connected to i_1 under the true type profile, then he cannot profitably misreport a type that keeps him strongly connected to i_1. If misreporting makes him not strongly connected to i_1, it is not profitable either since he will then lose the treatment. On the other hand, the single-seed mechanism is UA through iterative reasoning starting from the agents closer to i_1. Its working hence hinges on the ability of the social

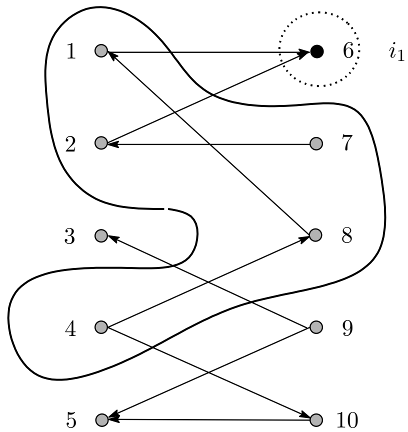

Figure 2: Single-seed mechanism based on $i_1 = 6$

Agents encircled by the solid curve are assigned the treatment with no subsidy as they are strongly connected to $i_1 = 6$.

planner to make public the reported type profile θ along with the subsidy profile $y(\theta)$ (Miklós-Thal & Shaffer, 2017).

Proposition 3. *For any $i_1 \in I$, the single-seed mechanism Γ^s based on i_1 is UE.*

The single-seed mechanism Γ^s is typically inefficient since only a few agents are strongly connected to i_1 with non-negligible probability. In Figure 2, for example, a single-seed mechanism based on $i_1 = 4$, 7 or 9 ends up treating no additional agent. As the population grows, however, such a problem with the treatment size occurs less frequently as seen below. Regardless of the population, the surprising property of the single-seed mechanism concerns the independence of the number of treated agents from the reciprocal property of the spillover network X. This in particular implies that the expected revenue from the single-seed mechanism is also independent of the degree of reciprocity ρ. Formally, the percolation theorem of McDiarmid (1981, Theorem 4.2) shows that for any $J \subset I \setminus \{i_1\}$, the probability that all agents in J are strongly connected to i_1 is the same regardless of the joint distribution of (X_{ij}, X_{ji}) as long as the link occurrence between different pairs of agents is independent. The proof of the percolation theorem based on a clutter is involved, but we can gain some intuition by looking at the case where n is small.[4] Suppose that $I = \{1, 2, 3\}$ and that $i_1 = 1$. Consider the probability that both

[4]A clutter is a collection of the minimal sets possessing some property. In the example below, $\{Z_1, Z_2, Z_3\}$ is a clutter with the property that both 2 and 3 are strongly connected to 1.

agents 2 and 3 are strongly connected to 1. If we define

$$\begin{aligned} Z_1 &= \{\theta :\ X_{21}(\theta) = X_{31}(\theta) = 1\}, \\ Z_2 &= \{\theta :\ X_{21}(\theta) = X_{32}(\theta) = 1\}, \\ Z_3 &= \{\theta :\ X_{31}(\theta) = X_{23}(\theta) = 1\}, \end{aligned}$$

then

$$\begin{aligned} \Pr(\theta :\ 2,3 \rightsquigarrow_\theta 1) &= \Pr(Z_1 \cup Z_2 \cup Z_3) \\ &= \Pr(Z_1) + \Pr(Z_2) + \Pr(Z_3) \\ &\quad - \Pr(Z_1 \cap Z_2) - \Pr(Z_2 \cap Z_3) - \Pr(Z_3 \cap Z_1) + \Pr(Z_1 \cap Z_2 \cap Z_3). \end{aligned}$$

Since $Z_2 \cap Z_3 = Z_1 \cap Z_2 \cap Z_3$ and since no opposite links appear in the definitions of Z_1, Z_2, Z_3, $Z_1 \cap Z_2$, and $Z_3 \cap Z_1$, we conclude from the independence of link occurrence between different pairs that

$$\begin{aligned} \Pr(Z_1 \cup Z_2 \cup Z_3) &= \Pr(Z_1) + \Pr(Z_2) + \Pr(Z_3) - \Pr(Z_1 \cap Z_2) - \Pr(Z_3 \cap Z_1) \\ &= 3p^2 - 2p^3, \end{aligned}$$

which is independent of the degree of reciprocity.

Proposition 4. *(Payoff equivalence) The social planner's expected payoff* $\Pi(\Gamma^s)$ *from a single-seed mechanism* Γ^s *is independent of the degree of reciprocity* ρ.

The percolation theorem applies more generally to the case where the joint distribution of (θ_i^j, θ_J^i) (or (X_{ij}, X_{ji})) is different for different pairs (i,j). This corresponds to considering ex ante heterogeneity across agents more in line with the econometric approach to treatment assignment. Although the social planner's payoff is again independent from the reciprocity of the spillover network as long as the seed agent is fixed, the choice of a seed agent determines the performance of the single-seed mechanism. The optimal selection of the seed agent under ex ante heterogeneity is an interesting topic of future research. We now investigate the performance of the single-seed mechanism as the population grows large. Define $\Theta^{\rightsquigarrow i_1}$ to be the set of type profiles such that every agent is strongly connected to i_1:

$$\Theta^{\rightsquigarrow i_1} = \{\theta :\ i \neq i_1 \Rightarrow i \rightsquigarrow_\theta i_1\}.$$

Lemma 5. *For every* $\varepsilon > 0$*, there exists* $N > 0$ *such that if* $n > N$*, then*

$$\Pr\big(\Theta^{\rightsquigarrow i_1}\big) > 1 - \varepsilon.$$

The intuition behind the result is as follows. When the spillover network is perfectly reciprocal ($\rho = 1$), the corresponding network X is such that $X_{ij} = 1$ if and only if $X_{ji} = 1$, so that we can identify $X(\theta)$ with an undirected graph $G(\theta)$ which has a link between i and j if and only if $X_{ij} = X_{ji} = 1$ in $X(\theta)$. In this case, every agent $j \neq i_1$ is strongly connected to i_1 in X if and only if G is connected. Furthermore, since the link occurrence is independent across different pairs, G is the standard Erdős-Rényi type random graph. Lemma 5 is then established if the probability that G is connected approaches one as $n \to \infty$. When $\Pr(X_{ij} = 1) = p$ is held constant, this holds as n increases (e.g. Diestel, 2000). Since $\Pr(\theta \in \Theta^{\rightsquigarrow i_1})$ is independent of the value of ρ by the percolation theorem, the same holds for any ρ.

Now denote by $\Theta^s_\varepsilon(n)$ the set of type profiles θ at which the expected revenue from the single-seed mechanism Γ^s described above is within ε of the optimal level $\Pi^*(\theta)$ under complete information:

$$\Theta^s_\varepsilon = \{\theta : \Pi(\theta \mid \Gamma^s) > \Pi^*(\theta) - \varepsilon\}.$$

We now observe that $\Theta^s_\varepsilon \supset \Theta^{\rightsquigarrow i_1}$ for any $\varepsilon > 0$ since if $\theta \in \Theta^{\rightsquigarrow i_1}$, then the single-seed mechanism Γ^s treats all agents with no subsidy but i_1 so that the principal's payoff can be made arbitrarily close to the first-best. This observation immediately implies the following result.

Proposition 6. *The single-seed mechanism Γ^s is asymptotically optimal: For every $\varepsilon > 0$, there exists $N > 0$ such that if $n > N$, then*

$$\Pr(\theta \in \Theta^s_\varepsilon) > 1 - \varepsilon. \tag{4}$$

As the population grows, the number of steps required to find out the iteratively dominant action in the participation game may increase indefinitely. In this case, the result requires unboundedness in the agents' cognitive abilities. On the other hand, if we take the alternative interpretation of the participation game and suppose that the agents move sequentially to take best responses, the required cognitive load is not large. According to the latter interpretation, participation in the treatment grows over time through a chain of spillovers. Such snowballing growth of treatment participation is of empirical relevance (Dahl et al., 2014) as mentioned in Section 2.

6 Conclusion

A divide-and-conquer strategy in a two-sided market typically subsidizes all agents on one side and charges a positive price to the other side. The single-seed mechanism we study employs a generalization of such a strategy in the specification of subsidies based not only on direct externalities but also on indirect externalities. Although we have assumed that an agent's valuation of a treatment is positive with a spillover from a single agent, it is possible to suppose that the valuation is positive for the first time with two or more spillovers. When two spillovers are necessary, for example, would subsidies to two agents suffice to obtain a similar conclusion? Answering this question is difficult. In our model, an agent finds participation in the treatment a dominant action if and only if there exists a single directed path from him to the seed agent i_1. Two spillovers would then imply the existence of two disjoint directed paths to the two subsidized seed agents. However, an agent having two disjoint paths does not necessarily finds participation to be a dominant action. For example, suppose that there exist four agents 1,...,4, and that $I_1 = \{1, 2\}$ are subsidized seed agents. If agents 1 and 4 have spillovers over agent 3, and agents 2 and 3 have spillovers over agent 4, then both agents 3 and 4 have two disjoint paths to I_1: $3 \to 1$ and $3 \to 4 \to 2$ for agent 3, and $4 \to 3 \to 1$ and $4 \to 2$ for agent 4. However, agent 3 would find participation dominant only if agent 4 also subscribes, and vice versa. In other words, unlike in the original problem, there is no straight connection between the connectivity of the network constructed from I_1 as above and participation being a iteratively dominant action. The problem is difficult because of this difference, and how exactly the mechanism can be made UE in this modified environment is an open question.

Appendix

Proof of Proposition 1. We first show that

$$\Pi^*(\theta) \leq |Y^*(\theta)|V + |F^*(\theta)|(V + v^0).$$

Suppose to the contrary that a UA mechanism $\hat{\Gamma}$ with $(\hat{t}, \hat{y})$ yields the payoff $\Pi(\theta \mid \hat{\Gamma}) > |Y^*(\theta)|V + |F^*(\theta)|(V + v^0)$. Let $\hat{Y} = \{i \in I : \hat{y}_i(\theta) < -v^0\}$ be the set of agents who are offered the treatment for a subsidy less than $-v^0$, and $\hat{F} = \{i \in I : \hat{y}_i(\theta) > -v^0\}$ be the set of agents who are offered the treatment for a subsidy above $-v^0$. Note first that there exists $i_1 \in \hat{Y}$ who is not strongly connected to $\hat{F}$ since otherwise, $(\hat{F}, \hat{Y})$ would be feasible in (1). Let $J_1 = \{i_1\}$, and let $J_2 \subset \hat{Y}$ be the set of agents in $\hat{Y}$ to whom i_1 is strongly connected: $J_2 = \{j \in \hat{Y} : i_1 \rightsquigarrow j\}$. Since i_1 is charged a positive price, IR implies that $J_2 \neq \emptyset$, and furthermore, since i_1 is not strongly connected to $\hat{F}$, no $j \in J_2$ is strongly connected to $\hat{F}$. Let then $J_3 \subset \hat{Y}$ be the set of agents j to whom agents in J_2 are strongly connected. In the same way, we can iteratively construct a sequence $J_4, J_5, \ldots$ of subsets of $\hat{Y}$ so that no agent in those subsets is strongly connected to $\hat{F}$. Since $\hat{Y}$ is finite, however, we will have

$J_{k+1} \subset \cup_{\ell=1}^{k} J_\ell$ for some k. We then have a contradiction to unique acceptability since then for agents i in the set $\cup_{\ell=1}^{k} J_\ell$, rejection $a_i = 0$ is a Nash equilibrium action since they are offered no subsidies.

The mechanism Γ defined in (2) and (3) is clearly UA: For $i \in F^*(\theta)$, $y_i(\theta) > -v^0$ so that acceptance $a_i = 1$ is a dominant action. For $j \in Y^*(\theta)$, $a_j = 1$ is an iteratively dominant action since $Y^*(\theta)$ consists of all agents who are strongly connected to some $i \in F^*(\theta)$: For any $j \in Y^*(\theta)$, either $\theta_j^i = 1$ for some $i \in F^*(\theta)$ or there exists $k \in Y(\theta)$ with $k \in C_{\rightsquigarrow F^*(\theta)}(\theta)$. Rejection is dominated in the second round of the iterative elimination procedure in the first case, whereas in the second case, it is iteratively dominated for j in one round after it is dominated for k. Finally, the social planner's payoff under Γ equals

$$\Pi(\theta \mid \Gamma) = |Y^*(\theta)|V + |F^*(\theta)|(V + v^0) - \varepsilon.$$

Since ε is arbitrary, $\Pi^*(\theta) \geq |Y^*(\theta)|V + |F^*(\theta)|(V + v^0)$. The conclusion then follows. $\square$

Proof of Proposition 2. Since $\rho \in (0,1)$, there exists $\delta > 0$ such that $P(\theta) \geq \delta$ for every θ. Let Θ^{sc} denote the set of type profiles θ such that $X(\theta)$ is strongly connected. Define

$$\kappa = \delta V |\Theta^{\text{sc}}| > 0.$$

For any $\theta \in \Theta^{\text{sc}}$, there exists an UA mechanism under complete information offers one agent a subsidy slightly above $-v^0$ and all other agents no subsidy. If follows that the first-best satisfies

$$\Pi^*(\theta) = (n-1)V + (V + v^0).$$

Fix any mechanism (f, y). For any θ, let $J(\theta) = \{j \in G(f(\theta)) : y_j(\theta) < -v^0\}$ be the set of agents who are treated for subsidies less than $-v^0$ under (f, y). Since Γ is UA,

$$R(\theta \mid f, y) \leq |J(\theta)|V + (V + v^0).$$

It follows that for any $\theta \in \Theta^{\text{sc}}$,

$$\Pi^*(\theta) - \Pi(\theta \mid f, y) \geq (n - |J(\theta)| - 1)V.$$

Let Θ^1 be the set of $\theta \in \Theta^{\text{sc}}$ for which $|J(\theta)| < n - 1$, we have

$$\Pi^*(\theta) - \Pi(\theta \mid f, y) \geq V \quad \text{for any } \theta \in \Theta^1. \tag{5}$$

Let now Θ^2 be the set of $\theta \in \Theta^{\text{sc}}$ for which $|J(\theta)| = n - 1$. Take any such θ and $j \in J(\theta)$, and consider now a profile $(\hat{\theta}_j, \theta_{-j})$ where $\hat{\theta}_j$ is such that $\hat{\theta}_j^k = 0$ for every

$k \neq j$. Clearly, $(\hat{\theta}_j, \theta_{-j}) \notin \Theta^{\text{sc}}$ but the first-best under complete information satisfies $\Pi^*(\hat{\theta}_j, \theta_{-j}) = (n-1)V + (V + v^0)$. Let K and $\hat{K}$ be the numbers of treated agents who have spillover on j under θ and $(\hat{\theta}_j, \theta_{-j})$, respectively:

$$K = |\{k : \theta_j^k = 1\} \cap G(f(\theta_j, \theta_{-j}))|,$$
$$\hat{K} = |\{k : \theta_j^k = 1\} \cap G(f(\hat{\theta}_j, \theta_{-j}))|.$$

If j is treated under $(\hat{\theta}_j, \theta_{-j})$ (i.e., $j \notin G(f(\hat{\theta}_j, \theta_{-j}))$) and $\hat{K} \geq K$, then agent j would have incentive to report $\hat{\theta}_j$ when his true type is θ_j: Since $y_j(\hat{\theta}_j, \theta_{-j}) \geq -v^0$ by UA, his utility from misreporting would satisfy

$$v^{\hat{K}} + y_j(\hat{\theta}_j, \theta_{-j}) \geq v^K - v^0 > v^K + y_j(\theta),$$

where the last term equals his payoff from truthful reporting. Hence, under $(\hat{\theta}_j, \theta_{-j})$, we must have either $j \notin G(f(\hat{\theta}_j, \theta_{-j}))$, or

$$\hat{K} < K.$$

It follows that $G(f(\hat{\theta}_j, \theta_{-j}))$ consists of at most $n-1$ agents, and hence UA implies that

$$\Pi(\hat{\theta}_j, \theta_{-j} \mid f, y) \leq (n-2)V + (V + v^0).$$

This further implies that

$$\Pi^*(\hat{\theta}_j, \theta_{-j}) - \Pi(\hat{\theta}_j, \theta_{-j} \mid f, y) \geq V \quad \text{for any } \theta \in \Theta^2 \text{ and } j \in J(\theta). \tag{6}$$

We then have from (5) and (6),

$$\begin{aligned}
&\Pi^* - \Pi(f, y) \\
&\geq \sum_{\theta \in \Theta^1} \Pr(\theta) \left\{\Pi^*(\theta) - \Pi(\theta \mid f, y)\right\} \\
&+ \sum_{\theta \in \Theta^2} \sum_{j \in J(\theta)} \Pr(\hat{\theta}_j, \theta_{-j}) \left\{\Pi^*(\hat{\theta}_j, \theta_{-j}) - \Pi(\hat{\theta}_j, \theta_{-j} \mid f, y)\right\} \\
&\geq \Pr(\theta \in \Theta^1)\, V + \sum_{\theta \in \Theta^2} \sum_{j \in J(\theta)} \Pr(\hat{\theta}_j, \theta_{-j})\, V \\
&\geq \delta V(|\Theta^1| + |\Theta^2|) = \kappa.
\end{aligned}$$

Since (f, y) is an arbitrary UE mechanism, the conclusion follows. □

Proof of Proposition 3. To see that Γ^s is SP, note first that agent i_1 has no incentive to misreport since the allocation $(f(\theta), y(\theta))$ is independent of his report. Take any $i \neq i_1$.

1) If $i \rightsquigarrow_\theta i_1$, his subsidy equals $y_i(\theta) = \frac{\varepsilon}{n}$ and hence his payoff is given by

$$u_i(f(\theta), \theta_i) + y_i(\theta) = v^K + \frac{\varepsilon}{n} > 0,$$

where $K = |G_i(f(\theta_i, \theta_{-i}), \theta_i)|$ is the number of treated agents under (θ_i, θ_{-i}) who have spillovers on i. If i reports θ_i' such that $i \not\rightsquigarrow_{(\theta_i', \theta_{-i})} i_1$, then his payoff equals just his subsidy: $y_i(\theta_i', \theta_{-i}) = \frac{\varepsilon}{n}$.

Suppose then that i reports θ_i' such that $i \rightsquigarrow_{(\theta_i', \theta_{-i})} i_1$. We will first show that for any $j \neq i$,

$$f_j(\theta_i', \theta_{-i}) = 1 \quad \Leftrightarrow \quad f_j(\theta_i, \theta_{-i}) = 1. \tag{7}$$

Suppose to the contrary that $f_j(\theta_i', \theta_{-i}) = 1$ but $f_j(\theta_i, \theta_{-i}) = 0$ for some $j \neq i$. By the definition of Γ^s, this implies that every directed path from j to the seed agent i_1 goes through i. It then follows that $j \rightsquigarrow_{(\theta_i, \theta_{-i})} i$ and also $j \rightsquigarrow_{(\theta_i', \theta_{-i})} i$ since neither of these properties depends on i's type. This however implies that $j \rightsquigarrow_{(\theta_i, \theta_{-i})} i \rightsquigarrow_{(\theta_i, \theta_{-i})} i_1$, which leads to the contradiction that $f_j(\theta_i, \theta_{-i}) = 1$. The symmetric argument switching θ_i and θ_i' then shows (7), which shows that the number of treated agents under (θ_i', θ_{-i}) who have spillovers on i also equals K: $K = |G(f(\theta_i', \theta_{-i}), \theta_i)|$. It follows that i's payoff is unchanged when he reports θ_i'.

2) If $i \not\rightsquigarrow_\theta i_1$, then his payoffs equals just his subsidy: $u_i(f(\theta), \theta_i) + y_i(\theta) = \frac{\varepsilon}{n}$. If he reports θ_i' such that $i \not\rightsquigarrow_{(\theta_i', \theta_{-i})} i_1$, then his payoff is unchanged. If he reports θ_i' such that $i \rightsquigarrow_{(\theta_i', \theta_{-i})} i_1$, then his payoff is unchanged and given by $\frac{\varepsilon}{n}$. In either case, i has no incentive to misreport.

We have hence shown that Γ^s is SP. To see that Γ^s is UA, note that $a_i = 1$ is a strictly dominant action for $i = i_1$, and $a_i = 1$ is iteratively strictly dominant for any i such that $i \rightsquigarrow_\theta i_1$. □

Proof of Proposition 4. For any $i_1 \in I$, consider the single-seed mechanism Γ^s based on i_1, and let $J \subset I \setminus \{i_1\}$ be any subset of agents other than i_1. Define Θ_J to be the set of type profiles such that agent $i \neq i_1$ is strongly connected to i_1 if and only if $i \in J$:

$$\Theta_J = \{\theta \in \Theta : i \rightsquigarrow_\theta i_1 \Leftrightarrow i \in J\}.$$

Since $(X_{ij})_{i \neq j}$ is independent, by Theorem 4.2 of McDiarmid (1981), for any $J \subset I \setminus \{i_1\}$, the probability $\Pr(\theta \in \Theta_J)$ is independent of the specification of the joint distribution of (X_{ij}, X_{ji}) $(i \neq j)$and depends only on the marginal distribution $\Pr(X_{ij} = 1) = p$. Since the expected revenue $\Pi(\Gamma^s)$ from Γ^s is given by

$$\Pi(\Gamma^s) = \sum_{J \subset I \setminus \{i_1\}} \Pr(\theta \in \Theta_J) \, |J| V + (V + v^0) - \varepsilon,$$

it depends only on p and is independent of ρ. □

Proof of Lemma 5. When $\rho = 1$, X is identified with an undirected graph G and $\theta \in \Theta^{\rightsquigarrow i_1}$ is implied by the connectedness of G, which is the standard Erdős-Rényi type random graph. With $\Pr(X_{ij} = 1) = p$ fixed, the probability that G is connected approaches one as $n \to \infty$ (e.g. Diestel, 2000, p. 239). By Proposition 4, then we also have $\Pr(\theta \in \Theta^{\rightsquigarrow i_1}) \to 1$ as $n \to 1$ even when $\rho < 1$. □

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