

**MINOR-FIRM SUPPORT
AND MACROECONOMIC OUTCOMES**

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Minor-Firm Support and Macroeconomic Outcomes*

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Abstract

The question of whether to support low-productivity firms is usually answered based on productivity because the possibility of aggregate demand shortages is assumed away. We revisit this question using a monetary general-equilibrium model that allows for persistent demand shortages, where one sector consists of Cournot oligopolists with varying levels of productivity and the rest is competitive. We find that improving the productivity of a low-productivity firm in the oligopoly sector and allowing it to remain in the market increase employment and consumption under stagnation, despite impeding the production activities of more efficient firms. This occurs because supporting a low-productivity firm reduces the economy's aggregate productivity, which narrows the deflationary gap and induces households to increase consumption. This mechanism is specific to demand-constrained economies; in the absence of aggregate demand shortages, the conventional efficiency-based prescription is restored, and low-productivity firms should be encouraged to exit. Thus, under stagnation, whether to support low-productivity firms should be judged not from an efficiency standpoint but from the perspective of stimulating consumption.

keywords: Demand shortage, Minor firm, Cournot oligopoly, Productivity improvement, Firm exit

JEL classification: E12, D43, L52

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1 Introduction

Should low-productivity firms be supported, or encouraged to exit? Governments often provide such firms with subsidies and cheap credit, arguing that this support protects employment, especially during periods of economic stagnation. In contrast, the conventional view in economics maintains that supporting low-productivity firms is costly because it prevents resources from being reallocated to more efficient producers. In line with this view, Lahiri and Ono (1988), using a partial-equilibrium Cournot oligopoly model, show that supporting a highly inefficient oligopolist reduces welfare. The literature on Schumpeterian cleansing further emphasizes that recessions induce outdated, low-productivity firms to exit, thereby raising industry-level productivity.¹ The debate over “zombie firms”—firms that survive mainly through bank forbearance—reflects the widely held concern that supporting these firms distorts resource allocation, depresses the profits of healthy firms, discourages investment, and ultimately harms the economy.²

However, this reasoning implicitly assumes that aggregate output is constrained by the economy’s production capacity. When aggregate output is instead constrained by aggregate demand—as in an economy mired in secular stagnation—the conventional logic no longer applies. Under demand shortages, the key question becomes how the presence of low-productivity firms affects aggregate consumption.

To address this issue, we analyze the effect of supporting low-productivity firms on consumption and employment using the secular stagnation model of Ono (1994, 2001), in which aggregate output is determined by aggregate demand rather than productive capacity.³ Using this model, Matsuzaki and Ono (2025) demonstrate that improvements in productivity and quality reduce consumption and employment—precisely the opposite of the conventional view. While these studies assume perfect competition, we incorporate a Cournot oligopoly sector with one relatively inefficient minor firm and more productive major firms into this type of model. We show that supporting the minor firm—either by raising its productivity or allowing it to remain in the market—decreases national product and consumption under full employment, but increases employment and consumption under stagnation.⁴ Thus, the efficiency-based prescription of culling

¹See Caballero and Hammour (1994, 1996) and Kozeniauskas et al. (2022).

²There are a lot of studies on zombie firms. See e.g. Caballero et al. (2008), McGowan et al. (2018), and Acharya et al. (2024). Antić and Ichev (2026) provide an extensive survey of the literature on zombie firms.

³See also Michau (2018), Michaillat and Saez (2021, 2022), and Hashimoto et al. (2023) for this line of research.

⁴Caballero and Hammour (1996) analyze how the interaction between business-cycle cleansing and search-matching unemployment shapes industry-level productivity dynamics. Their framework assumes exogenous de-

inefficient firms is reversed under stagnation.

The mechanism is the same one emphasized in the conventional literature, but it operates in the opposite direction under stagnation. Eliminating inefficient firms expands the full-employment national product by reallocating resources toward more efficient producers. Under full employment, national product equals consumption; hence, this expansion raises consumption, implying that minor firms should be removed, as argued by Lahiri and Ono (1988) and by the zombie-firm literature. Under stagnation, however, expanding the full-employment national product widens the deflationary gap, which depresses households' consumption. Conversely, supporting low-productivity firms reduces the full-employment national product, narrows the deflationary gap, and thereby stimulates consumption and employment. Thus, under stagnation, minor firms should be supported rather than culled.

The remainder of the paper is organized as follows. Section 2 presents the model in which one sector consists of Cournot oligopolists and the rest is competitive, and derives the key proposition linking the full-employment national product to consumption under secular stagnation. Section 3 examines the case of Cobb–Douglas preferences and analytically investigates how productivity improvements and firm exit affect the full-employment national product and consumption. Section 4 extends the analysis to CES preferences and numerically shows that essentially the same results hold over empirically relevant ranges of parameters such as the elasticity of substitution, the number of oligopolists, and the relative preference for oligopoly and competitive goods. The final section discusses the economic implications of the results.

2 The model

We consider an economy in which good x is supplied by N firms in a Cournot oligopoly, each with a different level of labor productivity. All other goods are supplied under perfect competition and are collectively denoted by y . Hence, we consider a two-sector economy consisting of a Cournot oligopoly sector and a competitive sector. Firms in both sectors use labor as the sole input. A representative household owns all firms and directly receives all their profits. For simplicity, we assume that the government keeps nominal money supply M^S constant and there

mand and focuses on frictional unemployment, without aggregate demand shortages. In contrast, our analysis explicitly incorporates aggregate demand shortages through the endogenous determination of consumption.

is no fiscal spending g ; hence, there is no tax-cum-subsidy z :

$$M^S = \text{constant}, \quad g = 0, \quad z = 0. \quad (1)$$

2.1 Households

The representative household consumes the two goods, x and y , whose real prices are p_x and p_y . Given real consumption c , it maximizes utility U :

$$\max_{x,y} U = u(x, y) \quad \text{subject to } p_x x + p_y y = c. \quad (2)$$

The utility function, $U = u(x, y)$, is assumed to be homothetic, which allows us to define a consumer price index P that is independent of real consumption c . The household's first-order optimal condition is given by

$$\frac{u_x(x, y)}{u_y(x, y)} = \frac{u_x\left(\frac{x}{y}, 1\right)}{u_y\left(\frac{x}{y}, 1\right)} = \omega \left(= \frac{p_x}{p_y} \right), \quad (3)$$

where ω is the relative price of x to y , and u_j is the partial derivative of u with respect to j ($j = x, y$). From the constraint in (2), and (3), we obtain

$$p_x(\omega)x = \delta(\omega)c, \quad p_y(\omega)y = (1 - \delta(\omega))c, \quad (4)$$

where $\delta(\omega)$ is the expenditure share of good x , and p_x , p_y , and δ depend only on ω because $u(x, y)$ is homothetic.

Substituting (4) into $u(x, y)$ yields

$$U = u\left(\frac{\delta c}{p_x}, \frac{(1 - \delta)\omega c}{p_x}\right). \quad (5)$$

The consumer price index P is defined such that changes in the relative price ω do not affect the utility level U in (5) for a given c . Consequently, U depends only on c :

$$U = U(c). \quad (6)$$

Having this property in mind, we differentiate the right-hand side of (5) with respect to ω , apply (3) to the result, set it equal to zero, and derive the following property of p_x :

$$\frac{dp_x}{d\omega} = \frac{(1-\delta)p_x}{\omega}. \quad (7)$$

Using $U = U(c)$, the representative household's lifetime utility is given by

$$\int_0^\infty (U(c) + v(m)) e^{-\rho t} dt,$$

where ρ is the subjective discount rate, $v(m)$ is the utility of holding real money m , and $U(c)$ and $v(m)$ satisfy the following:

$$U'(c) > 0, \quad U'(0) = \infty, \quad U''(c) < 0; \quad v'(m) > 0, \quad v'(0) = \infty, \quad v''(m) < 0.$$

The household's labor endowment is normalized to unity, but unemployment may arise. In that case, actual employment l is determined by labor demand and falls below unity. Therefore,

$$l \leq 1.$$

The household receives labor income wl , where w denotes the real wage per unit of labor, the real return $-\pi m$ on money holdings, where π denotes the inflation rate, and the real profits $\bar{\Pi}$ distributed by all firms. There is no tax-cum-subsidy, as assumed in (1). Therefore, the household's flow budget equation is as follows:

$$\dot{m} = wl - \pi m + \bar{\Pi} - c. \quad (8)$$

The Hamiltonian is

$$H = U(c) + v(m) + \lambda(wl - \pi m + \bar{\Pi} - c),$$

where λ is the co-state variable. Using (3), (5), (6), we obtain the first-order optimal condition:

$$\rho + \xi \frac{\dot{c}}{c} + \pi = \frac{v'(m)}{U'(c)}, \quad \text{where} \quad \xi \equiv -\frac{cU''(c)}{U'(c)}. \quad (9)$$

In (9), the left-hand side represents the intertemporal marginal rate of substitution in consumption c , while the right-hand side represents the marginal benefit from holding money m measured in terms of c . The transversality condition is

$$\lim_{t \rightarrow \infty} \lambda m \exp(-\rho t) = 0. \quad (10)$$

2.2 Firms

Good y is supplied by competitive firms with constant labor productivity θ_y . Given that demand for y is $(1 - \delta)c/p_y$ from (4), we have

$$p_y \theta_y = w, \quad \Pi_y = 0, \quad l_y = \frac{y}{\theta_y} = \frac{(1 - \delta)c}{p_y \theta_y}, \quad (11)$$

where Π_y and l_y represent the real profits and labor demand in sector y , respectively.

Good x is supplied under Cournot competition by N firms with different levels of labor productivity. Firm i ($i = 1, \dots, N$) determines output x_i so as to maximize real profit Π_i , leading to

$$\begin{aligned} \Pi_i &= p_x x_i - w \frac{x_i}{\theta_i}, \\ \text{the first-order condition: } &\left(x_i \frac{\partial p_x}{\partial x_i} + p_x \right) \theta_i = w, \end{aligned} \quad (12)$$

where from (4) p_x satisfies

$$p_x x = \delta \left(\frac{p_x}{p_y} \right) c, \quad \text{where } x \equiv \sum_{i=1}^N x_i. \quad (13)$$

Firm i takes c , p_y and the total output of the other firms as given, and assumes that changing its output x_i varies p_x according to (13); hence,

$$\frac{\partial p_x}{\partial x_i} (= \frac{\partial p_x}{\partial x}) = - \frac{p_x \delta}{x (\delta - \delta' \omega)} < 0.$$

Substituting this equation and $w = p_y \theta_y (= p_x \theta_y / \omega)$ given by (11) into the first-order condition

in (12) yields

$$-\frac{x_i}{x} \frac{\delta}{(\delta - \delta' \omega)} + 1 = \frac{\theta_y}{\omega \theta_i}, \quad i = 1, \dots, N. \quad (14)$$

Summing (14) over the N firms gives

$$N - \frac{\delta}{\delta - \delta' \omega} = \frac{\left(\sum_{i=1}^N \frac{1}{\theta_i} \right) \theta_y}{\omega} \Rightarrow \omega = \omega(N, \theta_1, \dots, \theta_N, \theta_y), \quad (15)$$

implying that ω depends only on the parameters $(N, \theta_1, \dots, \theta_N, \theta_y)$. From (13), (14) and (15), we obtain firm i 's output x_i and labor demand $l_i (= x_i/\theta_i)$ as follows:

$$x_i = \frac{1 - \frac{\theta_y}{\omega \theta_i}}{\sum_{j=1}^N \left(1 - \frac{\theta_y}{\omega \theta_j} \right)} \left(\frac{\delta c}{p_x} \right), \quad l_i = \frac{\frac{1}{\theta_i} \left(1 - \frac{\theta_y}{\omega \theta_i} \right)}{\sum_{j=1}^N \left(1 - \frac{\theta_y}{\omega \theta_j} \right)} \left(\frac{\delta c}{p_x} \right), \quad \text{for } i = 1, \dots, N. \quad (16)$$

Therefore, for all x_i to be non-negative, we must have

$$1 - \frac{\theta_y}{\omega \theta_i} \geq 0, \quad i = 1, \dots, N. \quad (17)$$

From l_y in (11) and l_i in (16), we obtain total labor demand l :

$$l (= \sum_{i=1}^N l_i + l_y) = \frac{c}{y^F}, \quad (18)$$

$$y^F = \frac{p_x}{\delta \left\{ \frac{\sum_{i=1}^N \frac{1}{\theta_i} \left(1 - \frac{\theta_y}{\omega \theta_i} \right)}{\sum_{i=1}^N \left(1 - \frac{\theta_y}{\omega \theta_i} \right)} \right\} + \frac{(1-\delta)\omega}{\theta_y}}, \quad (19)$$

where y^F represents the national product under full employment.

Competitive firms earn no profit, $\Pi_y = 0$, and firm i in the oligopoly sector earns positive profit Π_i given by (12). The representative household receives all the profits; hence, $\bar{\Pi}$ in (8) satisfies

$$\bar{\Pi} = \sum_{i=1}^N \Pi_i.$$

2.3 Dynamics and steady states

From (18), if full employment prevails ($l = 1$), $c = y^F$, where y^F , given by (19), is constant over time because ω is constant from (15). Assuming perfect adjustment of nominal wages and prices under full employment, (9) then implies that m remains constant over time and that c , m and P satisfy

$$c = y^F, \quad \rho = \frac{v'(m)}{U'(y^F)}, \quad m = \frac{M^S}{P} \quad \text{if } l = 1. \quad (20)$$

However, if $v'(m)$ has a positive lower bound β :

$$v'(\infty) = \beta > 0, \quad (21)$$

and if y^F is so large that

$$\rho \leq \frac{\beta}{U'(y^F)} \quad \left(< \frac{v'(M^S/P)}{U'(y^F)} \text{ for any } P \right), \quad (22)$$

there is no P that satisfies (20), implying that the full-employment steady state does not exist.⁵ In this case, the subjective discount rate ρ , which represents the desire for current consumption, falls short of $\beta/U'(y^F)$, which represents the desire for money accumulation when $c = y^F$. Therefore, the household decreases c below y^F , leading to aggregate demand shortages and unemployment.

In this state, we assume that the nominal wage W declines gradually in response to the deflationary gap:

$$\frac{\dot{W}}{W} (= \pi) = \alpha \left(\frac{c}{y^F} - 1 \right) < 0 \quad \text{if } \frac{c}{y^F} < 1, \quad (23)$$

where α denotes the speed of downward nominal wage adjustment. $\dot{W}/W = \pi$ because ω is constant over time from (15), implying that $p_y(\omega)$ and hence the real wage $w (= W/P)$ are constant over time from the first equation of (11).

⁵Ono (1994, 2001) first showed that, under this assumption, unemployment caused by aggregate demand shortages emerges. This assumption is also adopted by Ono and Yamada (2018), and Michailat and Saez (2021, 2022), who justify it by incorporating status preferences with respect to wealth when analyzing aggregate demand shortages. In the present analysis, money is the sole asset; hence, the preference for money can be interpreted as a preference for wealth.

The money-market equilibrium condition $M^S/P = m$ implies $\dot{m}/m = -\pi$ because the nominal money supply M^S is assumed to be constant in (1). Hence, from (9) and (23), the dynamics of c and m under aggregate demand shortages are described as follows:

$$\begin{aligned}\frac{\dot{c}}{c} &= \frac{1}{\xi} \left(\frac{v'(m)}{U'(c)} - \alpha \left(\frac{c}{y^F} - 1 \right) - \rho \right), \\ \frac{\dot{m}}{m} &= -\alpha \left(\frac{c}{y^F} - 1 \right) > 0 \quad \text{if } \frac{c}{y^F} < 1.\end{aligned}\tag{24}$$

Real money m continues to expand, which makes $v'(m) = \beta$, as shown by (21). Therefore, from (24), c in the stagnation steady state (where $\dot{c} = 0$) satisfies the following:

$$\rho + \alpha \left(\frac{c}{y^F} - 1 \right) = \frac{\beta}{U'(c)}.\tag{25}$$

From (24) and (25), we find

$$\frac{\dot{m}}{m} = \rho - \frac{\beta}{U'(c)} < \rho,$$

implying that the transversality condition (10) holds even though m continues to grow.

Given that (22) is valid in the present case, the left-hand side is smaller than the right-hand side in (25) when $c = y^F$. Therefore, for c in (25) to have a solution in $(0, y^F)$, the left-hand side must exceed the right-hand side at $c = 0$; hence,

$$\rho - \alpha > 0.\tag{26}$$

We also assume that c has a unique solution in (25). Therefore, at the equilibrium level of c , the derivative of the left-hand side with respect to c is smaller than that of the right-hand side —i.e.,

$$\frac{\alpha}{y^F} < -\frac{\beta U''}{(U')^2},\tag{27}$$

and

$$c < y^F, \quad \pi = \alpha \left(\frac{c}{y^F} - 1 \right) < 0.$$

We summarize the above results as follows:

Proposition 1. When $\rho > \beta/U'(y^F)$, full employment prevails ($c = y^F$) and $\pi = 0$ in the steady state. When $\rho \leq \beta/U'(y^F)$ and (26) is valid, unemployment occurs ($0 < c < y^F$), real money m continues to grow, and $\pi < 0$ in the steady state, where c is given by (25).

Let us next derive how consumption c responds to a change in y^F when the economy is initially in the stagnation steady state. In this state $v'(m) = \beta$; hence the dynamics of c given in (24) reduce to

$$\frac{\dot{c}}{c} = \frac{1}{\xi} \left(\frac{\beta}{U'(c)} - \alpha \left(\frac{c}{y^F} - 1 \right) - \rho \right). \quad (28)$$

From (27), this equation is unstable. Therefore, a change in y^F causes an instantaneous jump in c to the new steady state. From (25) and (27), this effect on c is given by

$$\frac{\partial c}{\partial y^F} = \frac{\frac{\alpha c}{(y^F)^2}}{\frac{\alpha}{y^F} + \frac{\beta U''}{(U')^2}} < 0.$$

This gives the following proposition:⁶

Proposition 2. In the stagnation steady state, an increase (decrease) in the full-employment national product y^F widens (narrows) the deflationary gap, thereby decreasing (increasing) consumption c and employment $l(= c/y^F)$.

3 Analytical results on minor-firm intervention

In this section, we analytically examine how improving a minor firm's productivity and forcing its exit in the oligopoly sector affect consumption and employment, using the following Cobb-Douglas utility function:

$$u(x, y) = \tilde{u}(x^\delta y^{1-\delta}); \quad \tilde{u}' > 0, \quad \tilde{u}'' < 0, \quad 0 < \delta < 1, \quad (29)$$

⁶Propositions 1 and 2 are basically the same as the results obtained by Ono (1994, 2001) under perfect competition.

where δ is constant. In this case, the expenditure share of good x , $\delta(\omega)$, equals δ :

$$\delta(\omega) = \delta = \text{constant}, \quad \delta'(\omega) = 0, \quad (30)$$

and real prices p_x and p_y are

$$p_x(\omega) = \omega^{1-\delta}, \quad p_y(\omega) = \omega^{-\delta}. \quad (31)$$

From (15) and (30), we obtain ω as

$$\omega = \frac{\left(\sum_{i=1}^N \frac{1}{\theta_i}\right) \theta_y}{N-1}. \quad (32)$$

In order for ω to have a positive finite solution, N must be larger than or equal to 2.⁷ Furthermore, we will analyze the effect of a firm exit; hence, we assume

$$N \geq 3. \quad (33)$$

Substituting ω given by (32) into (19) and (31) leads to

$$y^F = y^F(\theta_1; \theta_2, \dots, \theta_N) = \frac{p_x}{\left(\delta + \frac{1-\delta}{N-1}\right) \sum_{i=1}^N \frac{1}{\theta_i} - \delta(N-1) \frac{\sum_{i=1}^N \frac{1}{\theta_i^2}}{\sum_{i=1}^N \frac{1}{\theta_i}}},$$

where $p_x = \left(\frac{\left(\sum_{i=1}^N \frac{1}{\theta_i}\right) \theta_y}{N-1}\right)^{1-\delta}.$ (34)

Applying the same ω to (16) and solving θ_1 that makes $x_1 = 0$ gives the lowest possible level of θ_1 :

$$\theta_1^L = \frac{N-2}{\sum_{i=2}^N \frac{1}{\theta_i}}. \quad (35)$$

$y^F(\theta_1^L; \dots)$ equals the full-employment national product when firm 1 is removed from the market.

We also define θ_1^A that denotes firm 1's labor productivity such that its unit labor requirement

⁷If there is only one firm ($N = 1$), the monopolist's revenue $p_x x = \delta c$ is independent of output; hence, the optimal output is zero and $p_x = \infty$. We avoid this case by assuming that there are more than two firms after a firm's exit.

$(1/\theta_1^A)$ equals the average of those of all other firms in the oligopoly sector:

$$\theta_1^A = \frac{N-1}{\sum_{i=2}^N \frac{1}{\theta_i}}. \quad (36)$$

We examine the shape of $y^F(\theta_1; \dots)$ when θ_1 varies in the following range:

$$\theta_1^L \leq \theta_1 \leq \theta_1^A.$$

As proved in Appendix A, we obtain the shape of y^F as a function of θ_1 as mentioned below:

Lemma. As θ_1 increases from θ_1^L to θ_1^A , where θ_1^L is the lowest possible level of θ_1 and θ_1^A is the level at which the unit labor requirement is the average of those of all other oligopolists, $y^F(\theta_1; \dots)$ satisfies the following:

- (A) $y^F(\theta_1; \dots)$ is U-shaped if $CV < \frac{1}{N-2} \left(\frac{N-3+\delta}{2-\delta} \right)^{\frac{1}{2}}$,
- (B) $y^F(\theta_1; \dots)$ always increases if $CV \geq \frac{1}{N-2} \left(\frac{N-3+\delta}{2-\delta} \right)^{\frac{1}{2}}$,
- (C) $y^F(\theta_1^L; \dots) < y^F(\theta_1^A; \dots)$,

where the CV denotes the coefficient of variation of $(1/\theta_2, 1/\theta_3, \dots, 1/\theta_N)$:⁸

$$CV = \left((N-1) \frac{\sum_{i=2}^N \frac{1}{\theta_i^2}}{\left(\sum_{i=2}^N \frac{1}{\theta_i} \right)^2} - 1 \right)^{\frac{1}{2}}. \quad (37)$$

Remark. If the CV is small, the shape of $y^F(\theta_1; \dots)$ is as illustrated in Figure 1. There is $\hat{\theta}_1$ at which $dy^F(\theta_1; \dots)/d\theta_1$ turns from negative to positive. This is the case where the distribution of $(\theta_2, \theta_3, \dots, \theta_N)$ is not widely dispersed. The case where $\theta_2 = \theta_3 = \dots = \theta_N$ is an example because $CV = 0$ in this case. If the CV is large, which is the case where the distribution is dispersed, $y^F(\theta_1; \dots)$ always increases with θ_1 . From (C) in the Lemma, we find $\theta_1^A > \bar{\theta}_1$, where $\bar{\theta}_1$ is the level of θ_1 that makes y^F equal to y_o^F , the national product achieved if firm 1 were removed

⁸This is the standard deviation divided by the mean: $\left(\left(\frac{\sum_{i=2}^N \frac{1}{\theta_i^2}}{N-1} \right) - \left(\frac{\sum_{i=2}^N \frac{1}{\theta_i}}{N-1} \right)^2 \right)^{\frac{1}{2}} / \frac{\sum_{i=2}^N \frac{1}{\theta_i}}{N-1}$.

from the market.

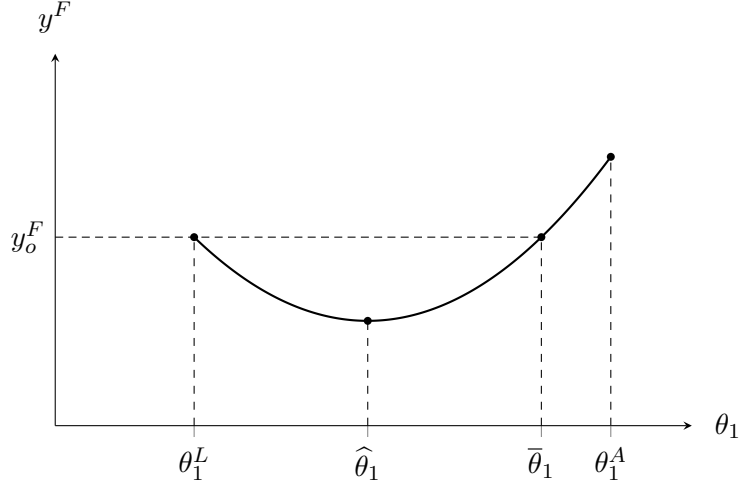


Figure 1: Relationship between θ_1 and y^F

Let us focus on the case where the distribution of θ_i ($i = 2, \dots, N$) is not widely dispersed, so that case (A) of the Lemma applies. Figure 1 and Proposition 2 directly lead to the following two propositions:

Proposition 3. Under full employment, an increase in a minor firm's labor productivity θ_1 lowers national product and consumption when the firm is substantially less productive than the other firms (i.e., $\theta_1 < \hat{\theta}_1$), but raises them otherwise (i.e., $\theta_1 > \hat{\theta}_1$). Under stagnation, an increase in a minor firm's labor productivity θ_1 raises consumption and employment when the firm is substantially less productive than the other firms (i.e., $\theta_1 < \hat{\theta}_1$), but lowers them otherwise (i.e., $\theta_1 > \hat{\theta}_1$).

Proposition 4. Under full employment, removing a minor firm from the market increases national product and consumption when the firm is substantially less productive than the other firms (i.e., $\theta_1 < \bar{\theta}_1$), but decreases them otherwise (i.e., $\theta_1 > \bar{\theta}_1$). Under stagnation, removing a minor firm from the market decreases consumption and employment when the firm is substantially less productive than the other firms (i.e., $\theta_1 < \bar{\theta}_1$), but increases them otherwise (i.e., $\theta_1 > \bar{\theta}_1$).

Supporting a firm—either by raising its productivity or allowing it to remain in the market—

has two opposing effects on industry-level productivity. The first is a direct positive effect through higher productivity or by keeping the firm's productive capacity in operation. The second is a negative effect arising from a reallocation of market share from more efficient firms to the supported firm. When the oligopoly sector consists of multiple major firms with similar productivity and a minor firm whose productivity is much lower than that of its rivals, the latter (negative) effect dominates the former (positive) effect. Consequently, supporting the minor firm reduces the sector's average productivity and lowers the full-employment national product. This result is a macroeconomic extension of the result of Lahiri and Ono (1988). They obtain the result using a partial equilibrium setting with linear demand, while we use a general equilibrium setting with Cobb-Douglas preference.

Moreover, we find that the result is the opposite under stagnation. Under stagnation, policy support for the minor firm—whether through productivity enhancement or survival assistance—generates new employment and stimulates consumption. This result stands in sharp contrast with conventional views, such as Lahiri and Ono (1988) and the literature on zombie firms, because these studies do not consider unemployment caused by aggregate demand shortages.

4 Numerical results under CES preference

Having analytically derived the properties under the Cobb-Douglas preference in (29), where the expenditure share δ is constant, we numerically examine the case of CES preference, under which $\delta(\omega)$ varies with the relative price ω . We show that the same qualitative properties as those under the Cobb-Douglas preference remain valid under the CES preference over wide ranges of various parameter values.

We assume the following CES utility function:

$$u(x, y) = \tilde{u} \left(\{ \kappa_x x^\sigma + \kappa_y y^\sigma \}^{\frac{1}{\sigma}} \right), \quad (38)$$

$$\tilde{u}' > 0, \quad \tilde{u}'' < 0, \quad \kappa_x + \kappa_y = 1, \quad \sigma < 1,$$

under which we have

$$\delta(\omega) = \frac{\kappa_x^{\frac{1}{1-\sigma}}}{\kappa_x^{\frac{1}{1-\sigma}} + \kappa_y^{\frac{1}{1-\sigma}} \omega^{\frac{\sigma}{1-\sigma}}}, \quad (39)$$

$$p_x = \left(\kappa_x^{\frac{1}{1-\sigma}} + \kappa_y^{\frac{1}{1-\sigma}} \omega^{\frac{\sigma}{1-\sigma}} \right)^{\frac{1-\sigma}{\sigma}}. \quad (40)$$

Applying $\delta(\omega)$ given by (39) to $\delta/(\delta - \delta'\omega)$ in (15) yields the following equation:

$$N - \left(\frac{\kappa_x^{\frac{1}{1-\sigma}} + \kappa_y^{\frac{1}{1-\sigma}} \omega^{\frac{\sigma}{1-\sigma}}}{\kappa_x^{\frac{1}{1-\sigma}} + \frac{1}{1-\sigma} \kappa_y^{\frac{1}{1-\sigma}} \omega^{\frac{\sigma}{1-\sigma}}} \right) = \frac{\left(\sum_{i=1}^N \frac{1}{\theta_i} \right) \theta_y}{\omega}, \quad (41)$$

which gives ω as a function of θ_1 . Substituting $\delta(\omega)$ in (39) and p_x in (40) into (19) gives

$$y^F = y^F(\theta_1) = \frac{\kappa_x^{\frac{1}{\sigma}} \left(1 + \left(\frac{\kappa_y}{\kappa_x} \right)^{\frac{1}{1-\sigma}} \omega^{\frac{\sigma}{1-\sigma}} \right)^{\frac{1}{\sigma}}}{\frac{\left(\frac{\kappa_y}{\kappa_x} \omega \right)^{\frac{1}{1-\sigma}}}{\theta_y} + \frac{\sum_{i=1}^N \frac{1}{\theta_i} \left(1 - \frac{\theta_y}{\omega \theta_i} \right)}{\sum_{i=1}^N \left(1 - \frac{\theta_y}{\omega \theta_i} \right)}}, \quad (42)$$

where ω is a function of θ_1 given by (41).⁹

From (17), θ_1^L , the lowest possible θ_1 , satisfies $\theta_1^L = \theta_y/\omega$. By substituting this value for θ_1 in (41) and (42), we find that at $\theta_1 = \theta_1^L$, $\omega (= \omega_o)$ and $y^F (= y_o^F)$ satisfy the following:

$$\begin{aligned} \theta_1^L &= \frac{\theta_y}{\omega_o}, \\ N - 1 - \left(\frac{\kappa_x^{\frac{1}{1-\sigma}} + \kappa_y^{\frac{1}{1-\sigma}} \omega_o^{\frac{\sigma}{1-\sigma}}}{\kappa_x^{\frac{1}{1-\sigma}} + \frac{1}{1-\sigma} \kappa_y^{\frac{1}{1-\sigma}} \omega_o^{\frac{\sigma}{1-\sigma}}} \right) &= \frac{\left(\sum_{i=2}^N \frac{1}{\theta_i} \right) \theta_y}{\omega_o}, \\ y_o^F &= \frac{\kappa_x^{\frac{1}{\sigma}} \left(1 + \left(\frac{\kappa_y}{\kappa_x} \right)^{\frac{1}{1-\sigma}} \omega_o^{\frac{\sigma}{1-\sigma}} \right)^{\frac{1}{\sigma}}}{\frac{\left(\frac{\kappa_y}{\kappa_x} \omega_o \right)^{\frac{1}{1-\sigma}}}{\theta_y} + \frac{\sum_{i=2}^N \frac{1}{\theta_i} \left(1 - \frac{\theta_y}{\omega_o \theta_i} \right)}{\sum_{i=2}^N \left(1 - \frac{\theta_y}{\omega_o \theta_i} \right)}}. \end{aligned} \quad (43)$$

y_o^F gives the full-employment national product in the case where firm 1 is removed.

⁹When $\sigma = 0$, (38) reduces to a Cobb-Douglas function, and equations (39), (40), (41), and (42) become equivalent to (30), (31), (32), and (34), respectively.

We consider the case where

$$\theta_1 < \theta_2 = \theta_3 = \dots = \theta_N;$$

namely, the oligopoly sector consists of $N - 1$ equally efficient major firms and one less efficient minor firm. We numerically examine whether y^F exhibits a shape similar to that in Figure 1. In this case, the average labor productivity of the efficient firms, θ_1^A , given by (36) is

$$\theta_1^A = \theta_2.$$

We impose the following normalization:

$$\sigma = 0.5, \quad \kappa_x = 0.5, \quad \kappa_y (= 1 - \kappa_x) = 0.5, \quad \theta_y = 1, \quad \theta_2 = \theta_3 = \dots = \theta_N = 1, \quad (44)$$

where the assumption $\sigma = 0.5$ follows Broda and Weinstein (2006) and Soderbery (2015).¹⁰ Under this setup, we numerically find that y^F is U-shaped, as illustrated in Figure 1. Here, $\hat{\theta}_1$ is the point where the derivative $dy^F(\theta_1)/d\theta_1$, derived from (42), switches from negative to positive, and $\bar{\theta}_1$ is the point where $y^F(\theta_1)$ in (42) equals y_o^F in (43).

Table 1 reports θ_1^L , $\hat{\theta}_1$, the minor firm's market share at $\theta_1 = \hat{\theta}_1$, $\bar{\theta}_1$, and the minor firm's market share at $\theta_1 = \bar{\theta}_1$, for $N - 1 \in \{2, 3, 5, 10\}$. Based on Table 1, let us look at the case with three major firms (i.e., $N - 1 = 3$), for example. If the minor firm's market share is less than 9.3% (with each major firm holding 30.2%), improving the minor firm's productivity decreases national product and consumption under full employment, but increases employment and consumption under stagnation. If the minor firm's market share is less than 18.6% (with each major firm holding 27.1%), eliminating the minor firm increases national product and consumption under full employment, but decreases employment and consumption under stagnation. Thus, in general, supporting a minor firm reduces consumption under full employment, whereas

¹⁰Broda and Weinstein (2006) estimate median elasticities of substitution of roughly 2 to 4 (in our notation, where the elasticity is given by $1/(1 - \sigma)$, this corresponds to σ values between 0.5 and 0.75), depending on the level of aggregation. The bias-corrected estimates of Soderbery (2015) are somewhat lower, with medians close to 2 ($\sigma = 0.5$ in our notation). We follow these studies and assume $\sigma = 0.5$. Estimates at broader levels of aggregation tend to be at or below unity: the structural-change literature finds broad sectors to be complements (e.g., Herrendorf et al., 2013), and Hottman et al. (2016) adopt a Cobb–Douglas upper tier across product groups. The values of σ in Table B1 span this empirical range, from complementarity ($\sigma = -0.3$) through the Cobb–Douglas case ($\sigma \rightarrow 0$) to strong substitutability ($\sigma = 0.9$).

Table 1: Threshold values and market shares under (44)

$N - 1$	θ_1^L	$\hat{\theta}_1$	x_1/x	$\bar{\theta}_1$	x_1/x
2	0.686	0.760	0.095	0.843	0.188
3	0.786	0.857	0.093	0.937	0.186
5	0.870	0.922	0.071	0.979	0.143
10	0.934	0.964	0.042	0.995	0.085

it stimulates consumption under stagnation.

Appendix B examines the robustness of our results. We examine the case of various values of σ ($\in \{-0.3, 0.001, 0.5, 0.9\}$) and summarize the findings in Table B1. Table B2 checks robustness with respect to κ_x and $\kappa_y (= 1 - \kappa_x)$, while Table B3 does so for different values of θ_y . These tables show that $y^F(\theta_1)$ is U-shaped for a wide range of values of the parameters. Thus, the results obtained under the Cobb-Douglas preference remain valid under the CES preference as well. A productivity improvement of a minor firm shifts market share from more efficient major firms to the less efficient minor firm. Under full employment, this reduces national product and consumption, whereas under stagnation it increases employment and consumption. Removing a minor firm increases national product and consumption under full employment, but reduces employment and consumption under stagnation. Hence, supporting minor firms is desirable for stimulating employment and consumption under aggregate demand stagnation.

5 Concluding remarks

We consider a monetary economy in which one sector consists of Cournot oligopolists with varying levels of productivity and the rest is competitive, and in which aggregate demand shortages may occur. In this setup, we analyze how supporting a low-productivity firm affects consumption under full employment and demand stagnation.

Supporting a firm—either by raising its productivity or allowing it to remain in the market—has two opposing effects on industry-level productivity. The first is a direct positive effect through higher productivity or by keeping the firm’s productive capacity in operation. The second is a negative effect arising from a reallocation of market share from more efficient firms to the supported firm. When the oligopoly sector consists of multiple major firms with similar productivity and a minor firm whose productivity is much lower than that of its rivals, the latter effect dominates the former. As a result, national product and consumption decrease under full

employment. Under stagnation, however, consumption and employment move inversely with the national product that would be achieved under full employment. A decrease in the full-employment national product narrows the deflationary gap, thereby stimulating consumption and employment.

By contrast, if the firm's productivity is close to that of the rivals, the former (positive) effect dominates the latter (negative) effect. Therefore, supporting the firm increases the full-employment national product, which naturally raises consumption in the absence of aggregate demand shortages. In the presence of aggregate demand shortages, however, this widens the deflationary gap, thereby reducing consumption and employment.

Our analytical results under Cobb–Douglas preferences—which yield a constant expenditure share—and our numerical results under CES preferences—under which the expenditure share depends on the relative price—indicate that these properties are robust across a wide range of preference and technology parameters as well as different numbers of firms.

Thus, in general, minor firms should be culled under full employment but supported under stagnation.

Appendices

A Proof of the Lemma

The shape of function $y^F(\theta_1; \dots)$

We examine the shape of $y^F(\theta_1; \theta_2, \dots, \theta_N)$ with respect to θ_1 under the Cobb–Douglas utility function given by (29). Partially differentiating $y^F(\theta_1; \theta_2, \dots, \theta_N)$ in (34) with respect to θ_1 gives

$$\frac{\partial y^F}{\partial \theta_1} = \frac{\delta p_x \Phi(\theta_1)}{\theta_1^4 \left(\sum_{i=1}^N \frac{1}{\theta_i} \right)^2 \left[\left(\delta + \frac{1-\delta}{N-1} \right) \sum_{i=1}^N \frac{1}{\theta_i} - \delta(N-1) \frac{\sum_{i=1}^N \frac{1}{\theta_i^2}}{\sum_{i=1}^N \frac{1}{\theta_i}} \right]^2}, \quad (45)$$

where

$$\begin{aligned}\Phi(\theta_1) = & \left[\left(\delta + \frac{1-\delta}{N-1} \right) \left(\sum_{i=2}^N \frac{1}{\theta_i} \right)^2 + (N-1)(2-\delta) \sum_{i=2}^N \frac{1}{\theta_i^2} \right] \theta_1^2 \\ & - 2 \left(\frac{(N-2)(N-\delta)}{N-1} \sum_{i=2}^N \frac{1}{\theta_i} \right) \theta_1 + \frac{1-\delta(N^2-3N+3)}{N-1}.\end{aligned}\quad (46)$$

Note that $\Phi(\theta_1)$ is a quadratic function of θ_1 .

By substituting θ_1^L in (35) into (46), we obtain

$$\Phi(\theta_1^L) = (N-1) \left\{ (N-2)^2(2-\delta) \frac{\sum_{i=2}^N \frac{1}{\theta_i^2}}{\left(\sum_{i=2}^N \frac{1}{\theta_i} \right)^2} - [(N-2) + (1-\delta)(N-3)] \right\}. \quad (47)$$

From (45) and (47), we find

$$\frac{\sum_{i=2}^N \frac{1}{\theta_i^2}}{\left(\sum_{i=2}^N \frac{1}{\theta_i} \right)^2} \begin{matrix} \leq \\ \geq \end{matrix} \frac{(N-2) + (1-\delta)(N-3)}{(N-2)^2(2-\delta)} \Leftrightarrow \frac{\partial y^F}{\partial \theta_1} \Big|_{\theta_1=\theta_1^L} \begin{matrix} \leq \\ \geq \end{matrix} 0. \quad (48)$$

The value of the quadratic function $\Phi(\theta_1)$ is minimized when θ_1 is

$$\theta_1^{min} = \frac{(N-2)(N-\delta)}{\left(\sum_{i=2}^N \frac{1}{\theta_i} \right) \left((N-1)\delta + 1 - \delta + (N-1)^2(2-\delta) \frac{\sum_{i=2}^N \frac{1}{\theta_i^2}}{\left(\sum_{i=2}^N \frac{1}{\theta_i} \right)^2} \right)}. \quad (49)$$

Given that

$$\frac{1}{N-1} \leq \frac{\sum_{i=2}^N \frac{1}{\theta_i^2}}{\left(\sum_{i=2}^N \frac{1}{\theta_i} \right)^2} < 1, \quad (50)$$

which is the Cauchy-Schwarz inequality, substituting $1/(N-1)$ for $\sum_{i=2}^N \frac{1}{\theta_i^2} / \left(\sum_{i=2}^N \frac{1}{\theta_i} \right)^2$ in (49) gives the maximum of θ_1^{min} . Comparing it with θ_1^L in (35) gives

$$\theta_1^L - \theta_1^{min} \geq \frac{(N-2)}{\left(\sum_{i=2}^N \frac{1}{\theta_i} \right)} \left(1 - \frac{N-\delta}{(N-\delta) + (N-1)} \right) > 0.$$

This implies that if $\partial y^F / \partial \theta_1 \Big|_{\theta_1=\theta_1^L} \geq 0$, then $\partial y^F / \partial \theta_1 > 0$ for $\theta_1 > \theta_1^L$. Therefore, from (48),

we find that

$$\text{if } \frac{\sum_{i=2}^N \frac{1}{\theta_i^2}}{\left(\sum_{i=2}^N \frac{1}{\theta_i}\right)^2} \geq \frac{(N-2) + (1-\delta)(N-3)}{(N-2)^2(2-\delta)}, \text{ then } \frac{\partial y^F}{\partial \theta_1} > 0 \text{ for } \theta_1 > \theta_1^L. \quad (51)$$

By substituting θ_1^A in (36) for θ_1 in (46), we obtain

$$\Phi(\theta_1^A) = (N-1)^3(2-\delta) \frac{\sum_{i=2}^N \frac{1}{\theta_i^2}}{\left(\sum_{i=2}^N \frac{1}{\theta_i}\right)^2} + \frac{N^2}{N-1} - 2(N-1)^2 + \delta \left(N^2 - 2N - \frac{1}{N-1} \right). \quad (52)$$

Since $\sum_{i=2}^N \frac{1}{\theta_i^2} / \left(\sum_{i=2}^N \frac{1}{\theta_i}\right)^2 \geq 1/(N-1)$ as shown by (50), by substituting $1/(N-1)$ for $\sum_{i=2}^N \frac{1}{\theta_i^2} / \left(\sum_{i=2}^N \frac{1}{\theta_i}\right)^2$ in the right-hand side of (52), rearranging the result, and using the property that $\partial y^F / \partial \theta_1$ has the same sign as $\Phi(\theta_1^A)$ from (45), we find

$$\Phi(\theta_1^A) \geq N + \frac{(1-\delta)N}{N-1} > 0 \Rightarrow \frac{\partial y^F}{\partial \theta_1} \Big|_{\theta_1=\theta_1^A} > 0. \quad (53)$$

This shows that y^F increases as θ_1 increases when $\theta_1 = \theta_1^A$.

Noting that $\Phi(\theta_1)$, given by (46), is a quadratic function of θ_1 with the coefficient of θ_1^2 being positive, from (48), (51) and (53), we find that $y^F(\theta_1; \dots)$ has the following property:

$$\begin{aligned} \text{(A) } y^F(\theta_1; \dots) \text{ forms a U-shape if } & \frac{\sum_{i=2}^N \frac{1}{\theta_i^2}}{\left(\sum_{i=2}^N \frac{1}{\theta_i}\right)^2} < \frac{(N-2) + (1-\delta)(N-3)}{(N-2)^2(2-\delta)}, \\ \text{(B) } y^F(\theta_1; \dots) \text{ always increases if } & \frac{\sum_{i=2}^N \frac{1}{\theta_i^2}}{\left(\sum_{i=2}^N \frac{1}{\theta_i}\right)^2} \geq \frac{(N-2) + (1-\delta)(N-3)}{(N-2)^2(2-\delta)}. \end{aligned} \quad (54)$$

Under (33) we have

$$\frac{1}{N-1} < \frac{(N-2) + (1-\delta)(N-3)}{(N-2)^2(2-\delta)} < 1. \quad (55)$$

From (50) and (55), cases (A) and (B) can both occur. By rewriting the conditions in (54) using the CV given by (37), we obtain the properties mentioned in (A) and (B) of the Lemma.

Proof of $y^F(\theta_1^L; \dots) < y^F(\theta_1^A; \dots)$

Let us next prove $y^F(\theta_1^L; \dots) < y^F(\theta_1^A; \dots)$; hence, $\theta_1^A > \bar{\theta}_1$, where $\bar{\theta}_1$ makes y^F equal to the national product achieved if firm 1 were removed from the market. From (34), (35) and (36), we obtain

$$\begin{aligned} y^F(\theta_1^L; \dots) - y^F(\theta_1^A; \dots) &= \frac{p_x(\omega(\theta_1^L))}{\Psi(\theta_1^L)} - \frac{p_x(\omega(\theta_1^A))}{\Psi(\theta_1^A)}, \\ \text{sgn}(y^F(\theta_1^L; \dots) - y^F(\theta_1^A; \dots)) &= \text{sgn}\Gamma, \end{aligned} \quad (56)$$

where

$$\begin{aligned} \Psi(\theta_1) &= \left(\delta + \frac{1-\delta}{N-1} \right) \sum_{i=1}^N \frac{1}{\theta_i} - \delta(N-1) \frac{\sum_{i=1}^N \frac{1}{\theta_i^2}}{\sum_{i=1}^N \frac{1}{\theta_i}}, \\ \Gamma &= (N-1)^{2(1-\delta)} \Psi(\theta_1^A) - (N(N-2))^{1-\delta} \Psi(\theta_1^L). \end{aligned}$$

Applying θ_1^L in (35) and θ_1^A in (36) to Γ in (56) yields

$$\begin{aligned} \text{sgn}\Gamma &= \text{sgn}\Lambda, \\ \Lambda &= N \left((N-1)^{-2\delta} - (N(N-2))^{-\delta} \right) + \delta \left[\frac{(N-1)^3}{N} \left(1 - \frac{(N-1) \sum_{i=2}^N \frac{1}{\theta_i^2}}{\left(\sum_{i=2}^N \frac{1}{\theta_i} \right)^2} \right) - 1 \right] (N-1)^{-2\delta} \\ &\quad - \delta N \left((N-3) - \frac{(N-2)^2 \sum_{i=2}^N \frac{1}{\theta_i^2}}{\left(\sum_{i=2}^N \frac{1}{\theta_i} \right)^2} \right) (N(N-2))^{-\delta}. \end{aligned} \quad (57)$$

The coefficient of $\sum_{i=2}^N \frac{1}{\theta_i^2} / \left(\sum_{i=2}^N \frac{1}{\theta_i} \right)^2$ in (57) reduces to

$$\frac{\delta}{N} \left(-[(N-1)^2]^{2-\delta} + [N(N-2)]^{2-\delta} \right) < 0,$$

which is negative because $N \geq 3$ from (33). We also have $1/(N-1) \leq \sum_{i=2}^N \frac{1}{\theta_i^2} / \left(\sum_{i=2}^N \frac{1}{\theta_i} \right)^2$ from (50). Therefore, substituting $1/(N-1)$ for $\sum_{i=2}^N \frac{1}{\theta_i^2} / \left(\sum_{i=2}^N \frac{1}{\theta_i} \right)^2$ in Λ of (57) gives the maximum of Λ , which we call Δ :

$$(\Lambda \leq) \Delta = \frac{N^{1-\delta}(N-\delta)(N-2)^{-\delta}}{N-1} \left[\left(\frac{N-1}{N} \right)^{1-\delta} \left(\frac{N-2}{N-1} \right)^{\delta} - \frac{N-1-\delta}{N-\delta} \right]. \quad (58)$$

Let us now prove that $\Delta < 0$. Given that $\log x$ is concave, we find

$$\begin{aligned} \log \left(\left(\frac{N-1}{N} \right)^{1-\delta} \left(\frac{N-2}{N-1} \right)^\delta \right) &= (1-\delta) \log \left(\frac{N-1}{N} \right) + \delta \log \left(\frac{N-2}{N-1} \right) \\ &< \log \left((1-\delta) \left(\frac{N-1}{N} \right) + \delta \left(\frac{N-2}{N-1} \right) \right) = \log \left(\frac{(N-1)^2 - \delta}{N(N-1)} \right), \end{aligned}$$

which implies

$$\left(\frac{N-1}{N} \right)^{1-\delta} \left(\frac{N-2}{N-1} \right)^\delta < \frac{(N-1)^2 - \delta}{N(N-1)}.$$

We also have

$$\frac{(N-1)^2 - \delta}{N(N-1)} - \frac{N-1-\delta}{N-\delta} = -\frac{\delta(1-\delta)}{N(N-1)(N-\delta)} < 0.$$

The two equations given above shows

$$\left(\frac{N-1}{N} \right)^{1-\delta} \left(\frac{N-2}{N-1} \right)^\delta < \frac{N-1-\delta}{N-\delta},$$

which implies $(\Lambda \leq) \Delta < 0$ in (58); hence, $\Gamma < 0$ in (57). Therefore, from (56), we find

$$y^F(\theta_1^L; \dots) < y^F(\theta_1^A; \dots),$$

as mentioned in (C) of the Lemma.

B Numerical thresholds under CES preferences

This appendix reports numerical results supporting Propositions 3 and 4 under the CES preference given by (38) for various values of the parameters. Table B1 displays robustness checks for the substitution parameter σ and the number of major firms $N - 1$. For various combinations of σ and $N - 1$, we report the lower bound θ_1^L , below which firm 1 cannot survive; the threshold value $\hat{\theta}_1$, at which the sign of $dy^F/d\theta_1$ changes from negative to positive; and the threshold value $\bar{\theta}_1$, at which the sign of $y_o^F - y^F$ changes from positive to negative, where y^F is the full-employment national product with firm 1 active and y_o^F is that with firm 1 removed.

Firm 1's market share x_1/x evaluated at each threshold is also given.

The table covers four representative values of σ (spanning complementarity, the Cobb–Douglas case, and substitutability) and four representative values of $N - 1$. The results confirm both propositions: $dy^F/d\theta_1 < 0$ and $y_o^F - y^F > 0$ when the minor firm is sufficiently inefficient, and both signs reverse when its productivity is sufficiently close to the average of those of the other firms.

Table B1: Robustness to σ					
$(\kappa_x = \kappa_y = 0.5, \theta_y = 1, \theta_2 = \dots = \theta_N = 1)$					
$\sigma = -0.3$					
$N - 1$	θ_1^L	$\hat{\theta}_1$	x_1/x	$\bar{\theta}_1$	x_1/x
2	0.442	0.559	0.100	0.722	0.206
3	0.626	0.746	0.099	0.907	0.202
5	0.775	0.864	0.075	0.973	0.150
10	0.887	0.938	0.043	0.994	0.087
$\sigma = 0.001$ ($\div$ Cobb–Douglas)					
$N - 1$	θ_1^L	$\hat{\theta}_1$	x_1/x	$\bar{\theta}_1$	x_1/x
2	0.500	0.602	0.092	0.732	0.188
3	0.667	0.773	0.096	0.909	0.195
5	0.800	0.879	0.073	0.973	0.148
10	0.900	0.945	0.043	0.994	0.086
$\sigma = 0.5$ (Table 1 in Section 4)					
$N - 1$	θ_1^L	$\hat{\theta}_1$	x_1/x	$\bar{\theta}_1$	x_1/x
2	0.686	0.760	0.095	0.843	0.188
3	0.786	0.857	0.093	0.937	0.186
5	0.870	0.922	0.071	0.979	0.143
10	0.934	0.964	0.042	0.995	0.085
$\sigma = 0.9$					
$N - 1$	θ_1^L	$\hat{\theta}_1$	x_1/x	$\bar{\theta}_1$	x_1/x
2	0.928	0.950	0.110	0.972	0.214
3	0.949	0.968	0.095	0.987	0.187
5	0.968	0.981	0.071	0.995	0.140
10	0.983	0.991	0.042	0.999	0.083

Table B2 and Table B3 check robustness to κ_x and θ_y respectively, with $\sigma = 0.5$ and $N - 1 = 3$.

In both tables, the ordering $\theta_1^L < \widehat{\theta}_1 < \bar{\theta}_1 < \theta_2$ and the sign pattern of Propositions 3 and 4 remains unchanged; only the threshold levels and the market shares differ. In Table B2, a larger κ_x , which strengthens the household preference for good x , increases the minor firm’s market share at both threshold values. In Table B3, θ_y matters little over a wide range.

The numerical results confirm that the qualitative results obtained under the Cobb-Douglas preference remain valid under the CES preference across wide ranges of values for various parameters.

Table B2: Robustness to κ_x					
$(\sigma = 0.5, N - 1 = 3, \theta_y = 1, \theta_2 = \dots = \theta_N = 1)$					
$(\kappa_x = 0.5 \text{ case is included in Table 1 of Section 4.})$					
κ_x	θ_1^L	$\widehat{\theta}_1$	x_1/x	$\bar{\theta}_1$	x_1/x
0.20	0.829	0.880	0.082	0.935	0.163
0.35	0.816	0.873	0.086	0.935	0.171
0.50	0.786	0.857	0.093	0.937	0.186
0.65	0.740	0.834	0.104	0.949	0.209
0.80	0.692	0.814	0.115	0.976	0.235

Table B3: Robustness to θ_y					
$(\sigma = 0.5, N - 1 = 3, \kappa_x = 0.5, \theta_2 = \dots = \theta_N = 1)$					
$(\theta_y = 1.0 \text{ case is included in Table 1 of Section 4.})$					
θ_y	θ_1^L	$\widehat{\theta}_1$	x_1/x	$\bar{\theta}_1$	x_1/x
0.3	0.741	0.835	0.104	0.948	0.209
0.5	0.761	0.844	0.099	0.942	0.199
1.0	0.786	0.857	0.093	0.937	0.186
2.0	0.805	0.867	0.089	0.935	0.176
3.0	0.813	0.872	0.086	0.935	0.172

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