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EFFECTS OF TRADE LIBERALIZATION WITH HETEROGENEOUS FIRMS UNDER STAGNATION

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Effects of trade liberalization with heterogeneous firms under stagnation*

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Abstract

We investigate the effect of trade liberalization in a stagnant economy characterized by firm-level productivity heterogeneity. By incorporating Melitz's (2003) framework into a model of economic stagnation driven by insufficient aggregate demand, we examine how trade opening affects employment and consumption. The outcomes deviate significantly from those of conventional models that assume full employment. Under conditions of demand deficiency, the selection effect induced by trade—whereby high-productivity firms expand and less efficient firms exit—leads to a reduction in total employment. This deterioration in labor market conditions suppresses aggregate income and consumption, suggesting that trade liberalization may inadvertently worsen economic stagnation.

Key Words: heterogeneous firms, trade liberalization, demand shortage, unemployment, stagnation

***JEL* Classification**: E24, E31, F13, F41, J20

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1 Introduction

Firms exhibit significant heterogeneity in productivity, and a key feature of this heterogeneity is that only high-productivity firms tend to enter export markets in addition to the domestic market. Melitz (2003) shows that trade liberalization leads to the exit of low-productivity firms while allowing more efficient firms to survive. Through this selection effect, globalization intensifies market competition and facilitates a more efficient allocation of resources. As discussed in Melitz and Redding (2014), the gains from trade in models with heterogeneous firms are larger than those in Krugman's (1980) model, which assumes homogeneous firms.

Whether these theoretical results continue to hold when aggregate demand is insufficient—that is, under economic stagnation—remains unclear. This issue is particularly relevant for advanced economies, most notably Japan, which has experienced prolonged stagnation over the past few decades characterized by a liquidity trap and persistent demand shortages. In such an environment, the exit of less productive firms—although theoretically efficient—may, contrary to standard predictions, exacerbate involuntary unemployment and suppress aggregate consumption.

Recent research has begun to explore demand-side mechanisms in models with heterogeneous firms. For example, Bilbiie and Melitz (2022) develop a closed-economy framework in which firm entry and exit interact with aggregate demand through what they call an entry–exit multiplier. In their model, negative supply shocks induce firm exit, which reduces product variety and thereby depresses aggregate demand. While their analysis highlights an important demand-side channel in economies with heterogeneous firms, it does not focus on persistent demand deficiency or economic stagnation. Moreover, their framework abstracts from international trade and therefore does not address how trade liberalization interacts with demand constraints.

Against this background, this paper examines whether the standard selection mechanism emphasized in heterogeneous-firm trade models continues to improve macroeconomic outcomes in an economy characterized by demand shortages. In particular, we ask whether the firm selection induced by trade openness promotes aggregate demand and employment in a stagnant economy, and whether trade liberalization—implemented through a reduction in transportation costs—necessarily increases consumption.

These questions are also motivated by empirical evidence suggesting that globalization may have adverse effects on labor market outcomes in some settings. For example, studies such as Autor et al. (2013) and Dix-Carneiro and Kovak (2019) document reductions in employment and working hours in regions exposed to increased import competition.

To address these questions, we incorporate a Melitz-type heterogeneous-firm structure into a macroeconomic framework characterized by demand-driven stagnation. In the model, unemployment arises from insufficient aggregate demand. Households supply labor inelastically when employed, but realized working hours fall short of potential working hours when households exhibit a sufficiently strong preference for wealth accumulation. When the desire to accumulate wealth is strong, households reduce consumption demand, generating a persistent demand shortfall in the goods market.

Our analysis shows that the effects of trade liberalization differ fundamentally depending on whether the economy operates under full employment or demand shortage. The key mechanism is that firm selection raises potential productivity and expands supply, but when aggregate demand is constrained this generates a supply–demand imbalance that creates deflationary pressure. Under full employment, trade liberalization increases consumption by raising average productivity, consistent with the standard predictions of heterogeneous-firm trade models. By contrast, when demand is insufficient, the same productivity improvements generate a supply–demand imbalance in the goods market. As a result, trade liberalization creates deflationary pressure, reduces aggregate consumption, and worsens employment outcomes. In this sense, globalization can give rise to an outcome similar to the paradox of toil emphasized in stagnation models.

Related literature: This paper contributes to several strands of literature. First, it relates to studies that analyze unemployment through labor market frictions, such as the search-and-matching framework developed by Pissarides (2000). While recent work has incorporated labor market frictions into open-economy models (e.g., Moore and Ranjan, 2005; Helpman et al., 2010), these studies focus primarily on supply-side frictions and typically assume that goods markets clear through price adjustments. By contrast, our analysis emphasizes demand shortages and endogenous underemployment.

Second, our framework builds on the concept of wealth preference introduced by Ono (1994). The implications of wealth preferences for secular stagnation have been examined extensively in the literature. Notable examples in closed-economy settings include Michau (2018), Illing et al. (2018), Michaillat and Saez (2022), Hashimoto et al. (2023), and Matsuzaki and Ono (2025). Several open-economy stagnation models are also related to our study. For instance, Ono (2014) and Hashimoto (2015) analyze the effects of trade liberalization through tariff changes in a two-country framework, while Hashimoto et al. (2024) examine the relationship between globalization and employment using a Ricardian model with a continuum of goods. Johdo (2008) examines the economic integration under stagnation in a Krugman-style monopolistic competition framework.

Despite these contributions, existing studies do not account for the firm-level selection and market exit triggered by trade liberalization because they do not incorporate the firm heterogeneity introduced by Melitz (2003). By combining heterogeneous-firm trade theory with a stagnation framework, this paper provides a new perspective on how globalization interacts with demand constraints and labor market outcomes.

The remaining paper is organized as follows. To clarify the mechanism through which demand shortages interact with firm selection, we first analyze a closed-economy benchmark before introducing international trade. Section 2 develops this benchmark model. Section 3 extends the framework to an open economy and examines how trade liberalization—modeled as a reduction in transport costs—affects macroeconomic outcomes. Section 4 concludes.

2 The model

2.1 Consumer

Let L denote the number of consumers. Each consumer obtains instantaneous utilities $u(c)$ by consuming the consumption index c, defined below, and $v(m)$ by holding real money balances m. Then, a consumer's lifetime utility is given by:

$$U = \int_0^\infty (u(c) + v(m)) \exp(-\rho t)\mathrm{d}t, \tag{1}$$

where $\rho > 0$ is the subjective time preference rate. The utility of consumption and real money holdings satisfy: $u'(c) > 0, u''(c) < 0, v'(m) > 0$ and $v''(m) \leq 0$.

Following Ono (2001), assume that the marginal utility of money does not go down to zero but only to a positive value β, in which $v'(m)$ is bounded away from zero: $v'(\infty) = \beta > 0$, but $u'(c)$ satisfies the Inada condition: $u'(0) = \infty$ and $u'(\infty) = 0$. These properties play a key role in creating persistent demand shortages (see section 2.3).

The consumption index is defined by:

$$c = \left(\int_{\omega \in \Omega} c(\omega)^{\frac{\sigma-1}{\sigma}} \mathrm{d}\omega \right)^{\frac{\sigma}{\sigma-1}},$$

where Ω is the set of available varieties of goods, $c(\omega)$ is the demand for variety $\omega \in \Omega$, and $\sigma > 1$ represents the elasticity of substitution between varieties. For any given level of total expenditure E, the household minimizes the cost of attaining the consumption index c. Then, the corresponding consumer price index P is given by $P \equiv \left(\int_{\omega\in\Omega} P(\omega)^{1-\sigma} \mathrm{d}\omega \right)^{\frac{1}{1-\sigma}}$, where $P(\omega)$ represents the nominal price of variety ω. The total expenditure is expressed as $E = Pc = \int_{\omega\in\Omega} P(\omega)c(\omega)\mathrm{d}\omega$.

From the first-order conditions of this static optimization problem, the optimal demand for each variety ω is derived as

$$c(\omega) = p(\omega)^{-\sigma} c,$$

where $p(\omega) \equiv P(\omega)/P$ is the real price of variety ω.

Each consumer is endowed with one unit of labor time. Then, each consumer receives wage income wx, where $x \in [0, 1]$ is the actual employment of labor per capita. This is because the actual employment is determined on the short side of the labor market. Each consumer allocates their total asset a between interest-bearing bonds b and real money balances m: $a = b + m$. The flow budget constraint in real terms is given by

$$\dot{a} = ra + wx - c - Rm, \tag{2}$$

where r is the real interest rate, R is the nominal interest rate, and a dot denotes time differentiation.[1]

The household chooses the paths of $\{c, m\}$ to maximize (1) subject to (2). The optimality conditions yield the following Keynes–Ramsey rule and the money demand condition:

$$\eta\frac{\dot{c}}{c} + \rho + \pi = R = \frac{v'(m)}{u'(c)}, \tag{3}$$

where $\eta \equiv -u''(c)c/u'(c)$ is the elasticity of marginal utility with respect to consumption, and $\pi \equiv \dot{P}/P$ is the inflation rate. The transversality condition is satisfied when $\lim_{t\to\infty} u'(c)a\exp(-\rho t) = 0$.

2.2 Firm

Under monopolistic competition, each firm produces a differentiated good y using the labor as the fixed f_D and variable l_y inputs. Following Melitz and Redding (2014), at every point in time, firms must pay a fixed entry cost and then draw their productivity. The fixed amount of the labor required for entry is f_E. After entry, productivity φ is realized according to probability density function $g(\varphi)$ and cumulative distribution function $G(\varphi)$.

[1] The nominal flow budget equation is $\dot{A} = RPb + Wx - E$, where $A(\equiv Pa)$ denotes total asset holdings. Using $a = b + m$ and $r \equiv R - \pi$, where $\pi \equiv \dot{P}/P$ is the inflation rate, we obtain (2).

The production technology of firm with φ is expressed as

$$y(\varphi) = \varphi l_y,$$

and the profit of the firm is $\kappa(\varphi) = p(\varphi)y(\varphi) - w\left(y(\varphi)/\varphi + f_D\right)$. The aggregate demand for $y(\varphi)$ is:

$$y(\varphi) = c(\varphi)L = p(\varphi)^{-\sigma}cL. \tag{4}$$

Then, profit maximization problem implies that a firm with productivity φ sets its price as a constant markup over marginal cost:

$$p(\varphi) = \frac{\sigma}{\sigma-1}\frac{1}{\varphi}w. \tag{5}$$

Let φ^* denote the cutoff level of productivity level defined in the same manner as in Melitz (2003): $\kappa(\varphi^*) = 0$. Using the cutoff condition, we obtain the profit of firm with φ as follows:

$$\kappa(\varphi) = \frac{1}{\sigma}\left(\frac{\sigma-1}{\sigma}\frac{\varphi}{w}\right)^{\sigma-1} cL - wf_D = wf_D\left[\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} - 1\right]. \tag{6}$$

The free entry condition requires that the fixed entry cost equals the sum of the expected net profits:

$$wf_E = \int_{\varphi\geq\varphi^*} \kappa(\varphi)g(\varphi)\mathrm{d}\varphi. \tag{7}$$

Substituting (6) into (7) gives the φ^* as follows:

$$f_E = f_D J(\varphi^*) \quad \Rightarrow \quad \varphi^* = \varphi^*(\underbrace{f_E}_{-}, \underbrace{f_D}_{+}), \tag{8}$$

where $J(\varphi^*) \equiv \int_{\varphi\geq\varphi^*}\left[\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} - 1\right] g(\varphi)\mathrm{d}\varphi$ and $J'(\varphi^*) < 0$.

Using the zero-profit firm condition $\kappa(\varphi^*) = 0$ with (4) and (5), we obtain the following relationship:

$$w = \frac{(\sigma-1)^{\frac{\sigma-1}{\sigma}}}{\sigma}\left(\frac{L}{f_D}\right)^{\frac{1}{\sigma}}\varphi^{*\frac{\sigma-1}{\sigma}}c^{\frac{1}{\sigma}}. \tag{9}$$

2.3 Equilibrium

The asset markets adjust perfectly and instantaneously to ensure that the following conditions hold continuously:

$$\text{Money Market}: mL = \frac{\bar{M}}{P},$$

$$\text{Bond Market}: bL = 0,$$

where $\bar{M}$ represents the nominal money supply, which is assumed to be constant. Following the free-entry condition, the aggregate firm value becomes zero, resulting in zero supply of stock value. Using the asset markets, (2) can be rewritten as follows:

$$c = wx. \tag{10}$$

In our model, the value of the gross domestic product (GDP) is equivalent to $wx(= c)$.

In the labor market, the nominal wage adjustment is perfect if full employment ($x = 1$) prevails but sluggish if there is unemployment. In stagnation ($x < 1$), the wage adjustment is:[2]

$$\frac{\dot{W}}{W} = \alpha(x-1), \tag{11}$$

where $x \equiv L^d/L$ is the employment rate and L^d is the aggregate labor demand.[3]

From (3) and money market, using (9) and (10), we obtain an autonomous dynamic system with respect to c and m. In the steady state c stays constant. We relegate the stability analysis of these dynamics into Appendix B.

[2]This asymmetry in the inflation process is a key feature of recent models of stagnation, including Schmitt-Grohé and Uribe (2017), Michau (2018), Illing et al. (2018), and Hashimoto et al. (2023).

[3]See Appendix A for an explicit derivation of the aggregate labor demand in a closed economy.

Full employment ($x = 1$):

We start by considering the benchmark case, in which full employment prevails ($x = 1$). From (10) with (9), when the consumption level at full employment in a closed economy is denoted as c_a^F, the equilibrium value can be described as

$$c_a^F = \frac{\sigma - 1}{\sigma} \sigma^{\frac{\sigma}{\sigma - 1}} \left(\frac{L}{f_D} \right)^{\frac{1}{\sigma - 1}} \varphi^*. \tag{12}$$

Stagnation ($x < 1$):

We next consider the case of aggregate demand shortage. With unemployment, the nominal wage, nominal prices, and price level continue to decline according to

$$\pi \equiv \frac{\dot{P}}{P} = \frac{\dot{W}}{W} = \alpha(x - 1), \tag{13}$$

using (11), since real wage ($w \equiv W/P$) is constant. A falling price level continuously expands real balances. However, a continuous increase in real balances may fail to stimulate consumption spending and guide the economy to full employment if the desire for real money balances is insatiable. In this case, (3) may be invalid for any m. Specifically, there cannot be a steady-state equilibrium with full employment if

$$\rho < \frac{\beta}{u'(c_a^F)} \left(< \frac{v'(m)}{u'(c_a^F)} \quad \text{for any } m \right). \tag{14}$$

In this condition, the economy's production capacity exceeds the upper limit of consumption; then, the only steady-state equilibrium is one with a persistent shortage of demand. As proven by Ono (2001), when $0 < x < 1$, nominal price P continues to decline, and real money balances continue to increase, causing $v'(m)$ to converge to β. Consequently, in this economy, the interest rate satisfies $R = \beta/u'(c)$, implying that the economy is in a liquidity trap.[4]

[4]Empirical support for this property can be found in Ono (1994, pp. 34–38), who employs GMM, and Ono et al. (2004), who utilize parametric and nonparametric methods. More recently, Akesaka et al. (2024) provide further evidence using a nationally representative survey of Japanese households.

Thus, in the steady state ($\dot{c} = 0$), from (3) and (13), we obtain

$$\frac{\beta}{u'(c)} = \rho + \alpha(x - 1), \tag{15}$$

which leads to the following relationship:

$$\left[-u''(c)\beta/(u'(c))^2\right] \mathrm{d}c = \alpha \mathrm{d}x.$$

Therefore, employment rate x moves in the same direction as the consumption c does under stagnation.

Let c_a^S denote the consumption level at the stagnation equilibrium in a closed economy. As such, from (15) with (9) and (10), we obtain the following equation:

$$\frac{\beta}{u'(c_a^S)} = \rho + \alpha \left(x(c_a^S) - 1 \right), \tag{16}$$

$$\text{where} \quad x(c_a^S) \equiv \frac{\sigma}{(\sigma - 1)^{\frac{\sigma-1}{\sigma}}} \frac{1}{(\varphi^*)^{\frac{\sigma-1}{\sigma}}} \left(\frac{f_D}{L} \right)^{\frac{1}{\sigma}} {c_a^S}^{\frac{\sigma-1}{\sigma}}.$$

Thus, the steady-state level of consumption satisfies the intersection point of the left-hand side and the right-hand side of (16), as illustrated in Figure 1. In the steady state in which c is smaller than c_a^F, under (14), the left-hand side is located above the right-hand side; in order for c_a^S to exist, the following inequality must be satisfied:

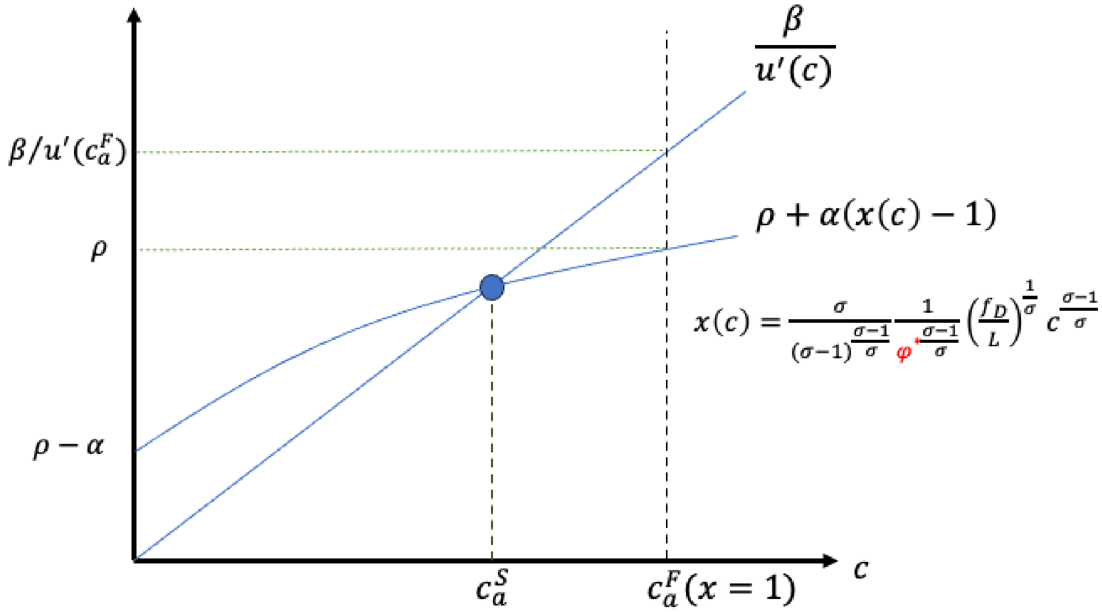
$$\frac{\beta}{u'(c_a^F)} > \rho > \alpha. \tag{17}$$

As is also evident from Figure 1, under (17), the left-hand side is more inclined than the right-hand side at the intersection point.[5] Then, from (12) and (16), we obtain the following lemma:

Lemma. *An increase in the productivity cutoff φ^* (and, thus, average productivity) enhances consumption under full employment but reduces consumption and the employment*

[5]This coincides with the dynamic stability condition. See Appendix B.

Figure 1: Equilibrium



rate under stagnation.

3 Open economy

In this part of the analysis, we extend from a closed economy to an open economy. We assume the existence of a completely symmetric foreign country. The setting where productivity is realized after paying entry costs is the same as in the closed economy. Sales to the domestic market require variable and fixed costs f_D. Exporting to foreign markets requires fixed costs f_X and iceberg-type transportation costs $\tau \geq 1$.

Let φ_D denote the productivity cutoff at which domestic market profit (κ_D) is zero in an open economy: $\kappa_D(\varphi_D) = 0$. Then, for the technology $l_{yD} = y_{yD}/\varphi$, similarly to a closed economy, the monopoly price is given by $p_D(\varphi) = \frac{\sigma}{\sigma-1}\frac{1}{\varphi}w$, and the domestic market profit of firm with productivity φ can be expressed as follows:

$$\kappa_D(\varphi) = \frac{1}{\sigma}\left(\frac{\sigma-1}{\sigma}\frac{\varphi}{w}\right)^{\sigma-1} cL - wf_D = wf_D\left[\left(\frac{\varphi}{\varphi_D}\right)^{\sigma-1} - 1\right]. \tag{18}$$

There exists a symmetric foreign market where foreign households' total consumption is also cL and prices are identical. The labor required for export for the firm with productivity φ is

$$l_{yX} = \frac{\tau y_X}{\varphi}.$$

Then, the export market profit (κ_X) is $\kappa_X(\varphi) = p_X y_X - w\left(\tau y_X/\varphi + f_X\right)$. Using the good market $y(\varphi) = p(\varphi)^{-\sigma} cL$, the markup price is given by

$$p_X(\varphi) = \frac{\sigma}{\sigma - 1}\frac{\tau}{\varphi}w. \tag{19}$$

Let φ_X denote the productivity cutoff at which the foreign market profit is zero in an open economy: $\kappa_X(\varphi_X) = 0$. Then, the export market profit of firm with productivity φ can be expressed as follows:

$$\kappa_X(\varphi) = \frac{1}{\sigma}\left(\frac{\sigma - 1}{\sigma}\frac{\varphi}{\tau w}\right)^{\sigma-1} cL - wf_X = wf_X\left[\left(\frac{\varphi}{\varphi_X}\right)^{\sigma-1} - 1\right]. \tag{20}$$

From the first equality of the two equations $\kappa_D(\varphi_D) = 0$ in (18) and $\kappa_X(\varphi_X) = 0$ in (20), we obtain the following relationship:

$$\left(\frac{\varphi_X}{\tau\varphi_D}\right)^{\sigma-1} = \frac{f_X}{f_D}. \tag{21}$$

Assuming $\tau^{\sigma-1} f_X > f_D$, we have $\varphi_X > \varphi_D$. Under this setting, firms must have sufficiently high productivity to become exporters.

The free entry condition in an open economy is given by

$$wf_E = \int_{\varphi \geq \varphi_D} \kappa_D(\varphi) g(\varphi) \mathrm{d}\varphi + \int_{\varphi \geq \varphi_X} \kappa_X(\varphi) g(\varphi) \mathrm{d}\varphi. \tag{22}$$

Substituting (18) and (20) into (22) yields the following relationship:

$$f_E = J(\varphi_D) f_D + J(\varphi_X) f_X, \tag{23}$$

where $J(\varphi_i) \equiv \int_{\varphi \geq \varphi_i} \left[\left(\frac{\varphi}{\varphi_i} \right)^{\sigma-1} - 1 \right] g(\varphi) \mathrm{d}\varphi$ and $J'(\varphi_i) < 0$ for $i = (D, X)$.

Equations (21) and (23) determine (φ_D, φ_X):

$$\varphi_D = \varphi_D(\underbrace{\tau}_{-}, \underbrace{f_E}_{-}, \underbrace{f_D}_{+}, \underbrace{f_X}_{-}), \quad \varphi_X = \varphi_X(\underbrace{\tau}_{+}, \underbrace{f_E}_{-}, \underbrace{f_D}_{-}, \underbrace{f_X}_{+}). \tag{24}$$

See Appendix C for explicit expressions of the above properties.

3.1 Comparison of closed and open economy

First, let us compare the cutoff productivity levels of domestic firms in closed (φ^*) and open (φ_D) economies. From (8) and (23), we verify the following relationship:

$$\varphi^* < \varphi_D \ (< \varphi_X). \tag{25}$$

Based on the Melitz (2003) model, this ranking confirms that average productivity improves when an economy opens up to trade.

Next, using the cutoff level condition, $\kappa_D(\varphi_D) = p(\varphi_D)y_D - w\left(y_D/\varphi_D + f_D\right) = 0$, $p_D(\varphi_D) = \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}w$, and $y_D(\varphi_D) = p_D(\varphi_D)^{-\sigma}cL$, we obtain the following relationship:

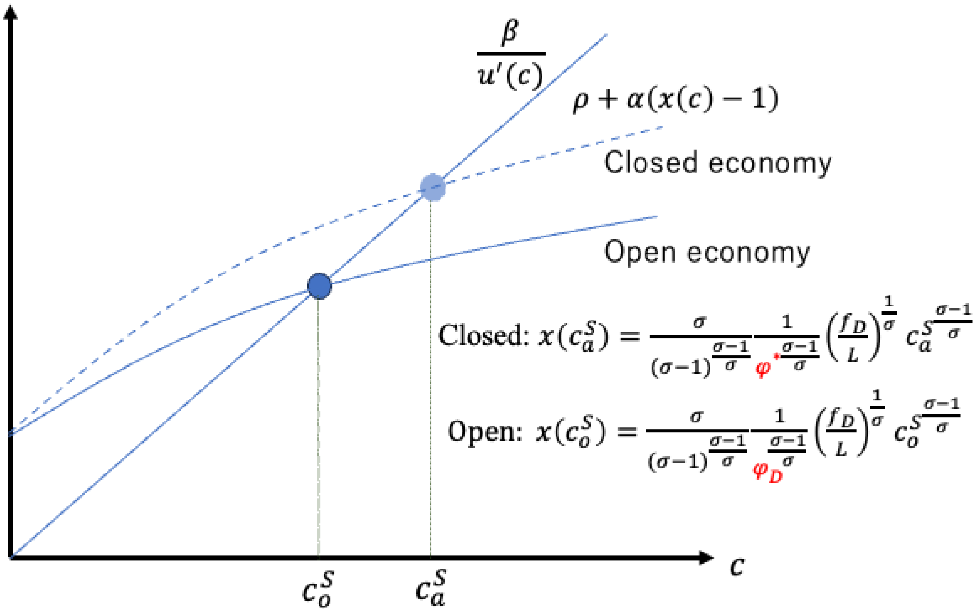
$$w = \frac{(\sigma-1)^{\frac{\sigma-1}{\sigma}}}{\sigma} \left(\frac{L}{f_D} \right)^{\frac{1}{\sigma}} \varphi_D^{\frac{\sigma-1}{\sigma}} c^{\frac{1}{\sigma}}. \tag{26}$$

On the one hand, using (10) and (26) when the consumption level at full employment ($x = 1$) in an open economy is denoted as c_o^F, the equilibrium value is given by

$$c_o^F = \frac{\sigma-1}{\sigma} \sigma^{\frac{\sigma}{\sigma-1}} \left(\frac{L}{f_D} \right)^{\frac{1}{\sigma-1}} \varphi_D. \tag{27}$$

On the other hand, given the consumption level under stagnation ($x < 1$) in an open economy as c_o^S, from (15) with (10) and (26), the following equation holds as a condition

Figure 2: Comparison of a closed and an open economy ($\varphi^* < \varphi_D$)



for equilibrium:[6]

$$\frac{\beta}{u'(c_o^S)} = \rho + \alpha\left(x(c_o^S) - 1\right), \tag{28}$$

$$\text{where} \quad x(c_o^S) \equiv \frac{\sigma}{(\sigma-1)^{\frac{\sigma-1}{\sigma}}} \frac{1}{\varphi_D^{\frac{\sigma-1}{\sigma}}} \left(\frac{f_D}{L}\right)^{\frac{1}{\sigma}} c_o^{S\,\frac{\sigma-1}{\sigma}}.$$

Then, using Lemma, (25), (27), and (28), as shown in Figure 2, we obtain the following proposition:

Proposition 1. *By increasing average productivity, the shift from a closed to an open economy promotes higher consumption under full employment but, conversely, suppresses consumption and employment rate under stagnation.*

3.2 Trade liberalization

The effects of trade liberalization (a decrease in τ) influence the macroeconomy through their effect on the cutoff level φ given in (24). A reduction in transport costs allows more

[6]See Appendix D for an explicit derivation of aggregate labor demand in an open economy.

firms to export; however, by intensifying market competition, it also forces less productive firms to exit, thereby raising the productivity cutoff. Consequently, lower transport costs lead to an overall increase in aggregate productivity:

$$\tau \downarrow \quad \Rightarrow \quad \varphi_D \uparrow .$$

Therefore, as is clear from (27) and (28), we obtain the following proposition:

Proposition 2. *While trade liberalization through lower transport costs results in higher consumption levels under full employment, it conversely reduces consumption and employment rate under stagnation.*

4 Conclusion

We extended Melitz's (2003) heterogeneous firm model to a stagnant economy framework in order to analyze the effects of trade liberalization under stagnation.

We found that, in conventional models that assume full employment, trade liberalization enhances aggregate productivity and increases consumption through the efficient reallocation of resources. However, in a stagnant economy characterized by chronic demand shortages, the productivity gains from trade induce the exit of low-productivity firms without a corresponding expansion in aggregate labor demand, thereby exacerbating involuntary unemployment.

This deterioration in labor market conditions further suppresses aggregate demand, potentially leading to a decline in both consumption and GDP—a result that aligns with the "paradox of toil" in stagnation models. These findings underscore the necessity of considering macroeconomic regimes, particularly the presence of demand-side constraints, when evaluating trade policies. Our analysis suggests that, in the absence of full employment, the effects of trade liberalization may differ fundamentally from the predictions of standard trade theory.

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Appendix

A Aggregate labor demand in a closed economy

Before discussing aggregate labor demand, we first define average productivity following Melitz (2003):

$$\tilde{\varphi} \equiv \left[\int_{\varphi \geq \varphi^*} \varphi^{\sigma-1} \frac{g(\varphi)}{1-G(\varphi^*)} \mathrm{d}\varphi\right]^{\frac{1}{\sigma-1}} \;\Rightarrow\; \tilde{\varphi} = \tilde{\varphi}(\varphi^*).$$

Using the price index $P \equiv \left(\int_{\omega\in\Omega} P(\omega)^{1-\sigma}\mathrm{d}\omega\right)^{\frac{1}{1-\sigma}}$ with $p(\omega) \equiv P(\omega)/P$ of (5), we have the following equation:

$$1 = \left(\int_{\omega\in\Omega} p(\omega)^{1-\sigma}\mathrm{d}\omega\right)^{\frac{1}{1-\sigma}} = \left(\int_{\omega\in\Omega} \left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi}w\right)^{1-\sigma} \mathrm{d}\omega\right)^{\frac{1}{1-\sigma}}$$

$$= \frac{\sigma}{\sigma-1} w \left(\int_{\omega\in\Omega} \varphi^{\sigma-1}\mathrm{d}\omega\right)^{\frac{1}{1-\sigma}} = \frac{\sigma}{\sigma-1} w \left(N\int_{\varphi\geq\varphi^*} \varphi^{\sigma-1} \frac{g(\varphi)}{1-G(\varphi^*)}\mathrm{d}\varphi\right)^{\frac{1}{1-\sigma}}.$$

Then, we obtain

$$w = \frac{\sigma-1}{\sigma}\tilde{\varphi}(\varphi^*)N^{\frac{1}{\sigma-1}}. \tag{29}$$

From the profit of a firm with the average productivity $\tilde{\varphi}$, substituting (29) into (6) yields the following relationship:

$$\tilde{\kappa} = \kappa(\tilde{\varphi}) = \frac{1}{\sigma}\frac{1}{N}cL - wf_D \;\Rightarrow\; N = \frac{1}{\sigma}\frac{cL}{\tilde{\kappa} + wf_D}, \tag{30}$$

where

$$\tilde{\kappa} \equiv \left[\int_{\varphi\geq\varphi^*} \kappa(\varphi)\frac{g(\varphi)}{1-G(\varphi^*)}\mathrm{d}\varphi\right].$$

Aggregate labor demand L^d is given by

$$L^d \equiv \int_{\omega\in\Omega} (l_y(\omega) + f_D)\mathrm{d}\omega + N_E f_E, \tag{31}$$

where N_E denotes the number of entering firms paying the entry cost. The relationship between the number of operating firms N and the number of entries N_E is $N = (1 - G(\varphi^*))N_E$.

Using the definition of $\tilde{\kappa}$, (7) can be rewritten as:

$$wf_E = \int_{\varphi\geq\varphi^*} \kappa(\varphi)g(\varphi)\mathrm{d}\varphi = (1-G(\varphi^*))\tilde{\kappa}, \tag{32}$$

and using $\kappa(\varphi) = p(\varphi)y(\varphi) - w(y(\varphi)/\varphi + f_D)$ with (5), the following condition holds:

$$l_y(\varphi) + f_D = \frac{1}{w}\left((\sigma-1)\kappa(\varphi) + \sigma w f_D\right). \tag{33}$$

Substituting (32) and (33) into (31) gives

$$L^d = \frac{N}{w}\sigma(\tilde{\kappa} + w f_D). \tag{34}$$

Using (30) and (34), we have

$$L^d = \frac{c}{w}L. \tag{35}$$

Equation (35) yields the employment rate as follows, which is equivalent to Equation (10):

$$x\left(\equiv \frac{L^d}{L}\right) = \frac{c}{w}.$$

B Dynamic stability

From $w \equiv W/P$, (9) and (11), the time derivative of w can be expressed as follows:

$$\frac{\dot{w}}{w} = \frac{\dot{W}}{W} - \frac{\dot{P}}{P} = \alpha(x-1) - \pi,$$

$$\frac{\dot{w}}{w} = \frac{1}{\sigma}\frac{\dot{c}}{c}.$$

Then, we obtain the inflation rate π is given by

$$\pi = \alpha(x-1) - \frac{1}{\sigma}\frac{\dot{c}}{c}. \tag{36}$$

Now, we examine the stability of the dynamics. Let $\xi \equiv 1/m$, substituting (36) into (3) and money market yield

$$\dot{c} = H_1(c,\xi) \equiv \frac{c}{\eta - \frac{1}{\sigma}}\left[\frac{v'(1/\xi)}{u'(c)} - \rho - \alpha(x(c)-1)\right],$$

$$\dot{\xi} = \xi\pi = H_2(c,\xi) \equiv \xi\left[\alpha(x(c)-1) - \frac{1}{\sigma}\frac{1}{c}H_1(c,\xi)\right],$$

where

$$x(c) \equiv \frac{\sigma}{(\sigma-1)^{\frac{\sigma-1}{\sigma}}}\frac{1}{(\varphi^*)^{\frac{\sigma-1}{\sigma}}}\left(\frac{f_D}{L}\right)^{\frac{1}{\sigma}} c^{\frac{\sigma-1}{\sigma}},$$

using (9) and (10). We assume $\eta > 1/\sigma$.[7]

Linearizing them in the neighbourhood of the steady state values $(\bar{c}, \bar{\xi})$ gives

$$\begin{pmatrix} \dot{c} \\ \dot{\xi} \end{pmatrix} = \begin{bmatrix} H_{1c} & H_{1\xi} \\ H_{2c} & H_{2\xi} \end{bmatrix} \begin{pmatrix} c - \bar{c} \\ \xi - \bar{\xi} \end{pmatrix},$$

where

$$H_{1c}\,(\equiv \partial\dot{c}/\partial c) = \frac{c}{\eta - \frac{1}{\sigma}} \left[\frac{-v'(1/\xi)u''(c)}{(u'(c))^2} - \alpha x'(c) \right],$$

$$H_{1\xi}\,(\equiv \partial\dot{c}/\partial\xi) = \frac{c}{\eta - \frac{1}{\sigma}} \frac{-v''(1/\xi)}{u'(c)\xi^2},$$

$$H_{2c}\left(\equiv \partial\dot{\xi}/\partial c\right) = \xi \left[\alpha x'(c) - \frac{1}{\sigma}\frac{1}{c} H_{1c} \right],$$

$$H_{2\xi}\left(\equiv \partial\dot{\xi}/\partial\xi\right) = \alpha(x-1) - \xi\frac{1}{\sigma}\frac{1}{c} H_{1\xi}.$$

Since the characteristic equation is

$$\Lambda(\lambda) = \lambda^2 - (H_{1c} + H_{2\xi})\lambda + (H_{1c}H_{2\xi} - H_{1\xi}H_{2c}),$$

eigenvalues λ_1 and λ_2 satisfy

$$\lambda_1\lambda_2 = H_{1c}H_{2\xi} - H_{1\xi}H_{2c} = H_{1c}\alpha(x-1) - H_{1\xi}\xi\alpha x'(c)$$

$$= \frac{c\alpha(x-1)}{\eta - \frac{1}{\sigma}} \left[\frac{-v'(1/\xi)u''(c)}{(u'(c))^2} - \alpha x'(c) \right] + \frac{c\alpha x'(c)}{\eta - \frac{1}{\sigma}} \frac{v''(1/\xi)}{u'(c)\xi}. \tag{37}$$

If full employment is reached in the steady state, $x = 1$ and hence the first term of (37) is zero. Since and $v''(\cdot) < 0$, the second term is negative. Thus, $\lambda_1\lambda_2 < 0$, implying that one of the eigenvalues is positive and the other is negative. Since c is jumpable and $\xi(\equiv 1/m)$ is non-jumpable, the dynamic path is saddle-point stable.

If there is no full-employment steady state and persistent stagnation arises, $x < 1$ in the steady state. Therefore, $\xi(\equiv 1/m)$ approaches zero and the value given by the square brackets of the first term of (37) equals the difference between the slopes of the left-hand side and the right-hand side of (16), which is positive under the condition of (17). Thus, the first term of (37) is negative. The second term is negative since $v''(\cdot) < 0$. Therefore, (37) is negative ($\lambda_1\lambda_2 < 0$), implying that the dynamic path is saddle-point stable.

C Derivation of the properties of (φ_D, φ_X)

From (21), we have

$$\varphi_X = \tau \left(\frac{f_X}{f_D} \right)^{\frac{1}{\sigma-1}} \varphi_D.$$

[7] This holds true if $\sigma(>1)$ is sufficiently large or if η is greater than 1. See Hall (1988), Guvenen (2006), and Chiappori and Paiella (2011) for empirical estimates of the intertemporal elasticity of substitution that are consistent with $1/\eta \in (0,1)$.

Substituting the above equation into (23) gives the equilibrium of $\varphi_D = \varphi_D(\tau, f_E, f_D, f_X)$; taking the derivative with respect to each parameter yields the following relationship:

$$\frac{\mathrm{d}\varphi_D}{\mathrm{d}\tau} = \frac{1}{\Phi_D}\left[J'(\varphi_X)\frac{\varphi_X}{\tau}f_X\right] < 0,$$

$$\frac{\mathrm{d}\varphi_D}{\mathrm{d}f_E} = -\frac{1}{\Phi_D} < 0,$$

$$\frac{\mathrm{d}\varphi_D}{\mathrm{d}f_D} = \frac{1}{\Phi_D}\left[J(\varphi_D) + \frac{-J'(\varphi_X)\varphi_X f_X}{(\sigma-1)f_D}\right] > 0,$$

$$\frac{\mathrm{d}\varphi_D}{\mathrm{d}f_X} = \frac{1}{\Phi_D}\left[J(\varphi_X) + \frac{\varphi_X}{\sigma-1}J'(\varphi_X)\right] = \frac{1}{\Phi_D}\left[J(\varphi_X) - \int_{\varphi \geq \varphi_X}\left(\frac{\varphi}{\varphi_X}\right)^{\sigma-1} g(\varphi)\mathrm{d}\varphi\right] < 0,$$

where

$$\Phi_D \equiv -J'(\varphi_D)f_D - J'(\varphi_X)\frac{\varphi_X}{\varphi_D}f_X > 0.$$

In a similar manner, the equilibrium properties of $\varphi_X = \varphi_X(\tau, f_E, f_D, f_X)$ can also be derived as follows:

$$\frac{\mathrm{d}\varphi_X}{\mathrm{d}\tau} = \frac{1}{\Phi_X}\left[-J'(\varphi_D)\frac{\varphi_D}{\tau}f_D\right] > 0,$$

$$\frac{\mathrm{d}\varphi_X}{\mathrm{d}f_E} = -\frac{1}{\Phi_X} < 0,$$

$$\frac{\mathrm{d}\varphi_X}{\mathrm{d}f_D} = \frac{1}{\Phi_X}\left[J(\varphi_D) + \frac{\varphi_D}{\sigma-1}J'(\varphi_D)\right] = \frac{1}{\Phi_X}\left[J(\varphi_D) - \int_{\varphi \geq \varphi_D}\left(\frac{\varphi}{\varphi_D}\right)^{\sigma-1} g(\varphi)\mathrm{d}\varphi\right] < 0,$$

$$\frac{\mathrm{d}\varphi_X}{\mathrm{d}f_X} = \frac{1}{\Phi_X}\left[J(\varphi_X) + \frac{-J'(\varphi_D)\varphi_D f_D}{(\sigma-1)f_X}\right] > 0,$$

where

$$\Phi_X \equiv -J'(\varphi_X)f_X - J'(\varphi_D)\frac{\varphi_D}{\varphi_X}f_D > 0.$$

D Aggregate labor demand in an open economy

Let N_X denote the number of exporting firms. Then, the relationship between the number of firms for domestic market (N), number of exporting firms (N_X), and number of firms entering the market (N_E) is

$$N = (1 - G(\varphi_D))N_E \quad \text{(firms for domestic market)}$$

$$N_X = (1 - G(\varphi_X))N_E = \frac{1 - G(\varphi_X)}{1 - G(\varphi_D)}N \quad \text{(firms for foreign market)}$$

As in Appendix A, from the price index: $P \equiv \left(\int_{\omega\in\Omega} P(\omega)^{1-\sigma}\mathrm{d}\omega\right)^{\frac{1}{1-\sigma}}$ with (5) and (19), we have the following equation:

$$
\begin{aligned}
1 &= N \int_{\varphi \geq \varphi_D} p_D(\varphi)^{1-\sigma} \frac{g(\varphi)}{1-G(\varphi_D)} \mathrm{d}\varphi + N_X \int_{\varphi \geq \varphi_X} p_X(\varphi)^{1-\sigma} \frac{g(\varphi)}{1-G(\varphi_X)} \mathrm{d}\varphi \\
&= N \left[\left(\frac{\sigma-1}{\sigma} \frac{\tilde{\varphi}_D}{w} \right)^{\sigma-1} + \left(\frac{\sigma-1}{\sigma} \frac{\tilde{\varphi}_X}{\tau w} \right)^{\sigma-1} \frac{1-G(\varphi_X)}{1-G(\varphi_D)} \right], \qquad (38)
\end{aligned}
$$

where

$$
\begin{aligned}
\tilde{\varphi}_D &\equiv \left[\int_{\varphi \geq \varphi_D} \varphi^{\sigma-1} \frac{g(\varphi)}{1-G(\varphi_D)} \mathrm{d}\varphi \right]^{\frac{1}{\sigma-1}}, \\
\tilde{\varphi}_X &\equiv \left[\int_{\varphi \geq \varphi_X} \varphi^{\sigma-1} \frac{g(\varphi)}{1-G(\varphi_X)} \mathrm{d}\varphi \right]^{\frac{1}{\sigma-1}}.
\end{aligned}
$$

Then, from (38) the real wage is

$$
w = \frac{\sigma-1}{\sigma} N^{\frac{1}{\sigma-1}} \left[\tilde{\varphi}_D^{\sigma-1} + \left(\frac{\tilde{\varphi}_X}{\tau} \right)^{\sigma-1} \frac{1-G(\varphi_X)}{1-G(\varphi_D)} \right]^{\frac{1}{\sigma-1}}. \qquad (39)
$$

Aggregate labor demand in an open economy is given by

$$
L^d = \int_{\omega \in \Omega} (l_{yD} + f_D) \mathrm{d}\omega + \int_{\omega \in \Omega} (l_{yX} + f_X) \mathrm{d}\omega + N_E f_E.
$$

Following the same procedure as in Appendix A, we derive the following condition:

$$
L^d = \frac{\sigma}{w} N \left[(\tilde{\kappa}_D + w f_D) + \frac{1-G(\varphi_X)}{1-G(\varphi_D)} (\tilde{\kappa}_X + w f_X) \right], \qquad (40)
$$

where

$$
\begin{aligned}
\tilde{\kappa}_D &= \frac{1}{\sigma} \left(\frac{\sigma-1}{\sigma} \frac{\tilde{\varphi}_D}{w} \right)^{\sigma-1} cL - w f_D, \\
\tilde{\kappa}_X &= \frac{1}{\sigma} \left(\frac{\sigma-1}{\sigma} \frac{\tilde{\varphi}_X}{\tau w} \right)^{\sigma-1} cL - w f_X.
\end{aligned}
$$

Using (39) and rearranging (40), we have:

$$
L^d = \frac{c}{w} L.
$$