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DELAYED RETIREMENT AND FERTILITY IN CHINA:THE ROLE OF ENDOGENOUS HEALTH INVESTMENT

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Delayed Retirement and Fertility in China: The Role of Endogenous Health Investment

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Abstract

We examine how health investment changes the fertility effects of delayed retirement in a three-period overlapping-generations model with grandparental childcare, calibrated to fertility data for Chinese cities and urban household data. We compare endogenous health investment with a counterfactual that fixes the medical expenditure-to-wage ratio at its pre-reform level. For a marginal permanent retirement-age increase from the common pre-reform steady state, adults making fertility decisions when reform begins have lower fertility with endogenous health investment than in the counterfactual. This fertility difference is more negative than for any generation reaching adulthood later. At implementation, young adults increase medical spending relative to wages, while their elderly parents' health reflects pre-reform investment. Later generations receive more childcare from healthier, longer-lived parents than in the counterfactual. Thus, for a marginal reform, steady-state comparisons understate this fertility reduction at implementation. In the calibrated model, raising the retirement age from 55 to 58 increases fertility among adults who are young at implementation by 0.29% with endogenous health investment and 1.27% with the medical expenditure-to-wage ratio fixed, relative to no reform. An illustrative increase from 55 to 62 instead lowers their fertility with endogenous health investment but raises it with medical spending fixed relative to wages, both relative to no reform. Relative to the counterfactual, endogenous health investment also narrows the retirement-age range over which calibrated steady-state fertility exceeds its no-reform level.

Keywords: Delayed retirement, Health investment, Grandparental childcare, Fertility, Intergenerational effects, China
JEL: D91, H55, I12, J13, J26

1. Introduction

China began gradually raising statutory retirement ages in January 2025.[1] Longer working lives can increase old-age earnings but reduce grandparents' childcare time. Grandparental childcare is widespread in urban China: in the urban sample of the China Health and Nutrition Survey (CHNS) studied by Du et al. (2019), 55.6% of children aged 0-6 received grandparental childcare in 2011.

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[1] Adopted in September 2024, the reform phases in increases over 15 years: from 60 to 63 for men, and from 50 to 55 and from 55 to 58 for women previously retiring at 50 and 55, respectively. It also calls for coordinated provision of elderly care and childcare (Standing Committee of the National People's Congress, 2024).

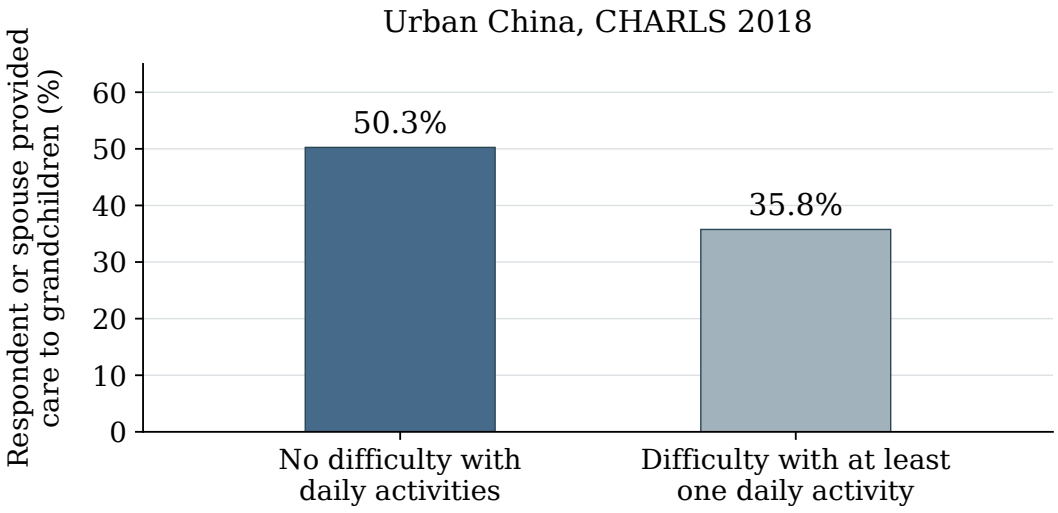

Figure 1. Physical capacity and grandparental childcare in urban China

Notes: Calculated from Yu et al. (2026), Table 2, using CHARLS 2018. The sample contains 1,872 urban-hukou adults aged 60 or older with grandchildren. Bars report the share stating that they or their spouse cared for a grandchild during the previous year. The health groups contain 1,377 respondents without difficulty in daily activities and 495 with at least one difficulty. Rates are unweighted and unadjusted. Appendix B.1 gives the calculation and definitions.

Grandparental childcare is also associated with physical capacity. Among urban-hukou grandparents in the 2018 China Health and Retirement Longitudinal Study (CHARLS) sample reported by Yu et al. (2026), 50.3% of those without difficulties in daily activities reported that they or their spouse had provided childcare during the previous year, compared with 35.8% of those with at least one difficulty (Figure 1). This descriptive association motivates considering grandparents' health alongside their available childcare time.

Retirement rules can also influence health behavior before retirement. Bertoni et al. (2018) find that middle-aged men affected by Italy's 2004 pension reform increased regular exercise in response to a longer expected working life. This evidence motivates allowing health investment to adjust when evaluating retirement reform.

Retirement and fertility models already identify a trade-off between additional earnings and reduced grandparental childcare (Cipriani and Fioroni, 2024; Dai et al., 2025; Huang, 2026). Research on China also studies health investment and retirement choices under pension incentives (Li and Cang, 2026).

Health investment entails current costs, while some of its benefits emerge later. If better health enables grandparents to provide more childcare, the benefits can extend to their adult children. We ask how health investment changes the fertility effects of retirement reform for adults making childbearing decisions at implementation and for generations reaching adulthood later. We also examine whether steady-state comparisons capture how health investment changes fertility at implementation.

We develop a three-period overlapping-generations model with endogenous fertility and health investment, grandparental childcare, capital accumulation, and pay-as-you-go pensions. Households choose fertility, saving, and health investment during working adulthood; health investment improves health and longevity in old age. We compare endogenous health investment

(Benchmark) with a counterfactual that fixes the medical expenditure-to-wage ratio at its pre-reform level (Frozen). Both start from the same pre-reform steady state and face the same retirement reform. Their fertility difference captures the effect of allowing health investment to respond; each specification's total reform effect is measured relative to no reform.

For a marginal permanent retirement-age increase from the common pre-reform steady state without old-age employment, adults making fertility decisions when reform begins have lower fertility in Benchmark than in Frozen. This fertility difference is more negative than for any generation reaching adulthood later. At implementation, young adults increase medical spending relative to wages while their elderly parents' health is fixed by pre-reform investment. Later generations receive more childcare from parents who invested under the new policy and are healthier and longer-lived than when medical spending is fixed relative to wages. The amount received by each household also depends on how many adult children share each grandparent's childcare. A steady-state comparison therefore understates the fertility reduction relative to Frozen among adults making fertility decisions at implementation.

We calibrate the model to fertility data for Chinese cities and urban household data, then simulate permanent increases in the retirement age. Raising the retirement age from 55 to 58, matching the statutory endpoints for women previously retiring at 55, increases fertility among adults who are young at implementation by 0.29% in Benchmark and 1.27% in Frozen, both relative to no reform. Higher expected old-age earnings reduce the saving needed for retirement, leaving more resources for childrearing. Higher medical spending relative to wages offsets part of this fertility gain. In the illustrative increase from 55 to 62, fertility for these adults falls by 1.62% in Benchmark but rises by 0.45% in Frozen, each relative to no reform. For these adults, Benchmark fertility is lower than Frozen fertility under both reforms in every admissible sensitivity specification examined. The decline in Benchmark and increase in Frozen at 62 persist in most specifications.

In the calibrated steady states, allowing health investment to respond lowers the retirement ages at which fertility peaks and returns to its no-reform level by about 0.7 and 1.9 years, respectively, compared with Frozen. This response narrows the range of retirement ages for which steady-state fertility exceeds its no-reform level.

Section 2 relates the analysis to the literature. Section 3 presents the model, and Section 4 derives the steady-state results. Section 5 describes calibration and quantitative steady-state comparisons. Section 6 analyzes fertility responses across generations, their sensitivity to calibration, and their capital and welfare implications. Section 7 concludes. Appendices A-D provide proofs, calibration details, and supplementary quantitative results.

2. Related literature

Models of retirement and family support connect fertility to old-age earnings and intergenerational transfers. Qiao and Wang (2019) analyze fertility constraints and life expectancy in an OLG model with endogenous fertility, old-age labor supply, and transfers from adult children to parents. With grandparental childcare, Miyazawa (2021) shows that the relationship between elderly labor participation and fertility depends on the pension system. Cipriani and Fioroni (2024) study pensions and child allowances under mandatory and individually chosen retirement. Their model links fertility to the number of adult children sharing each grandparent's childcare and can generate alternating convergence. Baldi et al. (2025) study elderly labor sup-

ply, endogenous grandparental childcare, fertility, education, and human and physical capital in an OLG economy.

Dai et al. (2025) combine empirical analysis with an OLG model of delayed retirement in China, linking fertility to grandparental childcare and bequest income; their calibrated reform reduces fertility. Huang (2026) uses a multiperiod OLG model with age-specific grandparenting and age discrimination. The calibrated reform initially raises fertility, but larger retirement delays reduce it. Our three-period model aggregates employment and grandparenting within life stages and also permits fertility gains for moderate extensions. We compare the fertility effects of a given retirement extension with endogenous health investment and with the medical expenditure-to-wage ratio fixed at its pre-reform level.

Research on health investment connects health to retirement incentives and household resource allocation. Grossman's (1972) distinction between health capital and medical inputs provides a basis for modeling health investment. Galama et al. (2013) analyze health investment, wealth accumulation, and retirement, while Aísa et al. (2015) relate preventive health investment to elderly labor supply. For China, Li and Cang (2026) model endogenous health accumulation and retirement under the Basic Old-Age Insurance system, emphasizing pension incentives and heterogeneous retirement responses. Zhang et al. (2006) show that policies promoting health and longevity can reduce capital accumulation in a life-cycle general-equilibrium model.

Grandparents' health has also been linked directly to fertility. Zhang and Luh (2018) incorporate grandparents' health into a family fertility model and find age-dependent associations in Taiwanese household data. Cipriani and Fioroni (2025) distinguish healthy grandparents who provide childcare from unhealthy parents who require care from adult children. In our model, households' medical spending determines their subsequent health and longevity, linking their current resource allocation to the childcare available to their adult children. This link makes health investment relevant to fertility responses across generations.

Empirical evidence links parental retirement to adult children's fertility and employment. Ilciukas (2023) exploits the 2006 Dutch pension reform and finds lower fertility among women whose mothers faced delayed retirement, consistent with reduced grandparental childcare. Using a survey experiment in China, Ma et al. (2026) find that parental retirement delays lower adult children's fertility intentions, with reduced grandparental childcare serving as a key mediating channel. Related European studies examine maternal employment (Bratti et al., 2018) and adult children's fertility decisions (Battistin et al., 2014; Frattola, 2025). These studies concern parental retirement; the permanent reform analyzed here also changes young adults' own expected working lives.

Evidence on childcare provision complements these retirement studies. Feng and Zhang (2018) study retirement and grandparental childcare in urban China, and Du et al. (2019) connect grandparental childcare to maternal labor participation. Zhang and Emery (2023) distinguish its association with the likelihood of a second birth from its association with the timing of that birth. Luo et al. (2012) find that grandparents with fewer functional limitations are more likely to begin or continue caring for nonresident grandchildren, linking physical capacity to childcare provision.

Evidence on saving before retirement supports the expected-income mechanism. Etgeton et al. (2023) exploit Germany's 1999 pension reform, which raised the early retirement age for affected women from 60 to at least 63. Exposed households reduced their saving rate before retirement

and increased leisure expenditure without a change in disposable income. The authors interpret this response as consumption smoothing in anticipation of higher lifetime labor income.

Retirement reforms can also change health behavior before retirement. Bertoni et al. (2018) find that Italy's 2004 pension reform extended affected middle-aged men's expected working lives and encouraged regular exercise. Using CHARLS data, Zhu et al. (2024) report higher medical expenditure and service use before retirement among respondents intending to delay retirement than among those without such intentions, with the ordering reversed after retirement. The authors interpret this pattern as anticipatory health investment. In a working paper, Geyer et al. (2026) find that a German reform extending women's working lives increased applications and approvals for work-related rehabilitation, but not completed treatment.

3. The model

The economy has three overlapping generations: children, young parents, and elderly grandparents. Children receive childcare from parents and grandparents. Young adults work and choose fertility, consumption, saving, and investment in their future health. Elderly adults work for the duration prescribed by the retirement rule and allocate their remaining lifetime between grandchild care and leisure. The model does not distinguish individuals by sex.

The subscript t indexes model periods. Young adulthood and potential old age each have unit length, with decisions within working adulthood aggregated over one period. In period t, π_t is the deterministic duration of old age and χ_t is statutory working time within that stage. Retirement lasts $\pi_t - \chi_t$, with $0 \leq \chi_t < \pi_t$.

Households within a cohort have identical preferences, endowments, and states, and make identical choices. Choices and resources are expressed per adult. Let N_t denote the number of young adults in period t and n_t fertility per adult. Population evolves according to

$$N_{t+1} = n_t N_t.$$

3.1 Health, grandparental childcare, and labor

Define the medical expenditure-to-wage ratio as

$$\widehat{m}_t \equiv \frac{m_t}{w_t}. \tag{1}$$

Here, m_t is medical spending in final goods and w_t is the pretax wage per unit of effective labor. We assume that old-age health and longevity depend on spending relative to the current wage, so attaining a given health outcome requires goods expenditure proportional to that wage. Health investment during young adulthood determines health capital and the duration of old age in the next period:

$$h_{t+1} = h(\widehat{m}_t), \qquad \pi_{t+1} = \pi(\widehat{m}_t). \tag{2}$$

We use the same health index h_t, bounded above by one, to scale elderly labor and childcare capacity per unit of time.

Assumption 1 (Health and longevity technologies)

The health-capital technology $h(\cdot)$ and longevity technology $\pi(\cdot)$ are twice continuously differentiable and satisfy

$$0 < h(0) < 1, \qquad 0 < \pi(0) < 1, \tag{3}$$

$$h'(\widehat{m}) > 0, \qquad h''(\widehat{m}) < 0, \qquad \pi'(\widehat{m}) > 0, \qquad \pi''(\widehat{m}) < 0, \tag{4}$$

and

$$\lim_{\widehat{m}\to\infty} h(\widehat{m}) = \lim_{\widehat{m}\to\infty} \pi(\widehat{m}) = 1. \tag{5}$$

Here, primes denote derivatives with respect to $\widehat{m}$.

Positive intercepts capture health determinants other than private medical investment. Monotonicity and concavity imply that medical investment improves health with diminishing marginal returns. The upper bound of one represents full labor and childcare capacity and the full potential old-age period, respectively.

Research links grandparents' health to childcare provision (Du et al., 2019; Cipriani and Fioroni, 2025). We represent effective childcare as the product of health capital and grandchild care time per elderly adult, μ_t. Assistance per young adult is therefore

$$g_t = \frac{\mu_t h_t}{n_{t-1}}. \tag{6}$$

Aggregate effective grandparental childcare is $N_{t-1}\mu_t h_t$, and the number of young adults receiving it is $N_t = n_{t-1}N_{t-1}$. Fertility in the preceding generation determines the number of recipients sharing this assistance; current fertility determines the number of children per young adult.[2] Young households take the childcare assistance supplied by their parents as given when choosing fertility.

Following the childcare-cost mechanism in Cipriani and Fioroni (2024), define the multiplier that reduces parental childrearing time as

$$\Phi_t = 1 + \eta g_t. \tag{7}$$

Here, $v > 0$ is parental time required per child without grandparental help, and $\eta > 0$ measures how effectively grandparental childcare saves parental time. With assistance, parental time per child falls to v/Φ_t, and total childrearing time is vn_t/Φ_t. The marginal time saving per child diminishes as g_t increases.

[2]Raw grandparental childcare time per child is $\mu_t/(n_{t-1}n_t)$, whereas g_t is effective care per young adult.

Effective labor supplied by the young and the elderly, both measured per young adult, is

$$\ell_t^y = 1 - \frac{vn_t}{\Phi_t}, \qquad \ell_t^o = \frac{\chi_t h_t}{n_{t-1}}, \tag{8}$$

so that

$$\ell_t = \ell_t^y + \ell_t^o. \tag{9}$$

The young time endowment is the time available for work and childrearing after fixed personal needs; young labor efficiency is normalized to one. Combined labor ℓ_t can exceed one because it includes elderly labor per young adult.

Let ζ_t denote leisure time per elderly adult. Old-age survival time is allocated among market work, grandchild care, and leisure:

$$\mu_t + \zeta_t = \pi_t - \chi_t. \tag{10}$$

Combining the two generations' time constraints, feasible allocations satisfy

$$0 \leq \frac{vn_t}{\Phi_t} \leq 1, \qquad \mu_t \geq 0, \quad \zeta_t \geq 0, \qquad 0 \leq \chi_t < \pi_t. \tag{11}$$

The childcare technology is subject to a time-substitution upper bound: parental time saved per young adult cannot exceed the raw grandparental childcare time received per young adult,

$$n_t \left(v - \frac{v}{\Phi_t} \right) \leq \frac{\mu_t}{n_{t-1}}. \tag{11a}$$

We study interior equilibria with positive consumption, saving, medical investment, fertility, young labor, grandchild care, and elderly leisure, and with (11a) strictly slack at every date.

3.2 Household preferences and optimal decisions

The lifetime utility of a representative household that is young in period t is

$$U_t = \log c_t + \theta \log n_t + \beta \left[\log d_{t+1} + \xi \log \mu_{t+1} + \gamma \log \zeta_{t+1} \right], \tag{12}$$

where c_t is consumption during young adulthood and d_{t+1} is total consumption over the next period's old-age stage.

The parameter $\theta > 0$ governs the preference for children, $\beta > 0$ weights old age relative to youth, and $\xi, \gamma > 0$ measure the private value of grandchild care and leisure, respectively. The preference for providing grandchild care follows Cipriani and Fioroni (2024).

When choosing fertility, saving, and medical investment in period t, young households expect the current retirement policy to continue next period:

$$\chi_{t+1}^e = \chi_t. \tag{13}$$

Conditional on this retirement policy and the exogenous productivity path, expected prices and pension benefits are consistent with equilibrium in the continuation economy. Individual households take the expected wage w_{t+1}^e, gross capital return R_{t+1}^e, and pension benefit per unit

of retirement time b^e_{t+1} as given. The contribution rate $0 \leq \tau < 1$ applies uniformly to young and elderly labor income.

In old age, households choose grandchild care and leisure after observing the realized policy and prices. Their saving s_t and health investment m_t are predetermined:

$$V^o_{t+1} = \max_{\mu_{t+1},\zeta_{t+1}} \left\{\log d_{t+1} + \xi \log \mu_{t+1} + \gamma \log \zeta_{t+1}\right\}, \tag{14}$$

subject to the budget constraint

$$d_{t+1} = R_{t+1}s_t + (1-\tau)w_{t+1}\chi_{t+1}h_{t+1} + b_{t+1}(\pi_{t+1} - \chi_{t+1}), \tag{15}$$

and the time restrictions in Section 3.1.

The pension benefit per unit of retirement time, b_{t+1}, is determined by the PAYG budget in Section 3.3.

Old-age consumption is independent of how retirement time is divided between grandchild care and leisure. With (11a) strictly slack, logarithmic utility gives

$$\mu_t = \lambda_c(\pi_t - \chi_t), \qquad \zeta_t = (1-\lambda_c)(\pi_t - \chi_t), \tag{16}$$

where

$$\lambda_c \equiv \frac{\xi}{\gamma + \xi} \in (0,1). \tag{17}$$

For given longevity, delayed retirement reduces $\pi_t - \chi_t$, lowering grandchild care and leisure in the fixed proportions determined by λ_c.

Young households in period t choose $\{c_t, n_t, s_t, m_t\}$ to maximize current utility plus the expected optimized value of old age:

$$\max_{c_t,n_t,s_t,m_t} \left\{\log c_t + \theta \log n_t + \beta V^{o,e}_{t+1}\right\}, \tag{18}$$

subject to the budget constraint

$$c_t + s_t + m_t = (1-\tau)w_t\ell^y_t. \tag{19}$$

Here, $V^{o,e}_{t+1}$ is the old-age value in (14) evaluated at the expected policy, prices, and pension benefit rate. Expected consumption is

$$d^e_{t+1} = R^e_{t+1}s_t + (1-\tau)w^e_{t+1}\chi^e_{t+1}h(\widehat{m}_t) + b^e_{t+1}\left[\pi(\widehat{m}_t) - \chi^e_{t+1}\right], \tag{20}$$

and expected optimal time allocations are

$$\mu^e_{t+1} = \lambda_c\left[\pi(\widehat{m}_t) - \chi^e_{t+1}\right], \qquad \zeta^e_{t+1} = (1-\lambda_c)\left[\pi(\widehat{m}_t) - \chi^e_{t+1}\right]. \tag{21}$$

With parental childcare, prices, and pension benefit rates taken as given, the interior fertility and saving conditions are

$$\frac{\theta}{n_t} = \frac{(1-\tau)w_t v}{\Phi_t c_t}, \qquad \frac{1}{c_t} = \frac{\beta R^e_{t+1}}{d^e_{t+1}}.$$

Define wage-normalized consumption and saving as $\widehat{c}_t = c_t/w_t$ and $\widehat{s}_t = s_t/w_t$. Holding prices and pension benefit rates fixed, differentiation with respect to $\widehat{m}_t$ gives the medical investment condition

$$\frac{1}{\widehat{c}_t} = \beta \left[\frac{(1-\tau)w^e_{t+1}\chi^e_{t+1}h'(\widehat{m}_t) + b^e_{t+1}\pi'(\widehat{m}_t)}{d^e_{t+1}} + (\gamma+\xi)\frac{\pi'(\widehat{m}_t)}{\pi(\widehat{m}_t) - \chi^e_{t+1}} \right].$$

The fertility condition equates the marginal utility of an additional child to the utility cost of the after-tax earnings forgone through childrearing. The saving condition is the Euler equation.

In the medical condition, the left-hand side is the marginal utility cost of increasing $\widehat{m}_t$ during young adulthood. The right-hand side comprises returns from higher elderly earnings through health capital, longer pension receipt through longevity, and additional time for grandchild care and leisure. Households value providing grandchild care but do not internalize the resulting reduction in their adult children's childrearing costs.

3.3 Pay-as-you-go pensions

The government taxes all labor income at rate τ and uses current revenue to pay current retirees. Since b_t is the benefit per unit of retirement time, budget balance requires

$$b_t(\pi_t - \chi_t)N_{t-1} = \tau w_t N_t \ell_t. \tag{22}$$

Using the population identity $N_t = n_{t-1}N_{t-1}$ gives

$$b_t(\pi_t - \chi_t) = \tau w_t n_{t-1} \ell_t. \tag{23}$$

Households take the benefit rate as given when choosing health investment, so longer survival extends pension receipt. In equilibrium, this common rate adjusts to balance the budget as cohort longevity and the tax base change.

Under the next-period retirement policy χ^e_{t+1} expected by households, the continuation economy also satisfies pension budget balance:

$$b^e_{t+1}\left[\pi(\widehat{m}_t) - \chi^e_{t+1}\right] = \tau w^e_{t+1} n_t \ell^e_{t+1}. \tag{24}$$

3.4 Production and capital accumulation

Production is given by

$$Y_t = A_t K_t^{\alpha}(N_t \ell_t)^{1-\alpha}, \qquad 0 < \alpha < 1. \tag{25}$$

Here, Y_t denotes aggregate final output, K_t physical capital, and $N_t\ell_t$ effective labor supplied by both generations. Hicks-neutral total factor productivity $A_t > 0$ is exogenous, and α is the capital income share under perfect competition.

Define labor-augmenting productivity as

$$Z_t \equiv A_t^{1/(1-\alpha)}, \tag{26}$$

and capital per young adult, normalized by productivity, as

$$k_t \equiv \frac{K_t}{Z_t N_t}. \tag{27}$$

Current capital and population are determined by previous saving and fertility. Thus, given Z_t, normalized capital k_t is predetermined. Current labor adjustments change k_t/ℓ_t, the capital stock per unit of productivity-adjusted effective labor, and hence wages and returns.

Output per young adult, normalized by productivity, is

$$\frac{Y_t}{Z_t N_t} = k_t^{\alpha} \ell_t^{1-\alpha}, \tag{28}$$

Capital depreciates fully over a model period. The wage and gross capital return are

$$\widehat{w}_t^Z \equiv \frac{w_t}{Z_t} = (1-\alpha)\left(\frac{k_t}{\ell_t}\right)^{\alpha}, \tag{29}$$

$$R_t = \alpha\left(\frac{k_t}{\ell_t}\right)^{\alpha-1}. \tag{30}$$

The variables A_t and Z_t represent the same technology. The wage $\widehat{w}_t^Z$ is normalized by productivity Z_t, while medical spending, consumption, and saving are normalized by the wage w_t.

Define the one-period growth factor of labor-augmenting productivity as

$$G_{t+1}^Z \equiv \frac{Z_{t+1}}{Z_t}. \tag{31}$$

With full depreciation and $K_{t+1} = N_t s_t$, capital evolves according to

$$G_{t+1}^Z n_t k_{t+1} = (1-\alpha)\left(\frac{k_t}{\ell_t}\right)^{\alpha} \widehat{s}_t. \tag{32}$$

In (32), current saving determines next-period physical capital, fertility determines the number of descendants sharing it, and productivity growth determines its normalization.

3.5 Competitive equilibrium

Definition 1 (Competitive equilibrium under policy-continuation expectations)

Let $t = 1$ be the initial period of the path under consideration. Given $N_0, N_1, K_1 > 0$, the previous generation's medical expenditure-to-wage ratio $\widehat{m}_0$, the realized retirement path $\{\chi_t\}_{t\geq 1}$, productivity path $\{A_t\}_{t\geq 1}$, and contribution rate τ, initial conditions satisfy $n_0 = N_1/N_0$, $s_0 = K_1/N_0$, $h_1 = h(\widehat{m}_0)$, and $\pi_1 = \pi(\widehat{m}_0)$. A competitive equilibrium consists of

household allocations by cohort, population and capital paths, factor prices $\{w_t, R_t\}$, pension benefits b_t, and corresponding next-period expectations satisfying the following conditions.

1. Given prices, pension benefits, and childcare assistance from their parents, young households choose $\{c_t, n_t, s_t, m_t\}$ to solve (18), subject to their budget, health technology, and the time restrictions in (11)-(11a), under the policy-continuation expectation in (13). Elderly households solve (14) using realized health, policy, and prices to determine $\{\mu_t, \zeta_t, d_t\}$.
2. Firms produce final goods using K_t and $N_t\ell_t$. Profit maximization yields the wage and capital return in (29)-(30).
3. Population satisfies $N_{t+1} = n_t N_t$, and elderly health and longevity follow (2). Grandparental childcare, parental childrearing time, and labor satisfy (6)-(10). Allocations satisfy the positivity and time restrictions in Section 3.1, including strict slackness of (11a).
4. Pensions satisfy period-by-period budget balance in (22). Aggregate household saving finances next-period capital, $K_{t+1} = N_t s_t$, and firms' labor demand equals the effective labor supplied by both generations. The final-goods resource constraint is

$$Y_t = N_t(c_t + m_t) + N_{t-1}d_t + K_{t+1}. \tag{33}$$

5. Expectations satisfy (13) and equilibrium conditions 1–4 in the corresponding continuation economy, conditional on the exogenous productivity path. If policy continues as expected, expected prices and allocations are realized. An unanticipated policy change leaves inherited saving, health investment, and population fixed; elderly allocations and current prices adjust to the realized policy.

Equation (33) follows from aggregating household budgets, using pension budget balance and competitive factor pricing.

The following result separates wage-normalized household decisions from the determination of capital and prices along the transition.

Lemma 1. *Under the maintained model specification and Assumption 1, fix model parameters, initial population and capital $N_0, N_1, K_1 > 0$, the inherited medical expenditure-to-wage ratio $\widehat{m}_0$, and a feasible permanent retirement policy $\chi_t = \chi$ for $t \geq 1$. Suppose post-reform expectations are realized. Changing the strictly positive exogenous productivity path leaves unchanged the set of interior equilibrium paths for fertility and wage-normalized medical expenditure, consumption, and saving. Health, longevity, and time allocations are also identical across corresponding equilibria. Capital and factor prices are recovered conditional on productivity and inherited capital.*

Logarithmic preferences, Cobb-Douglas prices, full depreciation, and the PAYG budget allow capital and productivity to be eliminated from the normalized household conditions. Childrearing entails forgone wages, while health depends on medical expenditure relative to wages. Appendix A.1 derives the reduced dynamic system and shows how each feasible interior solution determines a full competitive equilibrium.

4. Steady-state analysis

This section compares steady states under different permanent retirement ages and examines how endogenous health investment changes fertility through household resources and grandparental childcare. The model specification and parameter restrictions in Section 3 apply throughout.

4.1 Steady-state equilibrium

Definition 2 (Balanced-growth equilibrium)

Given a constant retirement-policy parameter χ and a constant one-period productivity growth factor $G^Z > 0$, a competitive equilibrium in Definition 1 is a balanced-growth equilibrium if fertility, health, longevity, and childcare and labor allocations per adult are constant, and k_t, w_t/Z_t, R_t, and the wage-normalized ratios of consumption, saving, medical expenditure, and pensions remain constant. We refer to this allocation as a steady state.

In balanced growth, $\chi^e_{t+1} = \chi_t = \chi_{t+1} = \chi$ and expected next-period wages, returns, and pensions equal their realizations. Population evolves according to $N_{t+1}/N_t = n$. Real wages and resource flows per household grow proportionally with Z_t, while capital and aggregate output grow with $Z_t N_t$.

To simplify the equilibrium conditions, define

$$\widetilde{n}_t \equiv \frac{n_t}{\Phi_t}. \tag{34}$$

Thus $\widetilde{n}_t$ measures parental childrearing time in units of v, with actual time $v\widetilde{n}_t$ and young labor $\ell^y_t = 1 - v\widetilde{n}_t$.

Equations (6)-(7), (16), and (34) imply that steady-state fertility satisfies

$$n^2 - n\widetilde{n} - \eta\lambda_c[\pi(\widehat{m}) - \chi]h(\widehat{m})\widetilde{n} = 0. \tag{35}$$

Given $\widehat{m}$ and $\widetilde{n}$, (35) determines fertility. Appendix A.2 combines this relationship with household optimality to determine fertility and the medical expenditure-to-wage ratio. As established in Lemma 1, these reduced equations are independent of productivity growth. Normalized capital also depends on G^Z.

We use two sufficient conditions to establish a unique interior candidate allocation. The first ensures positive medical investment by requiring a sufficiently large longevity return at zero expenditure:

$$\beta(\gamma + \xi)(1 - \tau)\frac{\pi'(0)}{\pi(0)} > 1 + \theta + \beta. \tag{E}$$

The second restricts the relative valuation of retirement time:

$$\frac{\beta(\gamma + \xi)}{1 + \theta} \geq \frac{\tau(1 - \alpha)}{\alpha}. \tag{H}$$

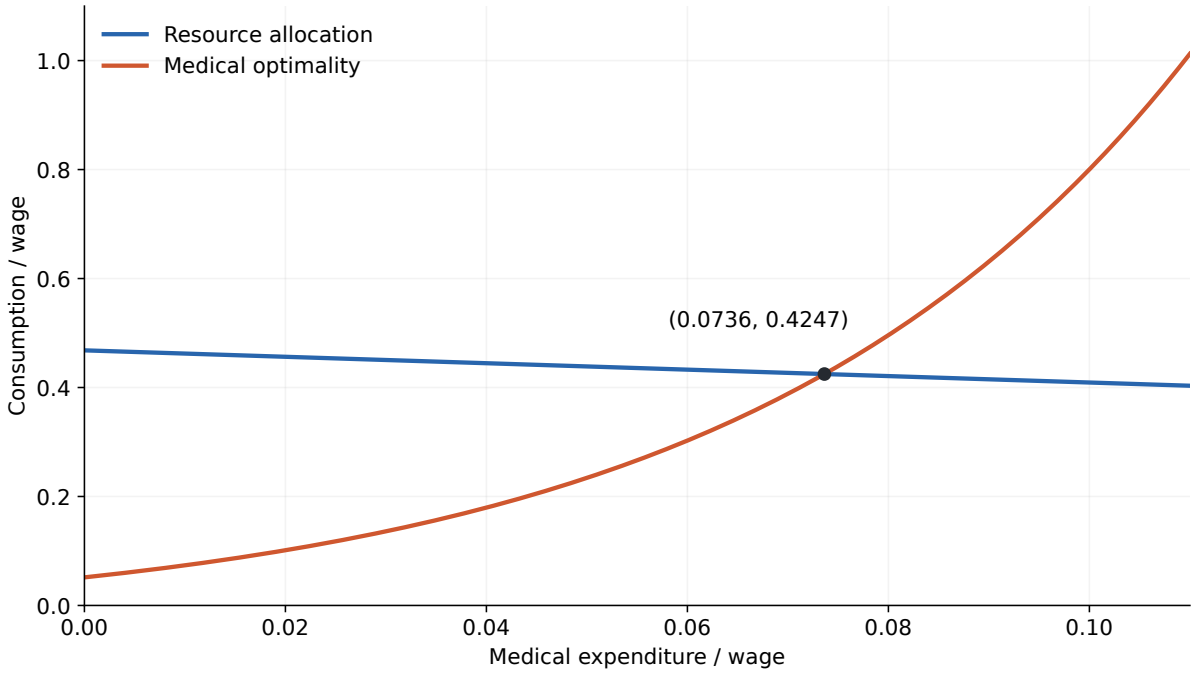

Figure 2. Pre-reform equilibrium: resource allocation and medical optimality

Notes: The horizontal and vertical axes show the medical expenditure-to-wage ratio $\widehat{m}$ and consumption-to-wage ratio $\widehat{c}$. The resource-allocation schedule slopes downward and the medical optimality schedule upward. Their intersection is approximately $(0.0736, 0.4247)$.

Condition H ensures that a higher elderly labor-income share does not reduce the weight on longevity returns in the reduced medical condition. It supports both uniqueness and the positive response of $\widehat{m}$ to delayed retirement established below. Since $\widehat{m} < 1 - \tau$ and retirement time is positive, an interior equilibrium requires $0 \leq \chi < \chi_{\max}$, where $\chi_{\max} \equiv \pi(1-\tau)$.

Lemma 2. *Under the parameter restrictions in Section 3, Assumption 1, and Conditions E and H, the reduced steady-state equations have a unique candidate allocation for each retirement-policy parameter $\chi \in [0, \chi_{\max})$. An interior balanced-growth equilibrium with (11a) strictly slack exists precisely when this candidate satisfies $\eta h(\widehat{m}) v \widetilde{n} < 1$. Whenever it exists, it is unique for each productivity growth factor $G^Z > 0$.*

The feasibility inequality in Lemma 2 is the steady-state form of strict slackness in (11a). Uniqueness concerns the normalized allocation; actual quantities also depend on initial productivity and population. Appendix A.2 provides the proof.

We take as the pre-reform benchmark the steady state with $\chi = 0$ and (11a) strictly slack. Overbars denote values in this benchmark. The resource-allocation and medical optimality conditions can be written in terms of the medical expenditure-to-wage and consumption-to-wage ratios as

$$\widehat{c}_R(\widehat{m}) = \frac{\lambda_r(1-\tau-\widehat{m})}{1+\theta\lambda_r}, \qquad \widehat{c}_H(\widehat{m}) = \frac{\pi(\widehat{m})}{\beta(\gamma+\xi+\tau/\omega)\pi'(\widehat{m})}, \quad 0 \leq \widehat{m} < 1-\tau, \tag{36}$$

where $\omega = \tau + \alpha/(1-\alpha)$ and $\lambda_r = \omega/[\omega + \beta\alpha/(1-\alpha)]$.

Higher $\widehat{m}$ reduces the consumption-to-wage ratio consistent with the resource constraint, so $\widehat{c}_R$ falls. Diminishing longevity returns require a lower marginal utility cost of medical investment to satisfy medical optimality. Since that cost is $1/\widehat{c}$, the medical schedule $\widehat{c}_H$ rises. Condition

E places the medical schedule below the resource schedule at zero medical investment; their opposite slopes and endpoint values yield the benchmark intersection in Figure 2. Fertility and capital then follow from household optimality and market clearing.

4.2 Health investment and childcare

Delayed retirement changes both the earnings return to health capital and the value of additional retirement time.

Proposition 1. *Under Assumption 1 and Condition H, the medical expenditure-to-wage ratio is strictly increasing in the retirement-policy parameter χ along any interval of interior balanced-growth equilibria satisfying (11a) strictly.*

Holding $\widehat{m}$ fixed, delayed retirement raises the elderly share of effective labor even after fertility and young labor adjust, strengthening the earnings component of medical returns. A shorter retirement period also raises the marginal value of time gained through longevity, while Condition H ensures that the weight on longevity returns does not fall. These effects raise the equilibrium medical expenditure-to-wage ratio. Appendix A.2 provides the proof.

Effective labor and childcare per elderly adult are, respectively,

$$W(\widehat{m},\chi) \equiv \chi h(\widehat{m}), \qquad E(\widehat{m},\chi) \equiv \lambda_c[\pi(\widehat{m}) - \chi]h(\widehat{m}). \tag{37}$$

Higher $\widehat{m}$ improves childcare capacity and lengthens the time available for childcare at a given χ. The bounds on health and longevity imply $E(\widehat{m},\chi) \leq \lambda_c(1-\chi)$, limiting the scope for these gains to offset the time lost to delayed retirement.

4.3 Fertility responses and health adjustment

Let subscripts χ denote total derivatives along an interior steady-state equilibrium branch. Differentiating (35) yields

$$n_\chi = \underbrace{\frac{n^2}{\widetilde{n}(2n-\widetilde{n})}\widetilde{n}_\chi}_{\text{parental time}} + \underbrace{\frac{\eta\widetilde{n}}{2n-\widetilde{n}}E_\chi}_{\text{effective grandparental care}}. \tag{38}$$

Both weights are positive. The first term captures the contribution of changes in young parents' own childrearing time, $v\widetilde{n}$. This time allocation is proportional to $\widehat{c}$ by the fertility condition and responds to expected income and medical costs. The second captures the contribution of changes in effective childcare per grandparent at a given allocation of parental time. Health investment affects both terms through young households' current resource costs and their parents' health and longevity. Appendix A.3 derives the decomposition.

Let H denote Benchmark, with endogenous health investment, and F denote Frozen, which fixes $\widehat{m}$ at its pre-reform level while retaining medical costs. Both share the pre-reform steady state. Define the baseline childcare relief share $\bar{\rho} = 1 - \overline{\widetilde{n}}/\bar{n} = 1 - 1/\bar{\Phi}$, the fraction of parental childrearing time saved through grandparental care, and write $\bar{\ell} = 1 - v\overline{\widetilde{n}}$.

Health-technology derivatives below are evaluated at $\overline{\widehat{m}}$. Let $\widehat{m}_\chi > 0$ denote the right derivative of the Benchmark medical expenditure-to-wage ratio at $\chi = 0$.

Proposition 2. *Maintain Assumption 1 and Condition H, and suppose an interior pre-reform steady state exists with (11a) strictly slack. At $\chi = 0$, the right derivative of Benchmark steady-state fertility satisfies*

$$(1+\bar{\rho})\frac{n_\chi^H(0)}{\bar{n}} = \frac{\bar{\widehat{s}}}{1-\tau-\bar{\widehat{m}}}\frac{(1-\tau)\bar{h}}{\omega\bar{n}\bar{\ell}} - \frac{\bar{\rho}}{\bar{\pi}} + \widehat{m}_\chi\left[\bar{\rho}\left(\frac{h'}{\bar{h}}+\frac{\pi'}{\bar{\pi}}\right) - \frac{1}{1-\tau-\bar{\widehat{m}}}\right].$$

The Frozen response $n_\chi^F(0)$ is obtained by setting $\widehat{m}_\chi = 0$. Consequently, when

$$\bar{\rho}\left(\frac{h'}{\bar{h}}+\frac{\pi'}{\bar{\pi}}\right) < \frac{1}{1-\tau-\bar{\widehat{m}}},$$

the marginal steady-state fertility response is lower in Benchmark than in Frozen. Equality gives identical responses, and reversing the inequality reverses the ranking.

Appendix A.3 provides the proof.

The first term captures the earnings channel. Its coefficient $\bar{\widehat{s}}/(1-\tau-\bar{\widehat{m}})$ is the ratio of baseline saving to after-tax full-time earnings net of medical spending. Higher expected elderly earnings reduce the saving needed for retirement, releasing resources for current consumption and childrearing. The second term, $-\bar{\rho}/\bar{\pi}$, captures the direct loss of grandparental childcare from shorter retirement time, weighted by the baseline childcare relief share. Both terms are common to Benchmark and Frozen at zero reform.[3]

The final term combines the resource cost of young households' own health investment with the childcare benefit of their parents' earlier investment. A higher medical expenditure-to-wage ratio leaves fewer resources for current consumption and childrearing. In the new steady state, their elderly parents' higher past investment raises childcare capacity and longevity. The inequality in Proposition 2 identifies when this childcare benefit is smaller than the resource cost and health investment reduces the marginal fertility response relative to Frozen. Benchmark can therefore have a smaller fertility increase than Frozen, with both increases measured relative to their common pre-reform steady state.

The factor $1/(1+\bar{\rho})$ reflects how grandparental childcare is shared in steady state: higher fertility spreads each grandparent's childcare across more adult children's households, whereas lower fertility concentrates it among fewer. For given changes in parental childrearing time and effective childcare per grandparent, this feedback dampens the fertility response.

[3]Health capital also raises old-age labor earnings. At zero reform, elderly working time is zero, so the earnings gain attributable to induced health is second order. Fertility and labor responses in the derivative of the elderly income share are also multiplied by χ and vanish at zero. When $\chi > 0$, health, fertility, and future labor all enter the income comparison.

5. Quantitative steady-state results

5.1 Calibration

The quantitative analysis uses exponential health technologies satisfying Assumption 1:

$$h(\widehat{m}) = 1 - (1 - h_0)e^{-\kappa_h \widehat{m}}, \qquad \pi(\widehat{m}) = 1 - (1 - \pi_0)e^{-\kappa_\pi \widehat{m}}. \tag{39}$$

The 2024 fertility target for Chinese cities is a total fertility rate of 1.01, calculated from the age-specific rates in the China Population and Employment Statistical Yearbook 2025 and rounded to two decimal places (National Bureau of Statistics, Department of Population and Employment Statistics, 2025). Assuming an equal sex ratio, we convert this target into fertility per adult, yielding $\bar{n} = 1.01/2 = 0.505$. Urban-sample childcare hours and expenditure-income ratios provide targets for childcare time and household resource allocation. Appendix B.1 documents population coverage and conversions.

Childhood, young adulthood, and potential old age correspond to ages 0-25, 25-55, and 55-85. Young adulthood and potential old age each last 30 years, and retirement occurs at $55 + 30\chi$. The main policy experiment takes as its institutional reference the increase from 55 to 58 for women previously subject to retirement at 55. The working-adult budget aggregates resources over ages 25-55, and grandparental childcare provision is aggregated over ages 55-85.[4] Table 1 reports the parameters and their calibration basis.

Because mothers and grandmothers play a central role in family childcare (Du et al., 2019), we use their childcare hours and female health and longevity data to calibrate the representative household's childcare provision. Childcare targets come from urban CHNS households with children of either sex aged 0-6 in 2004, 2006, 2009, and 2011. Childcare provided by mothers and linked co-resident grandmothers averages 23.35 and 11.70 hours per week per child, respectively. Appendix B.1 describes the sample and conversion to model time units. We combine maternal employment estimates from Du et al. (2019) with reported work and grandmother-care hours to infer the expected maternal work time released by grandparental childcare.

After adjustments for labor input and employer pension contributions, 2024 urban medical expenditure and wage income imply $\overline{\widehat{m}} = 0.0736$. Urban disposable income and expenditure data determine the split between saving and nonmedical consumption. Together with the household budget, these targets imply $\overline{\widehat{c}} = 0.4247$ and $\overline{\widehat{s}} = 0.2636$. These urban statistics cover all age groups and proxy young households' resource allocation.

We fit the health-capital technology to the shares of city women aged 60-84 classified as healthy or basically healthy in 2010 and 2020. Longevity targets are expected years lived between ages 55 and 85 from the 2010 life table and an extrapolated 2024 life table. Pairing same-year medical expenditure and old-age health uses a steady-state approximation, as explained in Appendix B.1.

We recover preference parameters from these targets and household optimality. The medical first-order condition determines $\gamma+\xi$, the sum of the preference weights on old-age childcare and leisure. Observed grandparental childcare determines its share of retirement time, identifying the two weights separately.

[4] Within-stage aggregation does not capture the overlap between specific retirement ages and grandchildren's childcare needs. If these needs are concentrated in the years affected by the extension, grandparental childcare could fall more than the model's proportional reduction in retirement time implies.

Table 1. Baseline parameters and calibration

Parameter	Baseline value	Method	Target or restriction
α	0.43882	External	Complement of China's 2023 labor share of 0.56118, PWT 11.0/FRED
τ	0.20690	Institutional conversion	Employer 16%, employee 8%; total labor-cost denominator: 0.24/1.16
β	0.78481	Internal	Saving and nonmedical consumption composition, capital clearing, and Euler condition
θ	0.073387	Internal	Baseline fertility $\bar{n} = 0.505$ and fertility first-order condition
γ	1.3665	Internal	Medical expenditure-to-wage ratio, longevity, childcare, and medical first-order condition
ξ	0.014878	Internal	Baseline grandparental care time and old-age time-allocation condition
h_0, κ_h	0.68393; 21.499	Two-point technology fit	Healthy or basically healthy shares of older city women in 2010 and 2020 and consistent medical expenditure-to-wage ratios
π_0, κ_π	0.59280; 22.631	Two-point technology fit	Survival duration between 55 and 85 from the 2010 and extrapolated 2024 life tables
v	0.10693	Partly external construction	CHNS maternal and grandmaternal care and expected maternal work time released
η	20.320	Joint calibration	Baseline parental childrearing time and effective grandparental care

Notes: Medical expenditure, nonmedical consumption, and saving are measured relative to potential total labor cost. Appendix B.1 documents data conversions and technology fitting.

The exponential specification links the fitted health levels to marginal returns and curvature, which also govern policy responses. Appendix C examines this dependence by varying health elasticities at fixed baseline targets, changing curvature at fixed levels and slopes, and recalibrating alternative health, medical-expenditure, and saving targets.

We infer a common productivity path by matching OECD BAU1 GDP-per-capita growth targets in the Benchmark transition with retirement at 58. BAU1 incorporates already-legislated retirement changes (OECD, 2025). All experiments use this productivity path to recover wages, capital, output, and consumption from the normalized allocations. By Lemma 1, normalized household allocations do not depend on this path.[5] Appendices B.2-B.4 describe the reconstruction and source data.

5.2 Fertility profiles and policy thresholds

Benchmark allows the medical expenditure-to-wage ratio to respond to reform, while Frozen fixes it at $\overline{\widehat{m}}$. Grandparental childcare, fertility, saving, and prices adjust in both specifications. Appendix D.1 separately examines the contribution of longevity.

The calibrated parameters satisfy the inequality in Proposition 2: at $\chi = 0$, the marginal steady-state fertility response is smaller in Benchmark than in Frozen. Appendix B.5 reports the numerical evaluation.

[5]Common productivity also cancels from reform-to-no-reform capital, output, and consumption ratios and hence from log-utility welfare comparisons; Appendix A.1 establishes this result.

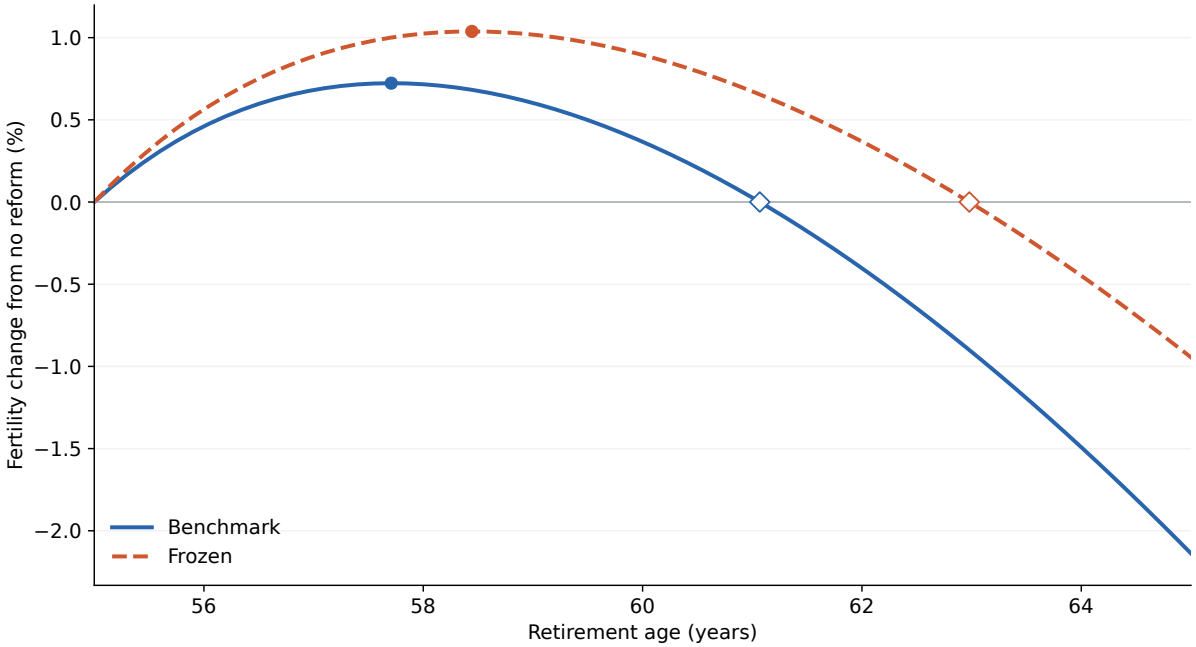

Figure 3. Steady-state fertility effects under alternative health specifications
Notes: The figure compares Benchmark and Frozen over retirement ages 55-65. The vertical axis reports $100[n^j(\chi)/n(0) - 1]$ in percent, for $j \in \{H, F\}$. Solid blue and dashed orange lines denote Benchmark and Frozen. Filled circles mark the peak χ_P; open diamonds mark the nonzero return-to-baseline threshold χ_L.

Figure 3 compares steady-state fertility under different permanent retirement ages with the common no-reform level $n(0) = 0.505$. Each profile has a peak at χ_P and equals the no-reform level again at $\chi_L > 0$. Between these thresholds, fertility falls as the retirement age rises but remains above its no-reform level.

Both profiles are inverted U-shaped. Section 5.3 decomposes them into the contributions of elderly labor income, reduced childcare time, and health investment.

Steady-state fertility peaks at retirement ages of approximately 57.7 in Benchmark and 58.4 in Frozen; it equals the no-reform level at approximately 61.1 and 63.0, respectively. Endogenous health investment thus lowers the peak and return-to-baseline ages by about 0.7 and 1.9 years. Between approximately 61.1 and 63.0, Benchmark predicts fertility below the no-reform level, while Frozen predicts fertility above it.

With permanent retirement at 58, Benchmark steady-state fertility is approximately 0.72% above the no-reform level, corresponding to a TFR increase from 1.010 to about 1.017 under the same population definition. At retirement ages 64 and 70, it is approximately 1.49% and 6.24% below that level.

5.3 Income, childcare, and health mechanisms

To quantify these channels, we construct a Time Compression (TC) counterfactual that fixes the medical expenditure-to-wage ratio, health, and longevity at their pre-reform levels and sets the elderly labor-income term to zero in the normalized household conditions. Childrearing time per young adult then remains at its pre-reform level.[6] TC fertility adjusts to shorter grandparental

[6]With $\widehat{m} = \overline{\widehat{m}}$, removing elderly labor income gives $(1+\theta)\widehat{c} + \widehat{s} = 1 - \tau - \overline{\widehat{m}}$ and $\widehat{s}/\widehat{c} = \beta(\omega - \tau)/\omega$. Thus $\widehat{c} = \overline{\widehat{c}}$, and fertility optimality implies $vn/\Phi = v\bar{n}/\bar{\Phi}$.

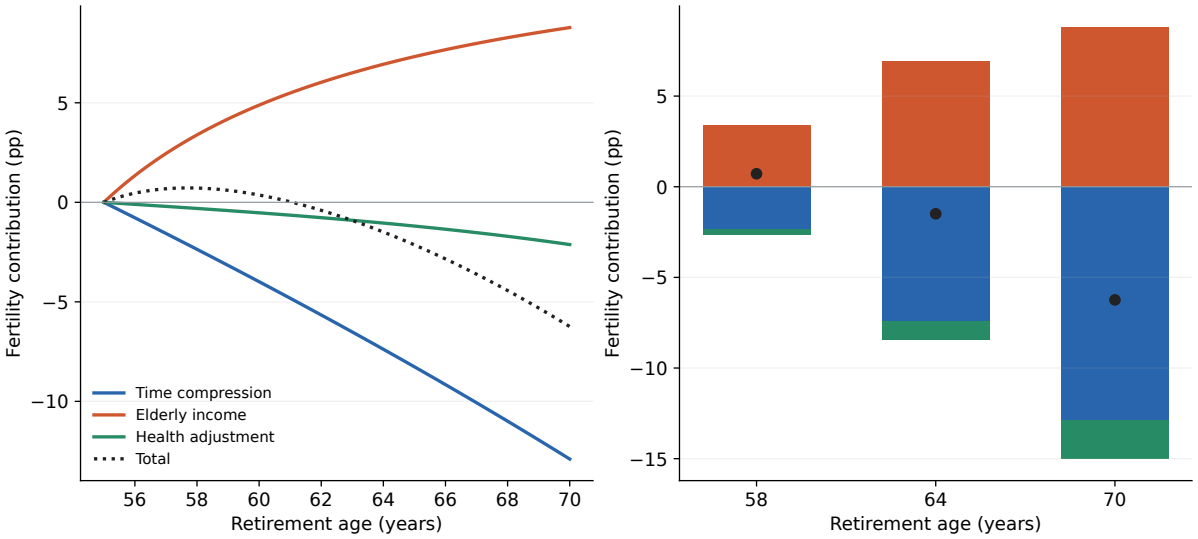

Figure 4. Time, resource, and health contributions to steady-state fertility

Notes: The left panel shows the nested contributions across retirement ages; the right reports reforms to 58, 64, and 70. Black dots denote total fertility effects and colored bars the three contributions. All contributions are normalized by common baseline fertility and expressed in percentage points. Orange denotes elderly labor income, blue time compression, and green the net endogenous-health contribution. Contributions are computed in the order TC, Frozen, Benchmark.

childcare time and changes in how that childcare is shared among adult children. Adding elderly labor income yields Frozen; allowing medical investment to respond then yields Benchmark. This sequential decomposition gives

$$n^H(\chi) - n(0) = [n^{TC}(\chi) - n(0)] + [n^F(\chi) - n^{TC}(\chi)] + [n^H(\chi) - n^F(\chi)]. \tag{40}$$

Dividing each contribution by common baseline fertility and multiplying by 100 yields the percentage-point decomposition in Figure 4.

The positive income contribution follows from households' consumption-saving allocation. Let $q = (1-\tau)\chi h/(n\ell)$ denote the elderly cohort's after-tax labor earnings relative to total pretax labor income in the steady state. Capital clearing, the pension budget, and household optimality imply

$$\frac{\widehat{s}}{\widehat{c}} = \frac{\beta(\omega - \tau)}{\omega + q}.$$

An increase in q lowers the saving-consumption ratio. At a fixed medical expenditure-to-wage ratio, this raises the consumption-to-wage ratio $\widehat{c}$. Fertility optimality makes childrearing time per young adult proportional to $\widehat{c}$, so the reallocation supports childbearing. This mechanism underlies the income contribution in Figure 4.

Time compression works through the cost per child. Grandparental childcare lowers parental time per child to $v/(1+\eta g)$; less childcare raises this requirement and permits fewer births from a given amount of parental time. The marginal time cost of losing childcare is larger when it is already scarce. In the calibrated Frozen economy, income gains initially outweigh childcare losses. Beyond its fertility peak, the additional loss from shorter childcare time exceeds the additional gain from elderly earnings.

When health investment can adjust, delayed retirement raises the medical expenditure-to-wage ratio. Holding q fixed, this lowers $\widehat{c}$, $\widehat{s}$, and childrearing time per young adult. In the calibrated steady states, young adults receive more effective grandparental childcare than under Frozen at the same retirement age. This reflects their parents' better health and longevity and differences in the number of adult children sharing each grandparent's childcare. Young adults' own health investment also affects their expected elderly earnings. The Benchmark-Frozen difference captures the net effect of these resource, childcare, and income changes. In Figure 4, this contribution is negative and becomes more negative as the retirement age rises.

For retirement at 58, the elderly labor income contribution is approximately 3.39 percentage points, the time-compression contribution is minus 2.36 points, and the net contribution of health adjustment is minus 0.31 points. Together, these yield a net fertility gain of about 0.72%. At 70, the time-compression contribution is minus 12.90 points, exceeding the 8.78-point income contribution in absolute value. Health adjustment contributes a further minus 2.12 points, bringing the total fertility effect to minus 6.24%.

Health improvements only partly offset lost retirement time relative to no reform. With permanent retirement at 58, Benchmark old-age duration increases by about 0.36 years, while retirement time falls by 2.64 years. Despite better health, effective grandparental childcare per young adult remains about 9.3% below the no-reform level. Appendix D.1 reports the health and time profiles.

Appendix C varies health returns, curvature, and calibration targets. Across all 26 specifications satisfying the maintained time-substitution restriction, Benchmark steady-state fertility peaks and returns to its no-reform level at lower retirement ages than under Frozen. The shifts vary in size: among the 18 cases that preserve baseline targets, the difference in return-to-baseline ages ranges from 0.58 to 7.17 years.

6. Health investment and intergenerational fertility

6.1 Policy experiments and equilibrium paths

The economy starts from a common pre-reform balanced-growth path. An unanticipated permanent retirement reform occurs in period 1:

$$\chi_0 = 0, \qquad \chi_t = \chi_R \quad (t \geq 1), \qquad \chi_R \in \{0.1, 7/30\}. \tag{41}$$

The experiments raise retirement from 55 to 58 and 62, respectively; the latter is an illustrative seven-year extension. Age 58 yields steady-state fertility gains in both Benchmark and Frozen, while age 62 lies between their return-to-baseline ages. In the figures, $t = 0$ is the baseline corresponding to the 2024 data, and $t = 1$ is the implementation period. Each model period represents 30 years.

Cohort t denotes adults who are young in period t. Cohort 1 chooses fertility, medical investment, and saving under the new policy. Cohort 0 is elderly at implementation and made these decisions in period 0 expecting no reform. Thus, $n_0 = \bar{n}$ and $\widehat{m}_0 = \overline{\widehat{m}}$. With $\bar{G}_0^Z$ denoting the pre-reform productivity growth factor, initial reform-period capital satisfies

$$G_1^Z n_0 k_1 = \widehat{w}_0^Z \widehat{s}_0, \qquad k_1 = \frac{\bar{G}_0^Z}{G_1^Z} k_0. \tag{42}$$

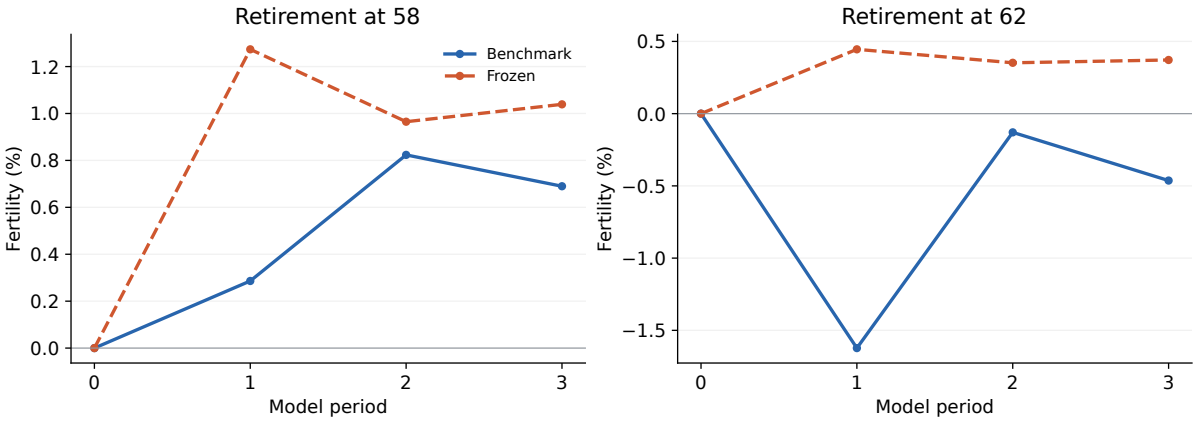

Figure 5. Health adjustment and the fertility transition following delayed retirement
Notes: The left and right panels show reforms from 55 to 58 and 62. The vertical axis is $P_t^j(n) = 100[n_t^{R,j}/n_t^{N,j} - 1]$. Blue denotes Benchmark, orange Frozen, and the gray zero line the common no-reform reference. Differences between reform paths are in percentage points. The horizontal axis shows model periods $t = 0, 1, 2, 3$, each lasting 30 years; $t = 0$ is the pre-reform baseline and $t = 1$ the implementation period. The final plotted point is period 3; long-run limits are reported separately in the text.

Post-reform cohorts correctly anticipate the permanent policy and future equilibrium prices. Benchmark and Frozen inherit identical assets, health, and population states and use the common productivity path calibrated in Section 5.1. Appendix B.4 describes the numerical solution and convergence checks.

Use H, F for Benchmark and Frozen, respectively, and R, N for reform and no reform. Define the percentage policy effect relative to no reform and the additional effect of health adjustment relative to Frozen as

$$P_t^j(X) = 100\left(\frac{X_t^{R,j}}{X_t^{N,j}} - 1\right), \qquad \mathcal{H}_t(X) = P_t^H(X) - P_t^F(X).$$

The former is measured in percent and the latter in percentage points. For the same retirement reform, the analytical comparisons use the Benchmark-Frozen level difference $\Delta X_t = X_t^H - X_t^F$ and its right derivative with respect to χ at zero reform, $\Delta X_t'$. The local results maintain Assumption 1 and Condition H and start from a common interior pre-reform steady state with (11a) strictly slack.[7]

6.2 Fertility across generations

Figure 5 reports fertility responses relative to no reform. Under Frozen, higher expected old-age earnings release current resources and initially outweigh the loss of parental childcare, so cohort 1's fertility rises in both experiments. The gain is smaller at 62 because more childcare time is lost. Additional births enlarge the next generation of young adults sharing each grandparent's childcare, tending to moderate its fertility gain. Fertility also changes next-period labor supply and the ratio of future after-tax elderly earnings to total pretax labor income. These interactions produce oscillating fertility responses under Frozen in the simulations.

[7] Appendix A.4.2 constructs differentiable Benchmark and Frozen branches and shows that this slackness persists uniformly across cohorts for sufficiently small reforms.

Table 3. Fertility policy effects with and without health adjustment

Age	Cohort	Benchmark (%)	Frozen (%)	Gap (pp)	TFR gap
58	1	+0.29	+1.27	-0.99	-0.0100
58	2	+0.82	+0.96	-0.14	-0.0014
58	Long run	+0.72	+1.02	-0.31	-0.0031
62	1	-1.62	+0.45	-2.07	-0.0209
62	2	-0.13	+0.35	-0.48	-0.0049
62	Long run	-0.40	+0.37	-0.77	-0.0078

Notes: Policy effects compare reform with the common no-reform path. Gap is Benchmark minus Frozen, calculated before rounding. TFR gap equals $2(n^H - n^F)$, in births per woman under the calibration mapping TFR $= 2n$. Long-run rows report the limits of the respective transition paths.

Table 3 reports the total reform effects and the Benchmark-Frozen differences. At retirement age 58, cohort 1's fertility rises by 0.29% in Benchmark and 1.27% in Frozen. The difference is -0.99 percentage points for cohort 1 and -0.31 points in the long run.

At retirement age 62, cohort 1's fertility falls by 1.62% in Benchmark but rises by 0.45% in Frozen, both relative to no reform. Benchmark fertility remains below no reform in cohorts 2 and 3, at -0.13% and -0.46%, and converges to -0.40%; the long-run Frozen effect is +0.37%.

The calibration uses TFR $= 2n$. The Benchmark–Frozen gap for cohort 1 therefore corresponds to about -0.010 births per woman at 58 and -0.021 at 62. Allowing health investment to respond can thus change the sign of the predicted fertility effect of retirement reform. Appendix C.4 distinguishes sign disagreement from changes in magnitude across all 26 admissible specifications.

In both reference reforms, the Benchmark-Frozen fertility difference is most negative for cohort 1. Later differences oscillate toward the long-run value. Section 6.3 separates the effects of parents' health from changes in the number of adult children sharing their childcare.

6.3 Medical costs and delayed childcare benefits

Cohort 1 raises medical spending relative to wages while its elderly parents' health is already fixed. Later cohorts receive childcare from parents who invested under the new policy. Under Assumption 1 and Condition H, the medical expenditure-to-wage ratio exceeds its pre-reform level for every post-reform young cohort along any interior Benchmark path starting from the pre-reform steady state, with $0 < \chi < \bar{\pi}$ and (11a) strictly slack. Appendix A.4.1 proves this result.

With inherited health and population fixed, retirement at 58 and 62 reduces period-1 effective grandparental childcare per young adult by approximately 10.83% and 25.28%, respectively, relative to no reform in both specifications. Under Frozen, childcare time per grandparent stays at its lower post-reform level; subsequent changes in effective childcare per young adult reflect the number of adult children sharing it.

In period 2, cohort 1 provides childcare as grandparents. Compared with Frozen, these grandparents have better health and longer lives in Benchmark, while their lower fertility in period 1 leaves fewer young adults sharing their childcare. At retirement age 58, the period-2 loss of effective childcare per young adult relative to no reform is about 8.87% in Benchmark, compared with 11.95% in Frozen. Childcare thus remains below its no-reform level in both specifications.

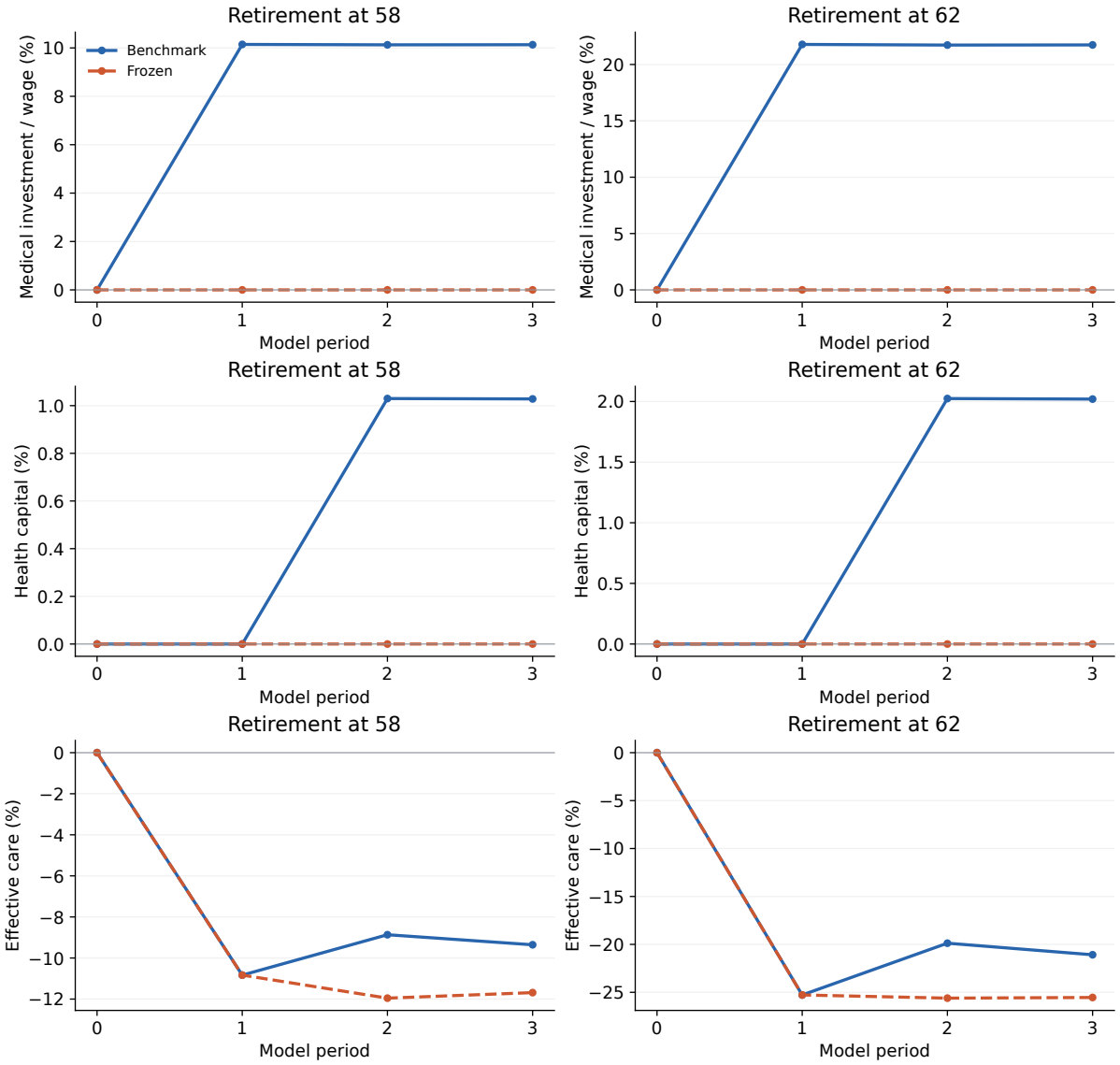

Figure 6. Immediate medical adjustment and delayed health and childcare returns

Notes: Left and right columns correspond to retirement at 58 and 62. Rows report policy percentage changes in $\widehat{m}_t = m_t/w_t$, $h_t = h(\widehat{m}_{t-1})$, and $g_t = \mu_t h_t / n_{t-1}$. Blue denotes Benchmark, orange Frozen, and gray the common no-reform reference. Under Frozen, the medical expenditure-to-wage ratio and health-capital effects are zero, but medical costs remain in the budget.

Writing $\Delta \log X_t = \log X_t^{R,H} - \log X_t^{R,F}$, the difference in effective grandparental childcare satisfies

$$\Delta \log g_t = \Delta \log \mu_t + \Delta \log h_t - \Delta \log n_{t-1}.$$

Proposition 3. *Maintain Assumption 1 and Condition H, and let Benchmark and Frozen inherit identical states from an interior pre-reform steady state satisfying (11a) strictly. Consider a marginal permanent retirement extension at zero reform. The additional fertility effect of allowing health investment to adjust, relative to Frozen, is negative for cohort 1 and more negative than for any later cohort:*

$$\Delta n_1' = -\frac{\bar{n}\widehat{m}_\chi}{1-\tau-\overline{\widehat{m}}} < 0, \qquad \Delta n_t' > \Delta n_1' \quad (t \geq 2).$$

Here, $\widehat{m}_\chi > 0$ is the right derivative of the Benchmark medical expenditure-to-wage ratio at zero reform, common to all post-reform young cohorts. The marginal Benchmark-Frozen fertility differences alternate around and converge to their long-run value.

Appendix A.4 provides the proof.

The local accounting behind Proposition 3 makes the channels explicit:

$$\frac{\Delta n_t'}{\bar{n}} = -\frac{\widehat{m}_\chi}{1-\tau-\overline{\widehat{m}}} + \bar{\rho}\left(\frac{h'}{\bar{h}} + \frac{\pi'}{\bar{\pi}}\right)\widehat{m}_\chi \mathbf{1}_{\{t\geq 2\}} - \bar{\rho}\frac{\Delta n_{t-1}'}{\bar{n}}, \qquad \Delta n_0' = 0. \tag{42a}$$

The three terms capture the cost of a higher medical expenditure-to-wage ratio, additional childcare from parents' health and longevity, and changes in the number of adult children sharing each grandparent's childcare. For cohort 1, parental health and population are predetermined. At zero reform, the direct first-order elderly income response is common to Benchmark and Frozen, leaving only the medical resource cost in their fertility difference. From cohort 2 onward, healthier, longer-lived parents supply more effective childcare per grandparent under Benchmark than under Frozen. If the preceding cohort has lower fertility in Benchmark, fewer young adults share this childcare; higher preceding-cohort fertility reverses that effect. The coefficient $-\bar{\rho}$ transmits deviations from the long-run difference, generating alternating convergence because $0 < \bar{\rho} < 1$.

Table B.3 quantifies these contributions and checks them against independently solved equilibrium responses. It separates gains from parental health and longevity from changes in how childcare is shared, a mechanism also present in fixed-health models. Proposition 3 establishes the cohort ordering; a negative long-run effect additionally requires the cost–benefit inequality in Proposition 2.

Equation (42a) is local to zero reform: at positive retirement shares, induced health also affects elderly earnings at first order. Appendix C.5 examines marginal extensions from initial retirement ages 58 and 62. Both specifications start from the corresponding Benchmark steady state, and Frozen holds the medical expenditure-to-wage ratio at its initial value. For the cohorts examined and the long-run limits, the Benchmark-Frozen marginal fertility difference remains most negative for cohort 1. These checks vary the pre-reform retirement age while retaining predetermined parental health and the 30-year investment lag.

6.4 Capital and lifetime welfare

For the reforms from $\chi_0 = 0$ in Section 6.1, Assumption 1 and Condition H imply lower saving-to-wage and saving-consumption ratios than under no reform in both specifications, along any interior path with $0 < \chi < \bar{\pi}$ and (11a) strictly slack. Expected elderly earnings lower the saving-consumption ratio. At a given elderly earnings ratio, a higher medical expenditure-to-wage ratio further reduces saving relative to wages. Appendix A.4.1 gives the exact identities.

In the calibrated experiments, reform raises effective labor at implementation while capital is predetermined, lowering the wage per effective unit. Under Frozen, retirement at 58 reduces cohort 1's saving-to-wage ratio by about 7.53% and its wage by 7.38%, both relative to no reform. These declines reduce the saving that finances next-period capital. Fertility determines how many young adults share that capital; its adjustment does not offset the saving decline in the computed paths.

Under the common productivity path, the exact Benchmark-Frozen log difference in capital per young adult, normalized by productivity, satisfies

$$\Delta \log k_{t+1} = \alpha \Delta \log k_t - \alpha \Delta \log \ell_t + \Delta \log \widehat{s}_t - \Delta \log n_t. \tag{43}$$

Proposition 4. *Maintain Assumption 1 and Condition H, and let Benchmark and Frozen inherit identical states from an interior pre-reform steady state satisfying (11a) strictly. Consider a marginal permanent retirement extension at zero reform. Health adjustment reduces normalized capital from period 2 onward relative to Frozen, leaving initial capital unchanged:*

$$\Delta(\ln k_1)' = 0, \qquad \Delta(\ln k_t)' < 0 \quad (t \geq 2).$$

Appendix A.5 provides the proof.

At zero reform, additional health investment reduces cohort 1's saving-to-wage ratio and fertility by the same proportion to first order, relative to Frozen. Their first-order contributions to (43) therefore cancel. Lower fertility releases young labor. With period-1 capital fixed, greater labor lowers the wage per effective unit and hence saving in goods. The first-order effect through $-\alpha \Delta \log \ell_1$ thus reduces normalized capital in period 2.

For later cohorts in this local comparison, the fertility difference is higher than for cohort 1, while the proportional saving-to-wage response is unchanged. The combined saving and fertility terms in (43) are therefore negative. Inherited capital losses and greater effective labor also lower wages, keeping the capital difference negative. Later cohorts thus receive more grandparental childcare than under Frozen while inheriting less capital.

By Lemma 1, capital does not enter the normalized fertility and medical-investment system. Capital changes therefore do not feed back into these decisions.

Lifetime welfare depends on consumption in both life stages, fertility, and old-age childcare and leisure. Childcare received from parents affects welfare through young adults' childrearing costs and choices. Appendix B.6 expresses welfare changes relative to no reform as equivalent proportional changes in consumption in both life stages. In the calibrated reform to age 62, cohort 1 loses welfare under both specifications. Its fertility is lower in Benchmark, but its consumption-equivalent welfare loss is approximately 0.28 percentage points smaller than in Frozen, chiefly because longer survival allows more leisure in its own old age. The long-run

Benchmark-Frozen welfare difference is negative in both calibrated reforms as lower capital reduces consumption resources. Appendix D.3 reports the numerical welfare decomposition.

Allowing health investment to adjust weakly raises optimized lifetime utility at given prices, pension benefits, and parental childcare. Changes in wages, capital returns, pensions, and parental childcare can offset this gain when all households adjust.

7. Conclusion

For a marginal permanent retirement extension from the pre-reform steady state without old-age employment, allowing health investment to adjust lowers fertility relative to Frozen most strongly among adults making fertility decisions at implementation. They raise medical spending relative to wages while their elderly parents' health is already fixed. Later generations receive more childcare than under Frozen from healthier, longer-lived parents; assistance per young adult also depends on the number of adult children sharing each grandparent's childcare. The marginal Benchmark–Frozen fertility differences alternate around their long-run value. This value is negative when the medical resource cost outweighs the additional parental childcare benefit, as in the reference calibration.

Relative to no reform, the calibrated extension from 55 to 58 raises fertility among adults young at implementation in both specifications, with a smaller gain in Benchmark. The illustrative extension from 55 to 62 instead lowers their fertility in Benchmark and raises it in Frozen. This sign disagreement holds in 22 of the 26 feasible sensitivity specifications. In the reference calibration, endogenous health investment also lowers the retirement ages at which steady-state fertility peaks and returns to its no-reform level by approximately 0.7 and 1.9 years, respectively, relative to Frozen.

The local analysis at zero reform also shows that generations reaching young adulthood after implementation receive more grandparental childcare but inherit less capital per young adult than under Frozen. Capital changes affect consumption and welfare without feeding back into fertility decisions. In the calibrated extension to 62, adults young at implementation have lower fertility in Benchmark but a smaller lifetime welfare loss relative to no reform than in Frozen.

Evaluating China's retirement reform requires distinguishing fertility responses at implementation from those of later generations. In the local comparison, steady-state analysis understates the fertility reduction from health investment relative to Frozen among adults young at implementation.

The model aggregates health investment and fertility over thirty years of working adulthood and fixes existing grandparents' health when reform begins. An age-specific model could assess how shorter investment lags, health adjustment during grandchild care, and gradual reform implementation change fertility responses across generations. The health technologies are calibrated to aggregate expenditure and health measures; the calibration does not identify causal returns to medical spending. Longitudinal evidence on these returns and the age overlap of employment and grandparental childcare would strengthen quantitative policy assessment.

Appendix A. Theoretical derivations and proofs

This appendix uses the notation and Assumption 1 from the main text. Wage-normalized medical investment is $\widehat{m} = m/w$. Normalized parental childrearing time is $\widetilde{n} = n/\Phi$, so actual parental time is $v\widetilde{n}$; $E = \mu h$ is effective grandparental childcare per elderly adult. Overbars denote common pre-reform values. Health-technology derivatives in local proofs are evaluated at $\overline{\widehat{m}}$. The baseline childcare relief share is $\bar{\rho} = 1 - \overline{\widetilde{n}}/\bar{n}$, and $\Delta X_t'$ is the right policy derivative defined in the main text.

A.1 Proof of Lemma 1: equilibrium separation

Fix inherited $N_0, N_1, K_1 > 0$ and $\widehat{m}_0$, a permanent policy $0 \le \chi < \pi(\widehat{m}_0)$, and a strictly positive productivity path. Expectations are realized. Define

$$\omega = \tau + \frac{\alpha}{1-\alpha}, \qquad q_{t+1} = \frac{(1-\tau)\chi h(\widehat{m}_t)}{n_t \ell_{t+1}}, \qquad z_{t+1} = \frac{q_{t+1}}{1-\tau}. \tag{A.1}$$

Here q_{t+1} is elderly after-tax labor income relative to total pretax labor income in period $t+1$, and z_{t+1} is the elderly share of effective labor in that period, when cohort t is old. The recurring resource-allocation denominator is

$$D(q) = (1+\theta)(\omega + q) + \beta(\omega - \tau). \tag{A.2}$$

Factor prices, capital clearing and the PAYG budget imply

$$\frac{w_{t+1}}{R_{t+1} w_t} = \frac{1-\alpha}{\alpha} \frac{\widehat{s}_t}{n_t \ell_{t+1}}, \qquad \frac{d_{t+1}}{w_{t+1}} = n_t \ell_{t+1} (\omega + q_{t+1}).$$

Consequently, the Euler and fertility conditions and the young budget become

$$\beta \widehat{c}_t = \widehat{s}_t \frac{\omega + q_{t+1}}{\omega - \tau}, \qquad (1-\tau) v n_t / \Phi_t = \theta \widehat{c}_t, \qquad (1+\theta)\widehat{c}_t + \widehat{s}_t + \widehat{m}_t = 1 - \tau.$$

Solving gives

$$\begin{aligned} \widehat{c}_t &= \frac{(1 - \tau - \widehat{m}_t)(\omega + q_{t+1})}{D(q_{t+1})}, \\ \widetilde{n}_t &= \frac{\theta(1 - \tau - \widehat{m}_t)(\omega + q_{t+1})}{v(1-\tau) D(q_{t+1})}, \\ \widehat{s}_t &= \frac{\beta(\omega - \tau)(1 - \tau - \widehat{m}_t)}{D(q_{t+1})}. \end{aligned} \tag{A.3}$$

After substituting factor prices and the pension budget, the ratio of marginal benefits to marginal cost in the medical investment condition is

$$\begin{aligned} \mathcal{R}(\widehat{m}_t, z_{t+1}, \chi) = \frac{\beta(1 - \tau - \widehat{m}_t)}{D(q_{t+1})} \Bigg\{ & q_{t+1} \frac{h'(\widehat{m}_t)}{h(\widehat{m}_t)} \\ & + \big[(\gamma + \xi)(\omega + q_{t+1}) + \tau\big] \frac{\pi'(\widehat{m}_t)}{\pi(\widehat{m}_t) - \chi} \Bigg\} = 1. \end{aligned} \tag{A.4}$$

This substitution follows private optimization at given prices and benefits: the private pension return remains $b_{t+1}\pi'(\widehat{m}_t)$, without internalizing changes in the common benefit rate. The

remaining equations are

$$\Phi_t = 1 + \frac{\eta\lambda_c[\pi(\widehat{m}_{t-1}) - \chi]h(\widehat{m}_{t-1})}{n_{t-1}}, \qquad \ell_{t+1} = 1 - \frac{vn_{t+1}}{\Phi_{t+1}} + \frac{\chi h(\widehat{m}_t)}{n_t}.$$

Together with $n_0 = N_1/N_0$, these form a closed system for $\{n_t, \widehat{m}_t\}_{t\geq 1}$, independent of productivity and physical capital.

Conversely, take a reduced solution with positive fertility, $0 < \widehat{m}_t < 1 - \tau$, positive retirement time and labor, and (11a) strictly slack. For any positive productivity path, reconstruct recursively

$$N_{t+1} = n_t N_t, \qquad w_t = (1-\alpha)A_t\left(\frac{K_t}{N_t\ell_t}\right)^{\alpha},$$
$$R_t = \alpha A_t\left(\frac{N_t\ell_t}{K_t}\right)^{1-\alpha}, \quad (c_t, s_t, m_t) = w_t(\widehat{c}_t, \widehat{s}_t, \widehat{m}_t), \quad K_{t+1} = N_t s_t.$$

Recover h_t, π_t from inherited health investment, $\mu_t = \lambda_c(\pi_t - \chi)$, $\zeta_t = (1-\lambda_c)(\pi_t - \chi)$, and

$$b_t = \frac{\tau w_t n_{t-1}\ell_t}{\pi_t - \chi}, \qquad d_t = R_t s_{t-1} + (1-\tau)w_t\chi h_t + b_t(\pi_t - \chi), \quad s_0 = K_1/N_0.$$

The reduced equations then reproduce all household budgets and first-order conditions. At given prices, pension benefits and childcare assistance from parents, the time-optimized objective is

$$\log c_t + \theta \log n_t + \beta \log d_{t+1} + \beta(\gamma + \xi)\log[\pi(\widehat{m}_t) - \chi] + \text{constant}.$$

Under Assumption 1, it is concave on the convex individual-budget domain before imposing (11a): young consumption is affine, and old-age consumption and retirement time are concave. Thus the first-order conditions identify a global maximizer. Strict satisfaction of (11a) preserves optimality on the restricted feasible set; the same argument applies to elderly time allocation (16). The initially old choose only current time, with assets and health inherited. Aggregating budgets cancels PAYG transfers and gives $Y_t = N_t(c_t + m_t) + N_{t-1}d_t + K_{t+1}$, so goods also clear. Expectations equal the reconstructed continuation prices.

Every full equilibrium therefore projects onto the same reduced system, and each strictly slack interior solution reconstructs an equilibrium under any positive productivity path. The normalized equilibrium set is therefore invariant to the productivity path. Frozen replaces medical optimality by the fixed medical expenditure-to-wage ratio. This proves Lemma 1. □

A further implication of Lemma 1 concerns reform-to-no-reform ratios. Within either Benchmark or Frozen, compare corresponding equilibrium branches with common inherited states and a common productivity path. Let $\Delta^P \log X_t$ denote log reform minus log no reform. Capital clearing gives

$$\Delta^P \log k_{t+1} = \alpha\Delta^P \log k_t - \alpha\Delta^P \log \ell_t + \Delta^P \log \widehat{s}_t - \Delta^P \log n_t, \qquad \Delta^P \log k_1 = 0.$$

The fertility, labor, and saving-to-wage differences are productivity-independent. Factor prices and $c_t = w_t\widehat{c}_t$, $d_{t+1} = \beta R_{t+1}c_t$ imply the same invariance for capital, price and consumption ratios. The same holds for output per capita: common productivity cancels, and the population

denominator $n_t + 1 + \pi_t/n_{t-1}$ is productivity-independent. For the initially old,

$$d_1 = w_1[\omega n_0 \ell_1 + (1-\tau)\chi h_1], \qquad h_1 = h(\widehat{m}_0),$$

which has the same ratio invariance. Log-utility differences, and hence consumption-equivalent changes, combine these consumption ratios with productivity-independent fertility and time ratios.

A.2 Lemma 2 and Proposition 1: steady-state equilibrium and medical responses

Fix $0 \le \chi < \chi_{\max}$ and $G^Z > 0$. Under Assumption 1 and Condition H, we solve for fertility and labor conditional on $\widehat{m}$, then show that $\mathcal{R}$ crosses one with a negative slope after accounting for the response of the labor share. Condition E establishes candidate existence in A.2.5; A.2.6 verifies the time-substitution restriction (11a). Health derivatives are evaluated at $\widehat{m}$.

A.2.1 Steady-state resource allocation and medical optimality

Suppress time subscripts on constant normalized allocations along a balanced-growth path. Equations (A.1)-(A.4) then give the steady-state allocation and medical condition $\mathcal{R}(\widehat{m}, z, \chi) = 1$. The elderly labor share is $z = (\chi h/n)/\ell$, with $q = (1-\tau)z$; positive young labor implies $0 \le z < 1$.

The quotient rule gives the derivatives of the two return weights:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}q}\frac{q}{D(q)} &= \frac{(1+\theta)\omega + \beta(\omega-\tau)}{D(q)^2} > 0, \\ \frac{\mathrm{d}}{\mathrm{d}q}\frac{(\gamma+\xi)(\omega+q)+\tau}{D(q)} &= \frac{\beta(\omega-\tau)(\gamma+\xi) - \tau(1+\theta)}{D(q)^2} \ge 0. \end{aligned} \tag{A.5}$$

Condition H makes the second weight nondecreasing; the first is strictly increasing, so $\mathcal{R}_z > 0$. Also $\mathcal{R}_\chi > 0$, since $\partial[\pi'/(\pi-\chi)]/\partial\chi = \pi'/(\pi-\chi)^2 > 0$.

A.2.2 Uniqueness of fertility and labor conditional on medical investment

Fix a feasible $\{\widehat{m}, \chi\}$. Equation (A.3) determines $\widetilde{n}$ for each candidate z. Using $D(q) > (1+\theta)(\omega+q)$ and $1-\tau-\widehat{m} < 1-\tau$ gives $0 < v\widetilde{n} < \theta/(1+\theta) < 1$. The implied young labor supply is

$$\ell^y = 1 - v\widetilde{n} > 0. \tag{A.6}$$

The fertility-care relationship is $n^2 = \widetilde{n}[n + \eta\lambda_c(\pi-\chi)h]$. Its constant term is negative, so it has exactly one positive root:

$$n = \frac{\widetilde{n} + \sqrt{\widetilde{n}^2 + 4\eta\lambda_c[\pi(\widehat{m}) - \chi]h(\widehat{m})\widetilde{n}}}{2}. \tag{A.7}$$

For the candidate z to equal the elderly labor share, it must satisfy

$$n\frac{z(1-v\widetilde{n})}{1-z} = \chi h(\widehat{m}). \tag{A.8}$$

We now show that the left-hand side is strictly increasing in z. Let $\rho = 1-\widetilde{n}/n \in (0,1)$ denote the share of parental childrearing time saved by grandparental childcare and ε_z the elasticity of $\widetilde{n}$ with respect to the elderly labor share. Log-differentiating (A.3) gives

$$\frac{\partial \log \widetilde{n}}{\partial z} = \frac{(1-\tau)\beta(\omega-\tau)}{(\omega+q)D(q)}, \qquad \varepsilon_z = \frac{q\beta(\omega-\tau)}{(\omega+q)D(q)}.$$

To bound the decline in young labor as parental childrearing time rises, use (A.3): $v\widetilde{n}/(1-v\widetilde{n}) \leq \theta(\omega+q)/[\omega+q+\beta(\omega-\tau)]$. Multiplying by the elasticity above gives

$$\frac{v\widetilde{n}}{1-v\widetilde{n}}\varepsilon_z \leq \frac{\theta\beta(\omega-\tau)q}{D(q)[\omega+q+\beta(\omega-\tau)]} \leq \frac{q}{\omega+q} < 1. \tag{A.9}$$

Differentiating the fertility quadratic and the labor term on the left of (A.8), holding $\widehat{m}$ and χ fixed, gives

$$\begin{aligned} \frac{n_z}{n} &= \frac{1}{1+\rho}\frac{\partial \log \widetilde{n}}{\partial z} > 0, \\ \frac{\partial}{\partial z}\left[\frac{z(1-v\widetilde{n})}{1-z}\right] &= \frac{1-v\widetilde{n}}{(1-z)^2}\left[1-(1-z)\frac{v\widetilde{n}}{1-v\widetilde{n}}\varepsilon_z\right] > 0. \end{aligned} \tag{A.10}$$

By (A.9)-(A.10), the left side of (A.8) is strictly increasing from zero to infinity as z goes from zero to one. Hence the conditional labor share is unique: zero at $\chi = 0$ and interior at $\chi > 0$.

A.2.3 Comparative statics of conditional fertility and labor

We differentiate allocations already satisfying (A.8). Their derivatives share the positive coefficient

$$\mathcal{J} = \frac{1+\rho}{1-z} + \left[1-(1+\rho)\frac{v\widetilde{n}}{1-v\widetilde{n}}\right]\varepsilon_z > 0. \tag{A.11}$$

Positivity follows directly from (A.10): the derivative of the left-hand side of (A.8) with respect to z, multiplied by $(1+\rho)(1-z)/[n(1-v\widetilde{n})]$, equals $\mathcal{J}$.

For $z > 0$, first write the three total-differential relationships:

$$\begin{aligned} \frac{\mathrm{d}\widetilde{n}}{\widetilde{n}} &= -\frac{\mathrm{d}\widehat{m}}{1-\tau-\widehat{m}} + \frac{\varepsilon_z}{z}\,\mathrm{d}z, \\ (1+\rho)\frac{\mathrm{d}n}{n} &= \frac{\mathrm{d}\widetilde{n}}{\widetilde{n}} + \rho\left[\left(\frac{h'}{h}+\frac{\pi'}{\pi-\chi}\right)\mathrm{d}\widehat{m} - \frac{\mathrm{d}\chi}{\pi-\chi}\right], \\ \frac{\mathrm{d}n}{n} + \frac{\mathrm{d}z}{z(1-z)} - \frac{v\widetilde{n}}{1-v\widetilde{n}}\frac{\mathrm{d}\widetilde{n}}{\widetilde{n}} &= \frac{\mathrm{d}\chi}{\chi} + \frac{h'}{h}\,\mathrm{d}\widehat{m}. \end{aligned}$$

Eliminating $\mathrm{d}n$ and $\mathrm{d}\widetilde{n}$ yields

$$\begin{aligned} z_{\widehat{m}} &= \frac{z}{\mathcal{J}}\left[\frac{h'}{h} - \rho\frac{\pi'}{\pi-\chi} + \frac{1-(1+\rho)v\widetilde{n}/(1-v\widetilde{n})}{1-\tau-\widehat{m}}\right], \\ z_{\chi} &= \frac{(1+\rho)(1-z)h/[n(1-v\widetilde{n})] + \rho z/(\pi-\chi)}{\mathcal{J}} > 0. \end{aligned} \tag{A.12}$$

In the second expression, (A.8) replaces z/χ by $(1-z)h/[n(1-v\widetilde{n})]$, allowing continuous extension to $z=0$, where $z_{\widehat{m}}=0$ and $z_\chi = h/[n(1-v\widetilde{n})] > 0$.

Hold $\widehat{m}$ fixed and substitute the conditional labor share $z(\widehat{m},\chi)$ into (A.4). Differentiating with respect to χ gives

$$\frac{\partial}{\partial\chi}\mathcal{R}\big(\widehat{m}, z(\widehat{m},\chi),\chi\big) = \mathcal{R}_\chi + \mathcal{R}_z z_\chi > 0.$$

Here subscripts on $\mathcal{R}$ denote partial derivatives of (A.4), while z_χ holds medical investment fixed. Both terms are positive by (A.5) and (A.12).

A.2.4 Negative slopes at medical-equilibrium intersections

At a medical-equilibrium root, define the earnings share of medical returns:

$$\nu = \frac{qh'/h}{qh'/h + [(\gamma+\xi)(\omega+q)+\tau]\pi'/(\pi-\chi)} \in [0,1). \tag{A.13}$$

Longevity returns are strictly positive, so $\nu < 1$. The root condition in (A.4) implies $q(1-\tau-\widehat{m})h'/h = D(q)\nu/\beta$. Substitution into the definition of ε_z gives

$$\begin{aligned} \varepsilon_z(1-\tau-\widehat{m})\frac{h'}{h} &= \frac{\omega-\tau}{\omega+q}\nu, \\ \varepsilon_z\left[1+(1-\tau-\widehat{m})\frac{h'}{h}\right] &= \varepsilon_z + \frac{\omega-\tau}{\omega+q}\nu \\ &< \frac{q+\omega-\tau}{\omega+q} \le 1. \end{aligned} \tag{A.14}$$

This uses $\varepsilon_z \le q/(\omega+q)$, $\nu<1$, and $\tau \ge 0$. The bound follows directly from the medical-equilibrium condition.

Equation (A.12) further implies

$$z_{\widehat{m}} \le \frac{z(1-z)}{1+\rho}\left(\frac{h'}{h} + \frac{1}{1-\tau-\widehat{m}}\right). \tag{A.15}$$

To verify the algebra, for $z>0$ cross-multiply the positive denominators. The required nonnegative difference expands as

$$\begin{aligned} &\frac{1+\rho}{1-z}\,\rho(1-\tau-\widehat{m})\frac{\pi'}{\pi-\chi} \\ &\quad + \varepsilon_z\left[1+(1-\tau-\widehat{m})\frac{h'}{h}\right] \\ &\quad + (1+\rho)\frac{v\widetilde{n}}{1-v\widetilde{n}}\left\{\frac{1+\rho}{1-z} - \varepsilon_z\left[1+(1-\tau-\widehat{m})\frac{h'}{h}\right]\right\}. \end{aligned}$$

The first two terms are nonnegative. Inside braces, the first term exceeds one and the second is below one by (A.14), so the third term is also nonnegative. At $z=0$, both sides of (A.15) are zero.

Concavity gives $(h'/h)' \le -(h'/h)^2$ and $[\pi'/(\pi-\chi)]' \le -[\pi'/(\pi-\chi)]^2$. Log-differentiating (A.4) at fixed z,χ therefore yields

$$\left.\frac{\partial \log \mathcal{R}}{\partial \widehat{m}}\right|_{z,\chi} \leq -\frac{1}{1-\tau-\widehat{m}} - \nu\frac{h'}{h} - (1-\nu)\frac{\pi'}{\pi-\chi}. \tag{A.16}$$

Differentiating with respect to q, using $q(1+\theta)/D = q/(\omega+q) - \varepsilon_z$ and the root condition, yields

$$\begin{aligned} 0 \leq \frac{z\mathcal{R}_z}{\mathcal{R}} &= \frac{\omega}{\omega+q}\nu + \varepsilon_z\left[1 - \frac{\tau(1-\tau-\widehat{m})}{\omega-\tau}\frac{\pi'}{\pi-\chi}\right] \\ &\leq \frac{\omega}{\omega+q}\nu + \varepsilon_z. \end{aligned} \tag{A.17}$$

Condition H ensures nonnegativity in (A.17). Combining (A.15)-(A.17), dropping the negative longevity term and using $1/(1+\rho) \leq 1$, gives

$$\begin{aligned} -(1-\tau-\widehat{m})\frac{\mathcal{R}_{\widehat{m}} + \mathcal{R}_z z_{\widehat{m}}}{\mathcal{R}} \geq 1 &+ \nu(1-\tau-\widehat{m})\frac{h'}{h} \\ &- (1-z)\left[\frac{\omega\nu}{\omega+q} + \varepsilon_z\right]\left[1 + (1-\tau-\widehat{m})\frac{h'}{h}\right]. \end{aligned} \tag{A.18}$$

Using (A.14) and $z(1-\tau-\widehat{m})h'/h = D(q)\nu/[\beta(1-\tau)]$ the right-hand side of (A.18) expands to

$$\begin{aligned} &1 - (1-z)\varepsilon_z - \frac{(1-z)[\omega + (\omega-\tau)]}{\omega+q}\nu \\ &\quad + \frac{D(q)[1+\omega-\tau]}{\beta(1-\tau)(\omega+q)}\nu^2 \\ &\geq \frac{\tau(1-\nu) + (\omega-\tau)(1-\nu)^2 + (\omega-\tau)\omega\nu^2/(1-\tau)}{\omega+q} > 0. \end{aligned} \tag{A.19}$$

For the lower bound, multiply by $\omega+q$ and use $\varepsilon_z \leq q/(\omega+q)$, $1-z \leq 1$, and $D(q)/\beta > \omega-\tau$. Expanding the square gives the numerator shown. Since $\omega-\tau > 0$ and $\nu < 1$, every root satisfies $\mathcal{R}_{\widehat{m}} + \mathcal{R}_z z_{\widehat{m}} < 0$. At $z = 0$, $\nu = \varepsilon_z = 0$, so the right-hand side of (A.18) equals one.

A.2.5 Endpoints, the unique intersection, and the policy response

With π^{-1} denoting inverse longevity, the feasible medical investment interval is

$$\begin{cases} 0 < \widehat{m} < 1-\tau, & 0 \leq \chi < \pi(0), \\ \pi^{-1}(\chi) < \widehat{m} < 1-\tau, & \pi(0) \leq \chi < \chi_{\max}. \end{cases}$$

Evaluate $\mathcal{R}$ at both endpoints along the conditional labor share $z = z(\widehat{m}, \chi)$. If $\chi < \pi(0)$, (A.5) and the shorter remaining retirement window imply

$$\mathcal{R}\big(0, z(0,\chi), \chi\big) \geq \mathcal{R}(0,0,0) \geq \frac{\beta(\gamma+\xi)}{1+\theta+\beta}(1-\tau)\frac{\pi'(0)}{\pi(0)} > 1,$$

Here $D(0) \leq (1+\theta+\beta)\omega$, and the strict bound is E. For $\chi \geq \pi(0)$, $\mathcal{R}$ instead diverges at the lower endpoint as $\pi - \chi \downarrow 0$, while resources and the longevity slope remain positive. As $\widehat{m} \uparrow 1-\tau$, $\mathcal{R}$ tends to zero: (A.3) gives $\widehat{c} \to 0$, so the marginal utility cost $1/\widehat{c}$ diverges, while

marginal benefits remain bounded because $\chi < \chi_{\max}$. Continuity gives an interior root. Every root crosses strictly downward by A.2.4, precluding a second root.

Since $\mathcal{R}_{\widehat{m}} + \mathcal{R}_z z_{\widehat{m}} < 0$ at the intersection, the implicit function theorem implies that candidate medical investment is locally differentiable with respect to χ. Differentiating the equilibrium identity $\mathcal{R}(\widehat{m}^{ss}(\chi), z(\widehat{m}^{ss}(\chi), \chi), \chi) = 1$ gives

$$\begin{aligned} \big(\mathcal{R}_{\widehat{m}} + \mathcal{R}_z z_{\widehat{m}}\big)\frac{\mathrm{d}\widehat{m}^{ss}}{\mathrm{d}\chi} + \mathcal{R}_\chi + \mathcal{R}_z z_\chi = 0, \\ \frac{\mathrm{d}\widehat{m}^{ss}}{\mathrm{d}\chi} = -\frac{\mathcal{R}_\chi + \mathcal{R}_z z_\chi}{\mathcal{R}_{\widehat{m}} + \mathcal{R}_z z_{\widehat{m}}} > 0. \end{aligned} \tag{A.20}$$

At equilibrium, A.2.3 makes the numerator positive and A.2.4 makes the denominator negative.

Pointwise uniqueness joins the candidate solutions into one differentiable branch. Its portions satisfying (11a) strictly are the equilibria studied here. Strict slackness persists in a neighborhood of each such policy by continuity; at $\chi = 0$, use a right neighborhood and right derivative.

A.2.6 Recovering the remaining equilibrium allocations

Given the medical root, A.2.2 uniquely determines fertility and labor, and (A.3) gives positive consumption and saving. The restrictions $v\widetilde{n} < 1$ and $\pi > \chi$ ensure positive young labor, care, and leisure. Capital clearing is $G^Z n k = (1-\alpha)(k/\ell)^\alpha \widehat{s}$. Divide by k^α and raise to the power $1/(1-\alpha)$ to obtain

$$k = \left[\frac{(1-\alpha)\widehat{s}}{G^Z n \ell^\alpha}\right]^{1/(1-\alpha)} > 0. \tag{A.21}$$

Factor pricing and the government budget then uniquely determine wages, capital returns, and pensions.

It remains to verify (11a). For positive care, (6)-(7) imply

$$\frac{n_t(v - v/\Phi_t)}{\mu_t/n_{t-1}} = \eta h_t v \widetilde{n}_t. \tag{A.21a}$$

In a steady state, (A.21a) makes strict slackness equivalent to $\eta h(\widehat{m}) v\widetilde{n} < 1$, evaluated at the unique candidate. If this inequality holds, Appendix A.1 verifies household optimality and market clearing for the recovered allocation. Conversely, every strictly slack interior balanced-growth equilibrium must solve the reduced system and hence coincide with this candidate. If the candidate fails the inequality, no equilibrium in that strictly slack class exists. This proves Lemma 2. At any existing interior equilibrium satisfying (11a) strictly, (A.20) establishes Proposition 1 under Assumption 1 and Condition H.

A.3 Proof of Proposition 2: total fertility effects and health adjustment

Differentiating $n^2 = \widetilde{n}(n + \eta E)$ along any interior steady-state branch gives

$$(2n - \widetilde{n})n_\chi = (n + \eta E)\widetilde{n}_\chi + \eta \widetilde{n} E_\chi, \qquad E_\chi = \lambda_c\{[(\pi - \chi)h' + \pi' h]\widehat{m}_\chi - h\}.$$

Since $2n - \widetilde{n} = n(1+\rho) > 0$, this yields the decomposition (38).

Maintain Assumption 1 and Condition H and take an interior pre-reform steady state with (11a) strictly slack. The nonsingularity argument in A.2 and baseline strict slackness yield local steady-state branches. All derivatives are evaluated at $\chi = 0^+$. The two specifications share the pre-reform allocation, with $D_0 = D(0)$, $\bar{\ell} = 1 - v\overline{\widetilde{n}}$, and $\bar{\rho} = 1 - \overline{\widetilde{n}}/\bar{n}$. The definition of the elderly income share first gives

$$q_\chi(0) = \frac{(1-\tau)\bar{h}}{\bar{n}\bar{\ell}}. \tag{A.22}$$

Derivatives of health, fertility, and labor are multiplied by χ and vanish at zero reform. Equation (A.22) is therefore identical for Benchmark and Frozen.

To determine the medical response, differentiate (A.4). Let $\overline{\mathcal{R}}_{\widehat{m}}$ denote its partial derivative with respect to medical investment at the common pre-reform root. At this root, the partial derivative with respect to χ, holding $\widehat{m}$ and z fixed, is $1/\bar{\pi}$. Thus

$$\begin{aligned} \overline{\mathcal{R}}_{\widehat{m}} &= -\frac{1}{1-\tau-\overline{\widehat{m}}} - \frac{\pi'}{\bar{\pi}} + \frac{\pi''}{\pi'} < 0, \\ \widehat{m}_\chi &= -\frac{1}{\overline{\mathcal{R}}_{\widehat{m}}} \left\{ \frac{1}{\bar{\pi}} + \frac{(1-\tau)\bar{h}}{\bar{n}\bar{\ell}} \left[\frac{h'/\bar{h} + (\gamma+\xi)\pi'/\bar{\pi}}{[(\gamma+\xi)\omega+\tau]\pi'/\bar{\pi}} - \frac{1+\theta}{D_0} \right] \right\} > 0. \end{aligned} \tag{A.23}$$

The bracketed term is the partial derivative of $\mathcal{R}$ with respect to q at the baseline; Condition H and (A.5) make it nonnegative. The negative derivative with respect to $\widehat{m}$ reflects the rising marginal utility cost of investment and diminishing relative marginal longevity returns, ensuring a finite medical response. Frozen fixes the medical expenditure-to-wage ratio, whose policy derivative is zero.

Differentiate the allocation in (A.3) and the steady-state childcare relationship $n^2 = \widetilde{n}(n + \eta E)$:

$$\begin{aligned} \frac{\widetilde{n}_\chi}{\overline{\widetilde{n}}} &= -\frac{\widehat{m}_\chi}{1-\tau-\overline{\widehat{m}}} + \frac{\beta(\omega-\tau)}{\omega D_0} q_\chi, \\ (1+\bar{\rho})\frac{n_\chi}{\bar{n}} &= \frac{\widetilde{n}_\chi}{\overline{\widetilde{n}}} + \bar{\rho}\left[\left(\frac{h'}{\bar{h}} + \frac{\pi'}{\bar{\pi}} \right) \widehat{m}_\chi - \frac{1}{\bar{\pi}} \right]. \end{aligned} \tag{A.24}$$

Substitute (A.22) and $\beta(\omega-\tau)/D_0 = \overline{\widetilde{s}}/(1-\tau-\overline{\widehat{m}})$ into (A.24) to obtain Proposition 2's total-response formula. Frozen sets $\widehat{m}_\chi = 0$; subtracting cancels the common income response and direct care loss:

$$\left.\frac{\mathrm{d}(n^H - n^F)}{\mathrm{d}\chi}\right|_{0^+} = \frac{\bar{n}\widehat{m}_\chi}{1+\bar{\rho}} \left[\bar{\rho}\left(\frac{h'}{\bar{h}} + \frac{\pi'}{\bar{\pi}} \right) - \frac{1}{1-\tau-\overline{\widehat{m}}} \right]. \tag{A.25}$$

The positive prefactor establishes Proposition 2's three sign cases; (A.22)-(A.24) give the total marginal policy effect.

A.4 Transition identities and proof of Proposition 3

Maintain Assumption 1 and Condition H, and take common initial states from an interior pre-reform steady state satisfying (11a) strictly. For a permanent reform $0 < \chi < \bar{\pi}$, A.4.1 derives medical-investment and saving responses along strictly slack interior paths. A.4.2 constructs the local branches used in Proposition 3.

A.4.1 Medical investment and saving after permanent reform

Along an interior Benchmark path, capital clearing, the pension budget, and household optimality imply (A.3)-(A.4) at every post-reform date, where

$$q_{t+1} = \frac{(1-\tau)\chi h(\widehat{m}_t)}{n_t \ell^y_{t+1} + \chi h(\widehat{m}_t)}, \qquad 0 < q_{t+1} < 1-\tau. \tag{A.26}$$

Positive next-period young labor ensures this range. Equation (A.5) and $\pi'/(\pi-\chi) > \pi'/\pi$ imply

$$\mathcal{R}\left(\widehat{m}_t, \frac{q_{t+1}}{1-\tau}, \chi\right) > \mathcal{R}(\widehat{m}_t, 0, 0). \tag{A.27}$$

The right-hand side is the marginal benefit–cost ratio at zero reform, strictly decreasing in $\widehat{m}_t$ and equal to one at $\overline{\widehat{m}}$. If $\widehat{m}_t \le \overline{\widehat{m}}$, the left-hand side of (A.27) exceeds one, contradicting medical optimality. Thus $\widehat{m}_t > \overline{\widehat{m}}$ for every post-reform young cohort on any such Benchmark path.

For either specification $j \in \{H, F\}$, (A.3) gives

$$\begin{aligned} \frac{\widehat{s}^j_t}{\overline{\widehat{s}}} &= \frac{1-\tau-\widehat{m}^j_t}{1-\tau-\overline{\widehat{m}}} \frac{D_0}{D(q^j_{t+1})} < 1, \\ \frac{\widehat{s}^j_t/\widehat{c}^j_t}{\overline{\widehat{s}}/\overline{\widehat{c}}} &= \frac{\omega}{\omega + q^j_{t+1}} < 1. \end{aligned}$$

Here $D(q^j_{t+1}) > D_0$, $\widehat{m}^H_t > \overline{\widehat{m}}$ and $\widehat{m}^F_t = \overline{\widehat{m}}$. Both saving ratios are therefore below their no-reform values at every post-reform date.

A.4.2 Local existence and cohort fertility responses

First establish local existence and differentiability from the strictly slack interior baseline. This construction uses Assumption 1; Condition H determines the sign of medical adjustment below. Write allocations near the baseline as $x_t = \log(n_t/\bar{n})$ and $\widetilde{m}_t = \widehat{m}_t - \overline{\widehat{m}}$, with $x_0 = \widetilde{m}_0 = 0$. The fertility-resource and medical residuals are

$$\begin{aligned} \mathcal{F}^n_t &= \log n_t - \log \Phi_t - \log \frac{\theta(1-\tau-\widehat{m}_t)(\omega + q_{t+1})}{v(1-\tau)D(q_{t+1})}, \\ \mathcal{F}^m_t &= \mathcal{R}\left(\widehat{m}_t, \frac{q_{t+1}}{1-\tau}, \chi\right) - 1. \end{aligned} \tag{A.28}$$

At $\chi = 0$, $q_{t+1} = 0$ for every nearby allocation. Differentiating (A.28) at the common pre-reform allocation, using $\overline{\mathcal{R}}_{\widehat{m}}$ from (A.23), gives

$$\begin{aligned} \mathrm{D}\mathcal{F}^n|_0 [x, \widetilde{m}]_t &= x_t + \bar{\rho} x_{t-1} + \frac{\widetilde{m}_t}{1-\tau-\overline{\widehat{m}}} - \bar{\rho}\left(\frac{h'}{\bar{h}} + \frac{\pi'}{\bar{\pi}}\right)\widetilde{m}_{t-1}, \\ \mathrm{D}\mathcal{F}^m|_0 [x, \widetilde{m}]_t &= \overline{\mathcal{R}}_{\widehat{m}} \widetilde{m}_t. \end{aligned} \tag{A.29}$$

Here D denotes the Fréchet derivative with respect to the allocation sequences, evaluated at the common pre-reform allocation and $\chi = 0$.

On the space of convergent sequences with the supremum norm, any convergent right-hand side first determines $\widetilde{m}_t$ from the second equation. The first then determines x_t with lag co-

efficient $-\bar{\rho}$. Since $0 < \bar{\rho} < 1$, the solution converges and has a uniform norm bound, so the linearization has a bounded inverse. Baseline interiority and twice continuously differentiable health technologies make the residual operator continuously differentiable in this neighborhood. Temporarily extend χ to an open interval around zero and apply the implicit function theorem. Restricting to $\chi \geq 0$ yields convergent sequences $\{n_t, \widehat{m}_t\}_{t\geq 1}$ that depend differentiably on χ. For Frozen, apply the same argument to the fertility residual with $\widetilde{m}_t = 0$ fixed.

At the baseline, strict slackness of (11a) gives $\eta\bar{h}v\overline{\widetilde{n}} < 1$. Continuity of both branches in the supremum norm implies that $\eta h_t v\widetilde{n}_t$ remains below one uniformly in t for sufficiently small nonnegative reforms. Hence (A.21a) preserves (11a) strictly along both entire paths. The remaining interiority inequalities also persist, and Appendix A.1 reconstructs the associated equilibria. The following right derivatives are taken along these branches.

Along these branches, the direct policy derivative of (A.26) and medical optimality imply

$$q'_{t+1}(0) = \frac{(1-\tau)\bar{h}}{\bar{n}\bar{\ell}}, \qquad (\widehat{m}_t^H)'(0) = \widehat{m}_\chi > 0, \qquad (\widehat{m}_t^F)'(0) = 0. \tag{A.30}$$

Responses of future labor and fertility are multiplied by χ and hence do not enter $q'_{t+1}(0)$. Differentiating (A.3) and $E_t = \lambda_c[\pi(\widehat{m}_{t-1}) - \chi]h(\widehat{m}_{t-1})$, then subtracting Frozen, cancels the common income response and direct retirement-time loss. Since $\widehat{m}_0$ is inherited,

$$\begin{aligned} \Delta\widetilde{n}'_t &= -\frac{\overline{\widetilde{n}}\widehat{m}_\chi}{1-\tau-\overline{\widehat{m}}}, \\ \Delta E'_1 &= 0, \qquad \Delta E'_t = \lambda_c(\pi'\bar{h} + \bar{\pi}h')\widehat{m}_\chi > 0 \quad (t \geq 2). \end{aligned} \tag{A.31}$$

Differentiate $n_t = \widetilde{n}_t(1 + \eta E_t/n_{t-1})$ and use $\eta\overline{\widetilde{n}}\bar{E}/\bar{n}^2 = \bar{\rho}$ to obtain

$$\Delta n'_t = -\frac{\bar{n}\widehat{m}_\chi}{1-\tau-\overline{\widehat{m}}} + \frac{\eta\overline{\widetilde{n}}}{\bar{n}}\Delta E'_t - \bar{\rho}\Delta n'_{t-1}, \qquad \Delta n'_0 = 0. \tag{A.32}$$

For cohort 1, which is young at implementation, $\Delta E'_1 = 0$, so

$$\Delta n'_1 = -\frac{\bar{n}\widehat{m}_\chi}{1-\tau-\overline{\widehat{m}}} < 0. \tag{A.33}$$

From the second cohort onward, the nonlagged term in (A.32) is constant and the lag coefficient has absolute value below one. The long-run value and comparison with the first cohort are

$$\begin{aligned} (1+\bar{\rho})(\Delta n'_\infty - \Delta n'_1) &= \frac{\bar{\rho}\bar{n}\widehat{m}_\chi}{1-\tau-\overline{\widehat{m}}} + \frac{\eta\overline{\widetilde{n}}}{\bar{n}}\Delta E'_2 > 0, \\ \Delta n'_t - \Delta n'_1 &= [1-(-\bar{\rho})^{t-1}](\Delta n'_\infty - \Delta n'_1) > 0 \quad (t \geq 2). \end{aligned} \tag{A.34}$$

The second line follows by iterating the recursion around its limit. Its positive bracket establishes Proposition 3. The limit $\Delta n'_\infty$ equals the steady-state derivative difference in (A.25).

A.5 Proof of Proposition 4: local effects on capital accumulation

Use the difference derivatives and common medical response from Appendix A.4.

In the log derivative of saving in (A.3), the $D(q_{t+1})$ terms cancel by (A.30). The direct first-order change in elderly labor is also common to both specifications; additional health and population responses are multiplied by $\chi = 0$. Hence $\Delta\ell'_t = -v\Delta\widetilde{n}'_t$, and (A.31) gives

$$\Delta(\ln \widehat{s}_t)' = -\frac{\widehat{m}_\chi}{1-\tau-\overline{\widehat{m}}}, \qquad \Delta(\ln \ell_t)' = \frac{v\overline{\widetilde{n}}}{\bar{\ell}}\frac{\widehat{m}_\chi}{1-\tau-\overline{\widehat{m}}} > 0. \tag{A.35}$$

Capital clearing and the common TFP path give

$$\Delta(\ln k_{t+1})' = \alpha\Delta(\ln k_t)' - \alpha\Delta(\ln \ell_t)' - \frac{\widehat{m}_\chi}{1-\tau-\overline{\widehat{m}}} - \frac{\Delta n'_t}{\bar{n}}. \tag{A.36}$$

Identical inherited capital implies $\Delta(\ln k_1)' = 0$. Substituting (A.33), the first cohort's medical-resource and fertility terms exactly cancel, giving $\Delta(\ln k_2)' = -\alpha\Delta(\ln \ell_1)' < 0$. Equation (A.34) ensures $\Delta n'_t \geq -\bar{n}\widehat{m}_\chi/(1-\tau-\overline{\widehat{m}})$, so the sum of the last two terms in (A.36) is nonpositive in every subsequent period. Iteration yields $\Delta(\ln k_t)' < 0$, $t \geq 2$.

This establishes Proposition 4.

Appendix B. Calibration and numerical methods

B.1 Target construction and data mapping

Table B.1 summarizes the targets and data mappings for the baseline calibration.

Table 2-45 of the China Population and Employment Statistical Yearbook 2025 reports city fertility by age and birth order for November 1, 2023 to October 31, 2024 (National Bureau of Statistics, Department of Population and Employment Statistics, 2025). Summing the all-birth-order rates for the 35 single ages from 15 to 49 gives 1013.78 per thousand, implying a total fertility rate of 1.01378. We round this value to two decimal places and use 1.01 as the calibration target. Under an equal sex ratio, division by two gives fertility per adult, $\bar{n} = 0.505$.

The CHNS individual-level files identify urban sampling sites by T2=1. We pool the 2004, 2006, 2009, and 2011 waves and select households with children of either sex aged 0-6, using reported survey age (China Health and Nutrition Survey, n.d.). Children, mothers, and counted grandmothers must be recorded as currently living in the household. Mother links use the wave-specific roster, with a unique family-relationship link filling a missing roster link. Conflicting links and unresolved child coverage are excluded. To allocate household care consistently, all resident children aged 0-6 must be linked to the same mother. The sample with valid maternal and co-resident grandmother care measures contains 579 mother-wave observations and 435 mothers.

Adult questionnaire items K12-K13 record participation and total hours spent caring for children aged six or younger in the household during the preceding week. A reported nonparticipation is zero; unknown or missing care is excluded. Grandmothers are counted once per mother-wave, and their hours are summed before division by the number of resident children aged 0-6. An absent linked co-resident grandmother contributes zero to this measured component. The unweighted mother-wave averages are $H_m = 23.346$ and $H_g = 11.695$ hours per week per child. The latter measures linked co-resident grandmother care; nonresident care is outside its coverage.

Reported hours allow care to overlap with other activities. We map them to model time using a 70-hour weekly activity budget, following Huang (2026), and seven years of intensive childcare within a 30-year life stage. The resulting maternal and raw grandmaternal time inputs per child are

$$t_m = \frac{H_m}{70}\frac{7}{30} = 0.077821, \qquad t_g = \frac{H_g}{70}\frac{7}{30} = 0.038984.$$

Let $t_{g,t}$ denote raw grandparental childcare time per child in period t. With $N_t n_t$ children and N_{t-1} elderly adults, total raw childcare satisfies $N_{t-1}\mu_t = N_t n_t t_{g,t}$. At the baseline steady state, $N_t = \bar{n}N_{t-1}$. Total parental childrearing time per young adult, $\bar{T}_p$, and childcare time per elderly adult are therefore

$$\bar{T}_p = \bar{n}t_m = 0.039300, \qquad \bar{\mu} = \bar{n}^2 t_g = 0.0099419.$$

We use the four two-stage residual inclusion estimates of the effect of grandparental childcare on maternal labor participation in Table 5 of Du et al. (2019): 0.367, 0.380, 0.409, and 0.437. Their equally weighted mean is 0.39825. Combining this participation-probability change with 45 weekly work hours and 24 weekly grandmother-care hours for working-mother families (Section

Table B.1. Principal baseline targets and model mappings

Object	Data and year	Calibration value	Definition
Fertility	2024 city age-specific fertility, Yearbook 2025, Table 2-45	$\bar{n} = 0.505$	TFR 1.01 divided by two under an equal sex ratio
Medical expenditure-to-wage ratio	2024 urban medical expenditure and wage income	0.073633	Labor-input correction and inclusion of employer pension contributions in the wage denominator
Maternal and grandmaternal care	CHNS 2004, 2006, 2009, 2011	23.346; 11.695	Maternal and co-resident grandmother hours/week/child; 70-hour budget and 7/30
Health capital	City women aged 60-84, 2010 and 2020 censuses	0.89427; 0.92656	Healthy or basically healthy share; model value for 2024: 0.93510
Longevity	Census-based 2010 and extrapolated 2024 life tables	0.87141; 0.92307	Expected survival years between 55 and 85 divided by 30
Saving composition	2024 urban income and expenditure	0.38295	Saving/(saving + nonmedical consumption)

2 and footnote 9 of that study) gives $q_{\text{release}} = 0.39825 \times 45/24 \simeq 0.74672$. We use this ratio to calibrate the expected maternal work time released per hour of grandmother childcare.

The implied childrearing time requirement without grandparental assistance is $v = t_m + q_{\text{release}} t_g = 0.10693$. Using the baseline health target $\bar{h}$ described below, $\bar{\Phi} = v/t_m = 1.3741$ and $\bar{g} = \bar{h}\bar{\mu}/\bar{n} = \bar{h}\bar{n}t_g = 0.018409$ give $\eta = (\bar{\Phi} - 1)/\bar{g} = 20.320$. Effective grandparental childcare is measured per young adult by g_t and per child by g_t/n_t.

For calendar year y, let M_y^{pc} and $Y_{w,y}^{pc}$ denote National Bureau of Statistics urban per capita medical expenditure and wage income, which use the same population denominator. Let e_y be the employer pension contribution rate and $\ell_y = 1 - n_y t_m$ the labor input used in the historical calibration. Medical expenditure relative to the model wage is

$$\widehat{m}_y = \frac{M_y^{pc}}{Y_{w,y}^{pc}} \frac{\ell_y}{1 + e_y}.$$

The factor $1 + e_y$ converts observed wage income to labor cost including employer pension contributions. Multiplication by ℓ_y then expresses medical spending relative to the wage per unit of effective labor.

For 2024, National Bureau of Statistics (2025) report medical expenditure of RMB 2,925 and wage income of RMB 32,899. Combined with $\ell = 0.96070$ and a 16% employer contribution, these imply $\widehat{m}_{2024} = 0.073633$. Consistently converted values for 2010 and 2020 are 0.050934 and 0.067888 (National Bureau of Statistics, 2011b,a, 2021). Employee pension contributions are included in observed gross cash wages (National Bureau of Statistics, 2023). Historical labor corrections hold maternal childcare time per child, t_m, fixed, impose $n_{2010} = 0.5$, and use $n_{2020} = 1.118/2$ from the city census TFR (National Bureau of Statistics, 2020a). Employer rates are represented by 20% in 2010 and the regular 16% rate in 2020.

The healthy or basically healthy shares among city women aged 60-84 in the 2010 and 2020 censuses are 0.89427 and 0.92656 (National Bureau of Statistics, 2010, 2020b). A two-point fit pairs these shares with the same-year medical expenditure-to-wage ratios to calibrate h_0, κ_h.

The fitted technology implies $\bar{h} = 0.93510$ at the 2024 expenditure ratio. Longevity targets are period-life-table expected years lived between ages 55 and 85, divided by 30: $26.1424/30 = 0.87141$ for 2010 and $27.6921/30 = 0.92307$ for 2024. For each age $x = 55, \ldots, 84$, the central death rate $M_{x,y}$ is female deaths divided by average female population in the city census table (National Bureau of Statistics, n.d.). Annual death probabilities are $Q_{x,y} = M_{x,y}/(1 + M_{x,y}/2)$, with survival initialized at age 55 and person-years calculated by the trapezoidal rule. For 2024, separate age-specific regressions of $\log M_{x,y}$ on calendar year using 2000, 2010, and 2020 supply extrapolated rates; no age smoothing or under-registration correction is applied. Pairing these targets with the 2010 and 2024 expenditure ratios calibrates π_0, κ_π.

The contemporaneous pairings use a steady-state approximation. When successive cohorts choose the same medical expenditure-to-wage ratio, health and longevity satisfy $\bar{h} = h(\overline{\widehat{m}})$ and $\bar{\pi} = \pi(\overline{\widehat{m}})$, allowing current expenditure and older cohorts' health to be matched jointly. The historical fit calibrates these aggregate steady-state relationships. The observations do not separately identify the causal effect of medical expenditure from changes in income, education, or medical technology. Along the reform transition, health investment affects the investor's health one period later.

Urban per capita disposable income of RMB 54,188, consumption of RMB 34,557, and medical expenditure of RMB 2,925 in 2024 imply saving of RMB 19,631 and nonmedical consumption of RMB 31,632. The composition ratio is

$$\lambda_s = \frac{19631}{19631 + 31632} = 0.38295.$$

Wage-normalized resources after medical spending are $(1 - \tau)\bar{\ell} - \overline{\widehat{m}} = 0.68830$. Allocating the fraction λ_s to saving gives $\overline{\widehat{s}} = 0.26358$ and $\overline{\widehat{c}} = 0.42472$. The saving and fertility conditions determine β and θ, respectively. Medical optimality fixes the total weight $\gamma + \xi$ on old-age childcare and leisure; combining it with the childcare share $\bar{\mu}/\bar{\pi} = \xi/(\gamma + \xi)$ yields γ and ξ.

The capital share is the complement of China's 2023 labor income share of 0.56118 from PWT 11.0 via FRED (Penn World Table and FRED, 2026). Medical and saving statistics cover urban residents of all ages and proxy young household resource allocation; city female health statistics for ages 60-84 proxy health capital in the model's 55-85 old-age stage.

Figure 1 is calculated from the published cell counts in Yu et al. (2026, Table 2). Among respondents without daily-activity difficulties, 692 report grandchild care and 685 do not; the corresponding counts for those with difficulties are 177 and 318. Dividing each care count by its health-group total gives 50.3% and 35.8%. Grandchild care refers to provision by the respondent or spouse during the preceding year. Physical difficulty follows the source's twelve basic and instrumental daily-activity items. The study's urban-hukou sample is restricted to respondents aged 60 or older with grandchildren and complete study variables, excluding those with a history of mental disorders or severe cognitive impairment.

B.2 Growth data, extrapolation, and the initial window

The population per young adult, P_t, includes children, young adults, and elderly adults weighted by survival time. GDP per capita is

$$P_t = n_t + 1 + \frac{\pi_t}{n_{t-1}}, \qquad y_t^{pc} = Z_t \frac{k_t^\alpha \ell_t^{1-\alpha}}{P_t}. \tag{B.1}$$

GDP per capita growth and productivity growth satisfy

$$G_t^Z = G_t^y \left(\frac{k_{t-1}}{k_t}\right)^{\alpha} \left(\frac{\ell_{t-1}}{\ell_t}\right)^{1-\alpha} \frac{P_t}{P_{t-1}}, \qquad G_t^A = (G_t^Z)^{1-\alpha}.$$

Here, $G_t^y = y_t^{pc}/y_{t-1}^{pc}$. Combining this relationship with capital clearing recovers the productivity path.

The OECD (2025) BAU1 methodology incorporates legislated future retirement-age changes into employment projections for ages 55 and above. We use the permanent reform from 55 to 58 in Benchmark as the reference experiment for matching these projections at the model's 30-year generational scale. National potential GDP per capita ratios determine productivity conditional on this experiment's labor, population, and capital adjustment; city and urban data determine the household calibration. The no-reform path and all other counterfactuals retain this productivity path, allowing their output to differ with retirement policy and health choices.

The initial growth calibration uses China's real GDP per capita growth over 2019-2024 to reflect recent economic conditions around the 2024 baseline. Data are from World Bank WDI series NY.GDP.PCAP.KD (World Bank, 2026), in constant 2015 US dollars (July 13, 2026 vintage). The 2019 and 2024 endpoints are 10356.29 and 13119.11, implying compound annual growth of 4.8431%. On the initial balanced-growth path, normalized capital, labor, and population composition are constant, so GDP per capita and labor-augmenting productivity grow at the same rate. Expressing this annual growth on the model's 30-year scale gives $\bar{G}_0^Z = 4.1324$, equivalent to annual Hicks-neutral TFP growth of 2.6896%.

Long-run targets use China's BAU1 scenario in OECD Economic Outlook 117 long-term scenarios (OECD, 2026), with potential GDP per capita in constant 2021 PPP US dollars. Endpoints for 2024, 2054, 2084, and 2100 are 23850.61, 49690.99, 70800.36, and 90782.98; their ratios define the growth targets for the reference experiment. Beyond 2100, use

$$g_{tail} = \left(\frac{90782.98}{70800.36}\right)^{1/16} - 1 = 1.5659\%, \qquad y_Y^{pc,target} = 90782.98(1 + g_{tail})^{Y-2100}.$$

This gives a 2114 target of 112843.69. The 30-year growth factors for 2024-2054 and 2054-2084 are approximately 2.0834 and 1.4248; the factor for 2084-2114 and later periods is 1.5938. The latter extrapolates the 2084-2100 compound growth rate.

Annual Hicks-neutral TFP growth is recovered from $100[(G_t^Z)^{(1-\alpha)/30} - 1]$. Lifetime welfare for the cohort young in 2114 requires old-age outcomes in 2144, so growth targets extend beyond plotted nodes.

The baseline saving Euler condition and pension budget imply

$$\beta = \frac{\bar{\hat{s}} + \bar{G}_0^Z \tau \bar{\ell} \bar{n}/R_0}{\bar{\hat{c}}}.$$

Capital clearing makes R_0 and $\bar{G}_0^Z$ move proportionally, so the inferred β is unchanged when the initial growth factor varies at fixed wage-normalized household targets. We normalize $Z_0 = 1$; wages, consumption, and output are expressed in model units.

B.3 Initial balanced growth and inherited capital

The initial state is a balanced-growth path with $\bar{G}_0^Z = 4.1324$. The common capital boundary is

$$\bar{G}_0^Z n_0 k_0 = \widehat{w}_0^Z \widehat{s}_0, \qquad G_1^Z n_0 k_1 = \widehat{w}_0^Z \widehat{s}_0,$$
$$k_1 = \frac{\bar{G}_0^Z}{G_1^Z} k_0,$$
$$k_0 = 0.0092322, \quad k_1 = 0.037240, \quad G_1^Z = 1.0245.$$

The values of G_1^Z and k_1 use the productivity path recovered for the Benchmark reform to age 58 in Appendix B.4. This path is shared by all experiments. Pre-reform saving fixes inherited physical capital; the difference between k_0 and k_1 reflects the change in productivity growth used to normalize it. Reform and no reform therefore have $k_1^R = k_1^N$, and the initial elderly hold assets $s_0 = Z_1 n_0 k_1$.

B.4 Solving the transition and recovering capital

The numerical solution uses the reduced transition system derived in Appendix A.1. Benchmark jointly solves the fertility-resource and medical optimality conditions in (A.28), with the income share given by (A.26). Frozen fixes $\widehat{m}_t = \overline{\widehat{m}}$ and solves the fertility equations. Since Φ_t contains the previous generation's state and q_{t+1} contains future fertility and labor, both specifications are two-point boundary-value problems.

The left boundary fixes $n_0, \widehat{m}_0$. For a solution over periods $1, \ldots, T$, the right boundary sets n_{T+1} to the respective post-reform steady-state fertility. The main horizon is $T = 20$, with $T = 12$ as a comparison. Positive unknowns are log-transformed and solved jointly by bounded nonlinear least squares. Given household paths, (A.3) recovers consumption and saving, and (32) recursively recovers capital from the inherited boundary in Appendix B.3.

For the Benchmark reform to age 58, substituting the GDP per capita targets into (B.1) and combining with capital clearing gives

$$k_{t+1} = \ell_{t+1} \left[\frac{(1-\alpha)\widehat{s}_t P_t}{n_t G_{t+1}^y \ell_t P_{t+1}} \right]^{1/(1-\alpha)}, \qquad G_{t+1}^Z = \frac{(1-\alpha)(k_t/\ell_t)^\alpha \widehat{s}_t}{n_t k_{t+1}}.$$

For the other experiments, this productivity path is held fixed and capital is recovered recursively from (32).

Computations use Python with NumPy, pandas, SciPy, mpmath, and Matplotlib. OpenAI ChatGPT and Codex assisted code development and debugging. The replication package provides the code, numerical inputs, and software requirements for reproducing the calculations and figures.

After solving, we check the original equilibrium conditions and sensitivity to the terminal horizon. Across the reported specifications, scaled residuals are below 10^{-12}. Relative differences between the 12- and 20-period solutions, measured against the latter, are below 10^{-13} over nodes $t = 0, \ldots, 8$ for fertility, the medical expenditure-to-wage ratio, capital, young consumption, and GDP per capita. Selected steady-state thresholds were also recomputed with higher-precision arithmetic, as described in Appendix C.2.

B.5 Baseline mechanism conditions and marginal responses

The common baseline satisfies Conditions E and H. The contributions of elderly labor income, lost grandparental childcare time, improved parental health and longevity, and medical expenditure to $(1+\bar{\rho})n_\chi^H(0)/\bar{n}$ in Proposition 2 are, respectively, 0.56634, −0.29492, 0.078642, and −0.11885. Their positive sum implies a positive marginal steady-state fertility effect in Benchmark. The common income gain also exceeds the direct childcare loss under Frozen. The childcare relief share is $\bar{\rho} = 0.27223$, and the two sides of the health-adjustment condition in Proposition 2 are

$$\bar{\rho}\left(\frac{h'}{\bar{h}} + \frac{\pi'}{\bar{\pi}}\right) = 0.91970 < \frac{1}{1-\tau-\overline{\widehat{m}}} = 1.3899.$$

Equations (A.23)-(A.25) and (A.33)-(A.34) give

$$\widehat{m}_\chi = 0.085508, \qquad n_\chi^H(0) = 0.091778, \qquad n_\chi^F(0) = 0.10774,$$
$$\Delta n_1' = -0.060019, \qquad \Delta n_\infty' = -0.015960.$$

All derivatives are with respect to the retirement share χ at zero reform; $n_\chi^H(0)$ and $n_\chi^F(0)$ are steady-state responses. The long-run difference $\Delta n_\infty'$ from (A.34) equals the steady-state derivative difference in (A.25). The computed transition paths for retirement at 58 and 62 also satisfy the saving inequalities in Appendix A.4.1.

B.6 Saving and welfare accounting

First solve household optimality at given prices and pension benefits, then impose pension budget balance. The young household's expected present value of old-age nonasset income, normalized by its current wage, is

$$B_t^e = \frac{w_{t+1}^e}{R_{t+1}^e w_t}\left[\chi_{t+1}^e(1-\tau)h_{t+1} + \tau n_t \ell_{t+1}^e\right]. \tag{B.2}$$

The young budget and Euler condition imply $\beta\widehat{c}_t = \widehat{s}_t + B_t^e$, hence

$$\widehat{s}_t = \frac{\beta[(1-\tau)\ell_t^y - \widehat{m}_t] - B_t^e}{1+\beta}. \tag{B.3}$$

Equation (B.3) gives the partial response of $\widehat{s}_t$ to B_t^e as $-1/(1+\beta)$, holding ℓ_t^y and $\widehat{m}_t$ fixed. Its partial response to $\widehat{m}_t$ is $-\beta/(1+\beta)$, holding ℓ_t^y and B_t^e fixed. Equilibrium saving also reflects the adjustment of these other terms.

For $j \in \{H, F\}$, let $\Delta^P U_t^j = U_t^{R,j} - U_t^{N,j}$ denote the policy change in lifetime utility. The elderly at implementation hold predetermined assets $s_0 = Z_1 n_0 k_1$ and realize consumption $d_1 = R_1 s_0 + (1-\tau)w_1\chi_1\bar{h} + \tau w_1 n_0 \ell_1$. Their young-age choices are unchanged, so their utility difference relative to no reform is, suppressing the specification index j,

$$\Delta^P U_0^{old} = \beta\left[\ln\frac{d_1^R}{d_1^N} + (\gamma+\xi)\ln\frac{\bar{\pi}-\chi_R}{\bar{\pi}}\right]. \tag{B.4}$$

At a given retirement policy, the initial elderly have identical assets, health, and time allocations in Benchmark and Frozen. If Benchmark fertility is lower at implementation, young labor is higher: $\ell_1^H - \ell_1^F = -v(n_1^H - n_1^F)/\Phi_1$. Holding the inherited states and policy fixed, differentiat-

ing d_1 in Appendix A.1 with respect to ℓ_1 gives a positive factor times $[\alpha+\tau(1-\alpha)]\ell_1^y+\tau\ell_1^o>0$. The initially old then gain $\beta\log(d_1^H/d_1^F)$ in lifetime utility relative to Frozen.

Within either specification, long-run welfare is the limit of same-cohort reform-to-no-reform comparisons along the respective balanced-growth paths:

$$\begin{aligned}\Delta^P U_\infty = \lim_{t\to\infty}\Bigg\{ &\ln\frac{c_t^R}{c_t^N}+\theta\ln\frac{n_t^R}{n_t^N}\\ &+\beta\left[\ln\frac{d_{t+1}^R}{d_{t+1}^N}+\xi\ln\frac{\mu_{t+1}^R}{\mu_{t+1}^N}+\gamma\ln\frac{\zeta_{t+1}^R}{\zeta_{t+1}^N}\right]\Bigg\}.\end{aligned} \tag{B.5}$$

The consumption-equivalent welfare change is

$$CEV_t^j = 100\left[\exp\left(\frac{\Delta^P U_t^j}{1+\beta}\right)-1\right]. \tag{B.6}$$

The CEV expresses the utility change as an equivalent proportional change in consumption in both life stages. We use the lifetime consumption weight $1+\beta$ for every cohort, including those already elderly at implementation. For the initial elderly, this is a full-lifetime normalization; an equivalent change in remaining old-age consumption alone would instead use β. Benchmark-Frozen CEV differences are measured in percentage points.

B.7 Lifetime utility decomposition

Because Benchmark and Frozen share the same no-reform path, define $\Delta\ln X = \ln X^{R,H} - \ln X^{R,F}$. Then

$$\begin{aligned}\mathcal{H}_t^U = &\Delta\ln c_t+\theta\Delta\ln n_t\\ &+\beta\big[\Delta\ln d_{t+1}+\xi\Delta\ln\mu_{t+1}+\gamma\Delta\ln\zeta_{t+1}\big].\end{aligned}$$

Table B.2 reports preference-weighted utility components, which sum to $\mathcal{H}_t^U = \Delta^P U_t^H - \Delta^P U_t^F$. The CEV definition (B.6) implies that the welfare percentage-point difference is

$$CEV_t^H - CEV_t^F = (100+CEV_t^F)\left[\exp\left(\frac{\mathcal{H}_t^U}{1+\beta}\right)-1\right].$$

Utility and CEV increments have the same sign, but the conversion magnitude also depends on Frozen's policy welfare level.

Greater longevity raises the old-age grandchild-care and leisure contributions in Table B.2 by extending retirement at a given retirement age. Childcare received from parents affects welfare through childrearing costs and the resulting fertility, consumption, and labor choices.

B.8 Local accounting of the intergenerational health effect

Table B.3 evaluates (42a) at the common pre-reform allocation. Each entry is a contribution to the difference between Benchmark and Frozen in the marginal percentage fertility response per additional year of retirement age. The medical resource term is the same for every post-reform cohort. Parental health contributes from cohort 2 onward, while care sharing depends on the preceding cohort's total fertility difference. The long-run row solves the fixed point of the recursion.

Table B.2. Lifetime utility decomposition of the incremental health effect

Retirement age (years)	Cohort	Young consumption	Fertility	Old-age consumption	Grandchild care	Leisure	Total utility
58	1	-0.009951	-0.000719	-0.005329	+0.000169	+0.015478	-0.000352
58	2	-0.012527	-0.000103	-0.003474	+0.000168	+0.015455	-0.000481
58	3	-0.016326	-0.000254	-0.005455	+0.000168	+0.015461	-0.006406
62	1	-0.021079	-0.001527	-0.008059	+0.000390	+0.035805	+0.005530
62	2	-0.029898	-0.000353	-0.007267	+0.000389	+0.035730	-0.001399
62	3	-0.037442	-0.000613	-0.010715	+0.000389	+0.035747	-0.012633

Notes: Entries are preference-weighted log differences between Benchmark and Frozen under the same retirement policy, in utility units. Cohort 1 is young at implementation. Grandchild care and leisure refer to the household's own old age.

Table B.3. Local accounting of the health contribution to fertility

Cohort	Medical resources	Parental health	Care sharing	Total
1	-0.3962	+0.0000	+0.0000	-0.3962
2	-0.3962	+0.2621	+0.1078	-0.0262
3	-0.3962	+0.2621	+0.0071	-0.1269
Long run	-0.3962	+0.2621	+0.0287	-0.1053

Notes: Entries are contributions to the Benchmark–Frozen difference in marginal percentage fertility responses per additional year of retirement age, evaluated at age 55. They equal the terms in (42a) multiplied by 100/30. Cohort 1 is young at implementation. Parental health includes both capacity and survival. Care sharing is the lagged fertility term. Table 3 reports the finite reforms to ages 58 and 62.

The Benchmark-Frozen marginal fertility difference becomes less negative from cohort 1 to the long run. The parental-health contribution to this narrowing is the direct health term in (42a) divided by $1 + \bar{\rho}$. The care-sharing contribution equals $\bar{\rho}/(1 + \bar{\rho})$ times the absolute medical resource term. Independently solving Benchmark and Frozen with policy steps 10^{-4} and 5×10^{-5} and applying Richardson extrapolation reproduces the first nine cohort derivatives to an absolute error below 2×10^{-6} per unit of χ.

Appendix C. Robustness analysis

Of 35 candidate specifications, 26 satisfy the calibration and feasibility requirements: 18 target-preserving elasticity cases, two alternative functional forms, and six alternative target calibrations. The nine exclusions violate the maintained time-substitution restriction (11a). Benchmark and Frozen share each specification's pre-reform economy.

C.1 Elasticity sensitivity with fixed 2024 baseline targets

The first exercise holds the 2024 baseline targets fixed while varying the elasticities of old-age health capital and survival duration with respect to the medical expenditure-to-wage ratio:

$$\varepsilon_h = \frac{\overline{\widehat{m}} h'(\overline{\widehat{m}})}{\bar{h}}, \qquad \varepsilon_\pi = \frac{\overline{\widehat{m}} \pi'(\overline{\widehat{m}})}{\bar{\pi}}.$$

The baseline elasticities are 0.10988 and 0.13888, respectively. We independently multiply each by 0.5, 1, or 1.5 and the parameter η, which governs parental time savings from grandparental childcare, by 0.75, 1, or 1.25, giving $3 \times 3 \times 3 = 27$ candidates. The nine with the η multiplier 1.25 violate (11a) at checked allocations and are excluded, leaving 18 cases.

For each technology $f \in \{h, \pi\}$, the recalibrated exponential specification f^r uses elasticity multiplier r_f:

$$\kappa_f^r = r_f \kappa_f, \qquad f_0^r = 1 - (1 - \bar{f}) \exp(\kappa_f^r \overline{\widehat{m}}).$$

This preserves $f^r(\overline{\widehat{m}}) = \bar{f}$ and multiplies the baseline elasticity by r_f. The recalibrated technologies preserve the 2024 baseline levels but need not reproduce the historical health and longevity moments used in the original calibration.

Changing the marginal longevity return alters households' incentives to spend on health. To keep their baseline choices unchanged, we preserve $\overline{\widehat{m}}, \bar{n}, \bar{h}, \bar{\pi}, \bar{\mu}, \overline{\widehat{c}}, \overline{\widehat{s}}$ and recalibrate the total preference weight on old-age childcare and leisure $S = \gamma + \xi$:

$$\omega = \tau + \frac{\alpha}{1-\alpha}, \quad S^r = \frac{\bar{\pi}}{\beta \overline{\widehat{c}} \, (\pi^r)'(\overline{\widehat{m}})} - \frac{\tau}{\omega}, \quad \lambda_c = \frac{\bar{\mu}}{\bar{\pi}}, \quad \gamma^r = (1 - \lambda_c) S^r, \quad \xi^r = \lambda_c S^r.$$

When the parameter converting grandparental childcare into parental time savings becomes $\eta^r = r_c \eta$, set

$$v^r = \frac{\bar{T}_p}{\bar{n}} \left(1 + \eta^r \frac{\bar{\mu} \bar{h}}{\bar{n}} \right), \qquad \bar{T}_p = 0.039300,$$

to preserve parental childrearing time per young adult. These preference and cost recalibrations keep the baseline allocation unchanged as health returns and childcare effectiveness vary. All other parameters retain their baseline values.

C.2 Functional forms, target moments, and steady-state outcomes

To distinguish slopes from curvature, consider two additional bounded, concave technologies:

$$f_{\text{pow}}(\widehat{m}) = 1 - (1 - \bar{f}) \left[1 + \frac{\kappa_f (\widehat{m} - \overline{\widehat{m}})}{\kappa_{\text{pow}}} \right]^{-\kappa_{\text{pow}}}, \qquad f = h, \pi; \quad \kappa_{\text{pow}} \in \{4, 8\}.$$

Table C.1. Reductions in retirement ages at steady-state fertility peaks and returns to no-reform fertility

Exercise	Cases	At fertility peak, years	At return to no-reform fertility, years	Objects held fixed or varied
Baseline	1	0.735	1.911	2024 reference calibration
Target-preserving elasticity sensitivity	18	0.211-2.717	0.580-7.170	Fixed baseline allocation; varying health returns and childcare effectiveness
Curvature increased by 12.5%	1	0.680	1.775	Fixed baseline levels, first-order slopes, and preferences
Curvature increased by 25%	1	0.633	1.659	Fixed baseline levels, first-order slopes, and preferences
Target recalibration	6	0.266-1.167	0.723-2.978	Alternative health scores, medical or saving targets

Notes: Each reported reduction is the retirement age in Frozen minus that in Benchmark at the fertility peak or the nonzero return to no-reform fertility. Positive values indicate lower retirement ages in Benchmark. The baseline is included among the 18 elasticity cases and is shown separately for reference. Together with two alternative functional forms and six target recalibrations, these give 26 distinct specifications.

Both satisfy the technology assumptions and preserve baseline levels and first-order slopes. At the baseline allocation, $-f''/f'$ rises by 25% and 12.5%, respectively, with preference parameters unchanged.

Six alternative target calibrations vary health scoring, the medical expenditure-to-wage ratio, or the saving share. Two replace the baseline health scores $(1, 1, 0, 0)$ with $(1, 2/3, 1/3, 0)$ and $(1, 0.75, 0.25, 0)$ to the four census categories: healthy, basically healthy, unhealthy but able to care for oneself, and unable to care for oneself. Two vary the 2024 medical expenditure-to-wage target by $\pm 10\%$, retaining the 2010 and 2020 expenditure ratios. Two vary λ_s, the saving share of saving plus nonmedical consumption defined in Appendix B.1, by ± 5 percentage points. Each case repeats the calibration procedure in Appendix B.1 with the specified target changed and the other data inputs retained.

All 26 retained specifications satisfy Conditions E and H. Consumption, saving, labor, and retirement leisure are positive, and (11a) is strictly slack at the checked steady states and computed transition nodes. Direct checks cover the 101-node retirement-age grid, refined thresholds, finite paths, and separately solved long-run limits. Within each specification, no-reform Benchmark and Frozen coincide and share initial capital.

On the 101-node grid over retirement ages 55-70, each retained Benchmark and Frozen fertility profile changes once from increasing to decreasing. Continuous peak ages are then located by solving $dn^{ss}/d\chi = 0$, and nonzero returns to no-reform fertility by solving $n^{ss}(\chi) = n^{ss}(0)$. Both retirement ages are lower in Benchmark in every case (Table C.1). For the specification with the smallest age differences, independent solutions at 40- and 70-digit precision confirm both rankings. The condition in Proposition 2 for a lower marginal fertility response under Benchmark also holds in all 26 cases.

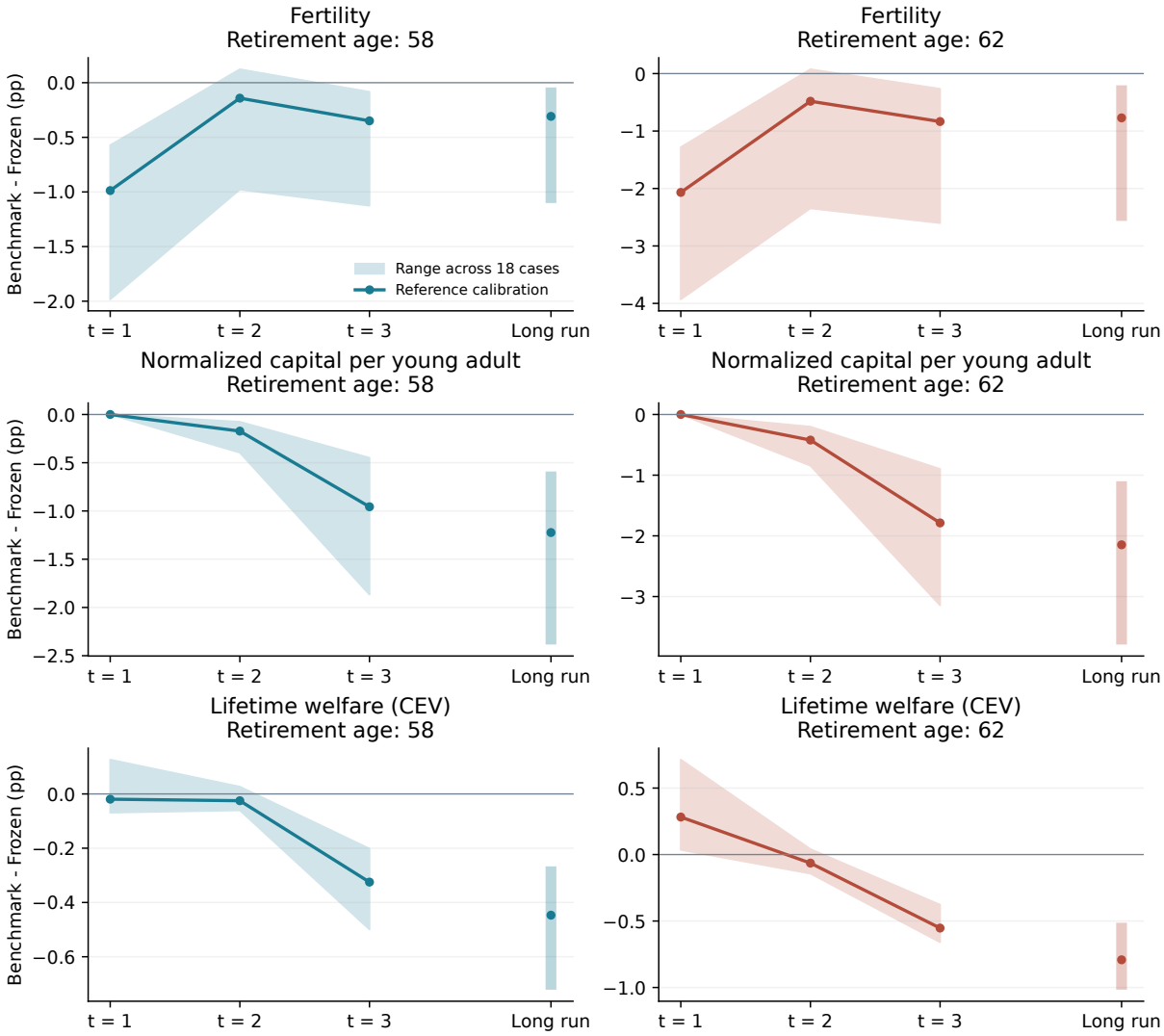

Figure C.1. Transition and welfare sensitivity with fixed baseline targets

Notes: Columns show retirement at 58 and 62. Rows report Benchmark-Frozen differences in percentage policy effects on fertility and normalized capital per young adult, k_t, and in lifetime CEVs. All differences are in percentage points. Solid lines show the reference calibration; shading gives the minimum and maximum across the 18 specifications at each plotted date. Separate points and vertical bars show long-run reference values and ranges.

C.3 Transition and welfare sensitivity

For each specification, we compare permanent retirement at 58 and 62 with its own no-reform path, using Appendix B.4's transition method and the common productivity path. Cohort 1 consists of adults who are young at implementation. Benchmark-Frozen differences in percentage policy effects and lifetime CEVs are reported in percentage points. Figure C.1 displays the 18 cases with fixed baseline targets, with long-run limits solved separately from the finite transition nodes.

Across all 26 retained specifications and both reforms, the Benchmark-Frozen fertility difference is most negative for cohort 1. The differences for subsequent computed cohorts and in the long run are less negative or positive. Normalized capital per young adult is lower in Benchmark at every computed date from period 2 onward and in the long run.

Across the 18 cases with fixed baseline targets, Benchmark-Frozen fertility differences for cohort 1 range from -1.98 to -0.57 percentage points at 58 and from -3.93 to -1.28 at 62. Fertility

Table C.2. Agreement on the sign of the total fertility policy effect

Retirement age	Cohort	$H+, F+$	$H-, F+$	$H-, F-$	$H+, F-$
58	1	18	8	0	0
58	Long run	26	0	0	0
62	1	3	22	1	0
62	Long run	10	15	1	0

Notes: Each row covers the same 26 retained specifications, including the reference case. Ages are in years. Cohort 1 is young at implementation. H and F denote Benchmark and Frozen; signs indicate their fertility effects relative to no reform. Counts summarize this sensitivity set and have no probability interpretation.

differences for cohort 2 are positive in some cases, whereas long-run differences are negative throughout these 18 cases.

Benchmark-Frozen lifetime CEV differences for cohort 1 range from -0.069 to +0.126 points at 58 and from +0.035 to +0.714 at 62. The CEV differences vary in sign for cohort 2 and are negative for cohort 3 and in the long run throughout these cases. Alternative health scoring can also change the sign of the cohort-1 CEV difference at 58.

C.4 Sign disagreement across admissible specifications

Table C.2 reports the signs of total fertility effects relative to no reform, separately from the incremental Benchmark–Frozen effect. It covers all 26 retained specifications, including alternative functional forms and target calibrations. At retirement age 58, both specifications predict higher long-run fertility in all cases. For cohort 1, however, Benchmark predicts lower fertility and Frozen higher fertility in 8 cases. At 62, this sign disagreement occurs in 22 cases for cohort 1 and 15 in the long run. The remaining cases agree on a positive or negative policy effect.

In every cohort-1 and long-run comparison in Table C.2, allowing health investment to adjust lowers fertility relative to Frozen. This ranking holds throughout the retained specifications, whereas disagreement on the sign of the total reform effect occurs in only a subset.

C.5 Local reforms from alternative initial retirement ages

The analytical transition results are local to $\chi = 0$, where induced health has no first-order effect on elderly labor earnings. To examine other starting points, we hold the reference parameters fixed and solve the Benchmark steady state at initial retirement ages 55, 58, and 62. For each initial allocation, both specifications inherit the same health, population, and capital, and Frozen fixes the medical expenditure-to-wage ratio at that allocation. We apply an unanticipated permanent marginal extension from each initial retirement age, using an identical productivity path for Benchmark and Frozen.

Using 24-period transitions and separately solved steady-state limits, Table C.3 reports Benchmark-Frozen differences in marginal fertility responses per additional year of retirement age. The difference is negative for cohort 1 at all three starting ages and higher for every checked later cohort, 2–12, and in the long run. The marginal response of normalized capital is also lower in Benchmark in period 2 and in the long run. We compute the derivatives from two retirement-share steps, 10^{-4} and 5×10^{-5}, using Richardson extrapolation. Original-equation residuals are below 3×10^{-14}, and (11a) remains strictly slack along all checked paths.

Table C.3. Marginal fertility effects of health adjustment from alternative initial retirement ages

Initial retirement age	Cohort 1	Long run	Minimum difference from cohort 1
55	-0.3962	-0.1053	+0.2693
58	-0.2974	-0.1048	+0.1808
62	-0.2783	-0.1245	+0.1462

Notes: Initial ages are in years. Each comparison starts from the Benchmark steady state at that age, with parameters and health technologies fixed. Frozen holds the medical expenditure-to-wage ratio at its initial level. Entries are Benchmark-Frozen differences in marginal percentage fertility responses per additional year of retirement age. The last column subtracts the cohort-1 entry from each response difference for cohorts 2–12 and the long-run limit, then takes the minimum. Cohort 1 is young at implementation.

The cohort ordering thus persists in these comparisons with initial elderly employment. The checks retain incumbent grandparents' predetermined health and the 30-year lag between health investment and its childcare benefits.

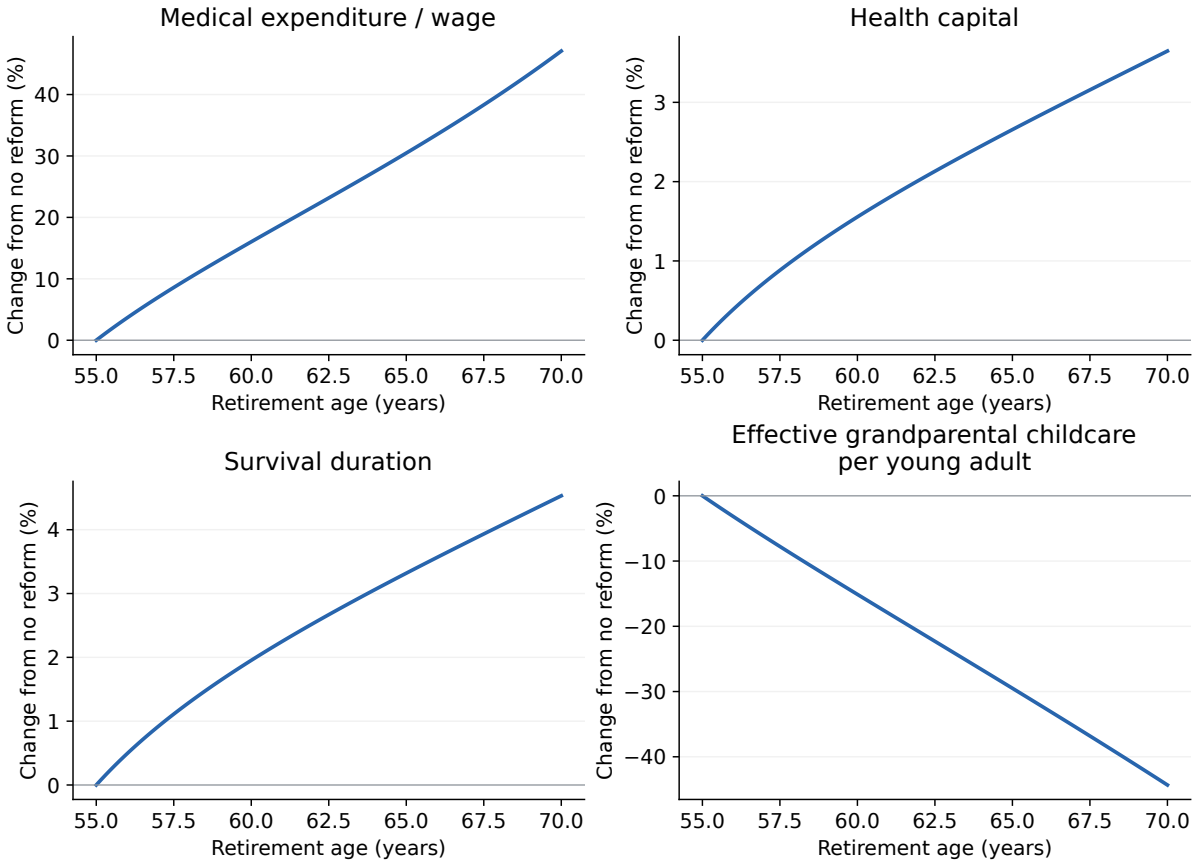

Figure D.1. Steady-state trade-offs between health improvements and childcare losses
Notes: Panels report $100[X(\chi)/X(0)-1]$ for Benchmark relative to the no-reform steady state, in percent. From top left to bottom right, the variables are the medical expenditure-to-wage ratio, health capital, survival duration, and effective grandparental childcare per young adult, $g = \mu h/n$.

Appendix D. Supplementary quantitative results

D.1 Health, longevity, and steady-state grandparental childcare

Figure D.1 reports Benchmark steady-state responses. At retirement age 58, the medical expenditure-to-wage ratio rises from approximately 0.074 to 0.081 and health capital from 0.935 to 0.945. Survival duration between ages 55 and 85 increases by about 0.36 years, below the three-year retirement extension, so time available for childcare and leisure contracts by about 2.64 years. Childcare time per elderly adult falls by approximately 9.5%. Higher health capital improves childcare capacity per hour by about 1.0%, while the 0.7% increase in steady-state fertility spreads each grandparent's childcare across more adult children's households. Combining these changes through $g = \mu h/n$ leaves effective grandparental childcare per young adult about 9.3% below its no-reform level.

Figure D.2 compares Benchmark and Frozen with Longevity-only, which fixes health capital at its pre-reform level while allowing the medical expenditure-to-wage ratio and longevity to adjust. Grandparental childcare, fertility, saving, and prices are re-equilibrated. All three specifications share the same pre-reform preferences and allocation.

Under Longevity-only, steady-state fertility peaks at a retirement age of approximately 57.5 and returns to its no-reform level at approximately 60.7. Compared with Frozen at the same retirement age, medical expenditure relative to wages rises, while longer survival extends the

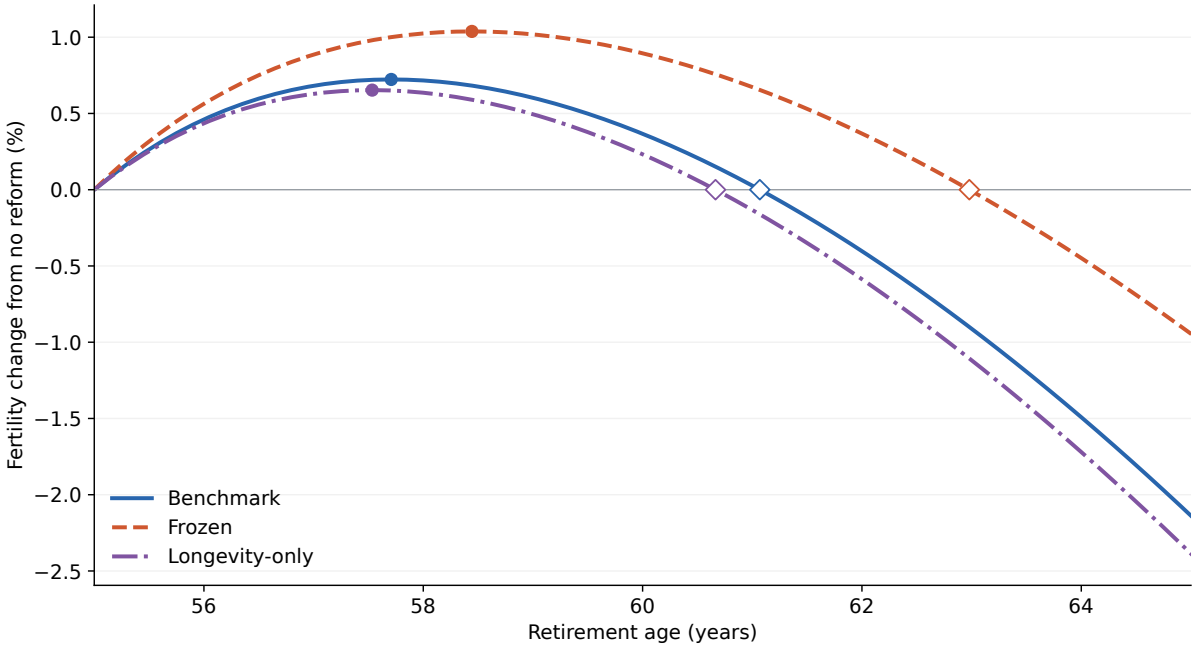

Figure D.2. Steady-state fertility effects under alternative health specifications
Notes: The figure compares Benchmark, Frozen, and Longevity-only over retirement ages 55-65. The vertical axis reports $100[n^j(\chi)/n(0) - 1]$ relative to the common no-reform steady state, in percent. Solid blue, dashed orange, and dash-dotted purple lines denote Benchmark, Frozen, and Longevity-only. Filled circles mark the peak χ_P; open diamonds mark the nonzero return-to-baseline threshold χ_L.

duration of pension receipt and grandparental childcare provision. The combined effect lowers steady-state fertility relative to Frozen in this calibration. Benchmark additionally allows health capital to respond, changing elderly earnings, childcare capacity, and optimal medical expenditure relative to wages. These adjustments raise the peak and return-to-baseline ages by approximately 0.2 and 0.4 years, respectively, relative to Longevity-only. Both ages remain below their Frozen values.

D.2 Capital, labor, and GDP per capita

Figure D.3 reports changes in normalized capital per young adult and GDP per capita relative to their same-period no-reform values. With retirement at 58, capital is initially unchanged, then is 15.60% lower in Benchmark and 15.43% lower in Frozen in period 2. The long-run reductions reach 26.64% and 25.41%. Lower inherited capital reduces wages and hence subsequent saving, carrying the initial loss forward. The long-run capital reduction is approximately 1.22 percentage points larger in Benchmark than in Frozen at retirement age 58, and 2.15 points larger at 62. Section 6.4 gives the underlying capital recursion.

In period 1, capital is identical to no reform, while elderly employment increases immediately. In both specifications, young households devote more time to childrearing and supply less labor than under no reform. Reduced grandparental childcare raises the parental time required per child, while higher expected old-age earnings support current consumption and childbearing. Even at retirement age 62, where Benchmark fertility falls, parental childrearing time per young adult rises. Additional elderly labor more than offsets the reduction in young labor: effective labor per young adult, ℓ_t, rises by roughly 19% with retirement at 58 and 45% with retirement at 62. Together with the population adjustment in (B.1), this yields first-period GDP-per-capita

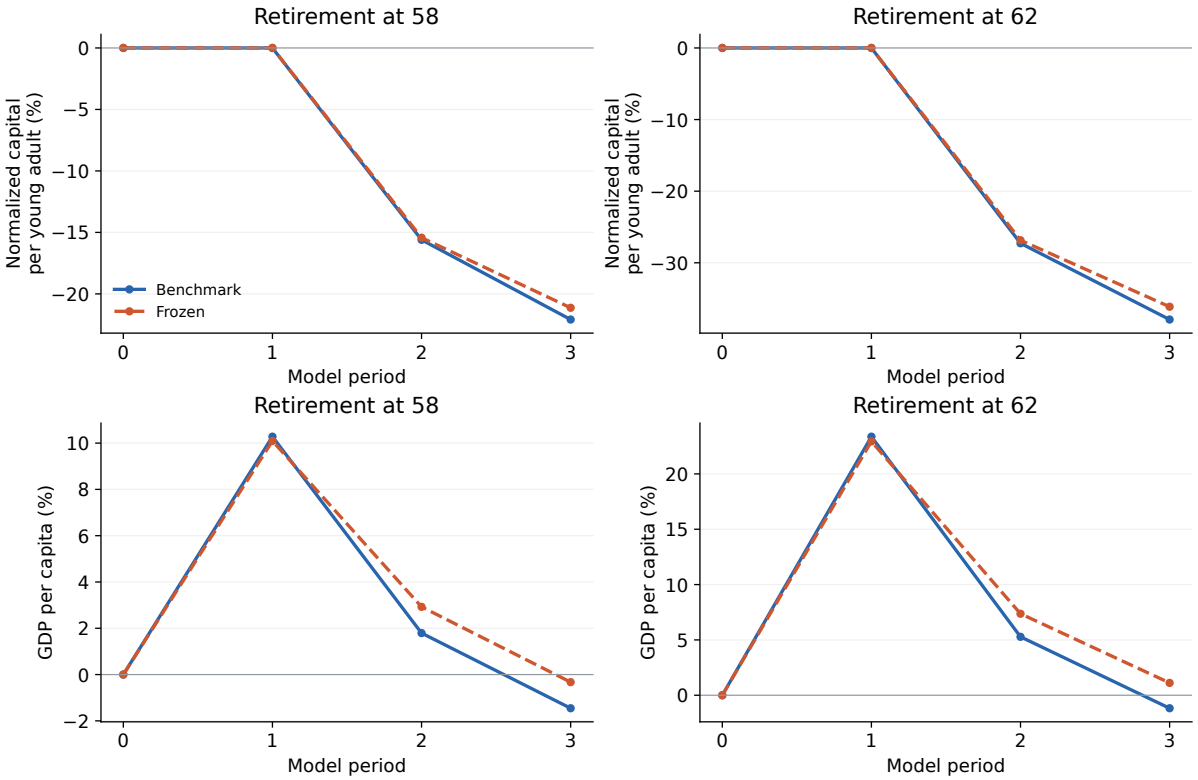

Figure D.3. Capital and GDP per capita following delayed retirement

Notes: The top row shows normalized capital per young adult, $k_t = K_t/(Z_t N_t)$, and the bottom GDP per capita, $y_t^{pc} = Z_t k_t^{\alpha} \ell_t^{1-\alpha}/P_t$, where P_t is population per young adult as defined in (B.1). Left and right columns correspond to retirement at 58 and 62. All changes are in percent relative to the same-period no-reform outcome. Benchmark and Frozen share the productivity path and inherited initial capital. Gray zero lines indicate no policy effect.

gains of approximately 10.28% and 10.09% in Benchmark and Frozen at 58, and 23.36% and 22.94% at 62.

The scale of the initial labor gain follows from the cohort accounting. At the baseline, the elderly-to-young headcount ratio is $1/\bar{n}$, and the survival-weighted ratio is $\bar{\pi}/\bar{n} \simeq 1.83$. With retirement at 58, elderly effective labor per young adult rises from zero to $0.1\bar{h}/\bar{n} \simeq 0.185$, compared with baseline young labor of about 0.961. This calculation applies the reference retirement rule to the representative population; its magnitude is governed by the model's closed-population cohort structure.

The labor gain remains substantial in later periods, while lower saving increases the capital shortfall relative to no reform. At retirement age 58, effective labor per young adult in Benchmark remains about 19.2% above no reform in period 3, but normalized capital is about 22.1% lower. GDP per capita is then 1.46% below no reform; the corresponding Frozen effect is -0.33%. The long-run effects are -4.08% and -2.71%, respectively. Changes in the population denominator in (B.1) also affect these magnitudes, but the initial gain and subsequent reversal primarily reflect immediate labor expansion and gradual capital loss. GDP-per-capita levels continue to grow with productivity along the calibrated paths.

Relative to Frozen, Benchmark has lower fertility and less parental childrearing time in cohort 1, the adults who are young at implementation. With parental childcare assistance identical at implementation, this raises young labor and reduces children per young adult in the population denominator. Initial capital is unchanged, so the Benchmark-Frozen difference in GDP-per-

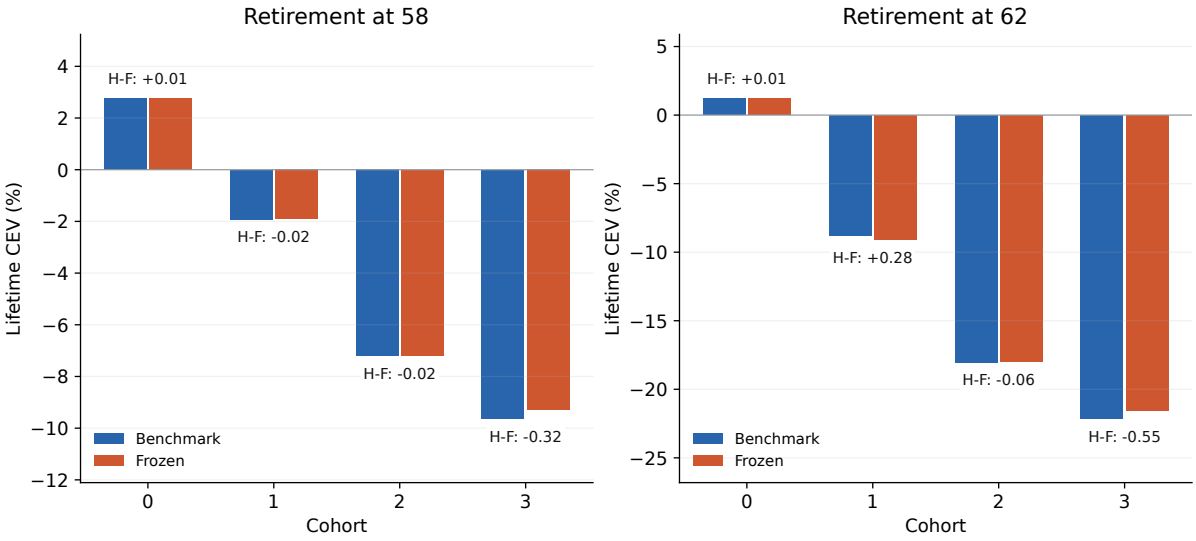

Figure D.4. Lifetime welfare effects of delayed retirement across cohorts

Notes: Left and right panels show retirement at 58 and 62. Blue and orange bars report lifetime CEVs for Benchmark and Frozen relative to the common no-reform path. Labels H-F show Benchmark minus Frozen in percentage points. The gray line denotes zero welfare change. Cohort 0 is elderly at implementation; cohort 1 is young at implementation, and cohorts 2 and 3 reach young adulthood later. Cohort 3's old-age outcomes use the numerical continuation to 2144.

capita policy effects is initially +0.19 and +0.43 percentage points at retirement ages 58 and 62. Later, the larger capital loss reduces production, while greater longevity increases the elderly population component, other things equal. After accounting for all labor and population responses, the differences turn negative and reach approximately -1.36 and -2.70 points in the long run.

The capital response also depends on how saving is represented across life stages. Only young households save to finance the next period's capital; earnings received during old age enter consumption within that stage. The model therefore does not separately represent saving out of earnings during the additional years of employment before full retirement.

D.3 Lifetime welfare across cohorts

Welfare is measured by the lifetime CEV defined in (B.6), relative to the same cohort's no-reform outcome. Cohort 0 is elderly at implementation, and cohort 1 is young at implementation.

In both calibrated reforms, welfare rises for the elderly at implementation and falls for the reported young cohorts in both specifications (Figure D.4). The initially old hold predetermined assets and health. More labor raises capital returns, and extended employment adds earnings; their consumption gain outweighs shorter childcare and leisure time. At retirement age 58, their lifetime CEV is approximately +2.76% in Benchmark and +2.75% in Frozen. The Benchmark-Frozen difference arises because lower fertility releases young labor, raising capital returns and the pension tax base. Their initial assets, health, and retirement-time allocations are identical across the two specifications.

Relative to no reform, cohort 1 faces lower current wages, less parental childcare assistance, and a shorter future retirement period despite health adjustment. At retirement age 58, its CEV is about -1.93% in Benchmark and -1.91% in Frozen. Later cohorts inherit less capital and face larger consumption losses; the long-run CEV reaches -11.41% and -10.96%, respectively. Welfare losses are larger for later cohorts in the reported paths.

Table D.1. Utility components of the total retirement-reform effect

Retirement age	Model	Cohort	$\Delta^P \log c$	$\theta\Delta^P \log n$	$\beta\Delta^P \log d$	Retirement time	Total	CEV (%)
58	Benchmark	1	-0.0440	+0.0002	+0.1178	-0.1087	-0.0347	-1.93
58	Benchmark	3	-0.1538	+0.0005	+0.0813	-0.1087	-0.1807	-9.63
58	Frozen	1	-0.0341	+0.0009	+0.1231	-0.1243	-0.0344	-1.91
58	Frozen	3	-0.1375	+0.0008	+0.0867	-0.1243	-0.1743	-9.30
62	Benchmark	1	-0.1073	-0.0012	+0.2240	-0.2797	-0.1643	-8.79
62	Benchmark	3	-0.3200	-0.0003	+0.1535	-0.2798	-0.4466	-22.14
62	Frozen	1	-0.0863	+0.0003	+0.2320	-0.3159	-0.1698	-9.08
62	Frozen	3	-0.2826	+0.0003	+0.1642	-0.3159	-0.4340	-21.58

Notes: Retirement ages are in years. Cohort 1 is young at implementation; cohort 3 reaches young adulthood in period 3. Each utility entry is reform minus the same cohort's no-reform value, including its own next-period old-age outcomes. Retirement time combines $\beta\xi\Delta^P \log \mu$ and $\beta\gamma\Delta^P \log \zeta$. The four preference-weighted components sum to total utility before rounding. CEV is calculated from total utility using $100[\exp(\Delta^P U/(1+\beta)) - 1]$; differences between reported CEVs are computed before rounding.

Table D.1 decomposes the change in lifetime utility relative to no reform into young consumption, fertility, old-age consumption, and retirement-time components. Higher old-age consumption does not offset the losses from young consumption and retirement time. For example, at retirement age 62, cohort 1 in Benchmark gains 0.2240 utility units from old-age consumption but loses 0.2797 from retirement time, with an additional loss from young consumption. The negative time contribution relative to no reform can coexist with the positive Benchmark-Frozen time contribution in Table B.2. Its magnitude reflects the old-age preference weights recovered from medical optimality and the childcare target. Total utility changes are then converted to CEVs using (B.6).

At retirement age 58, the Benchmark-Frozen difference in lifetime CEV is approximately -0.02 percentage points for cohort 1, -0.32 for cohort 3, and -0.45 in the long run. At retirement age 62, the corresponding differences are +0.28, -0.55, and -0.79 points. The positive difference for cohort 1 at 62 means that health adjustment reduces its welfare loss relative to no reform.

Additional longevity gives cohort 1 more time for grandchild care and leisure in its own old age than under Frozen. At retirement age 62, this utility gain, chiefly from leisure, outweighs the losses from consumption and fertility differences. The cohort-1 Benchmark-Frozen CEV difference changes sign at approximately retirement age 58.4, where these gains and losses balance. Later cohorts also receive more childcare assistance from their parents, but larger capital losses reduce wages and consumption. Appendix B.7 distinguishes utility from providing grandchild care in old age from the effects of parental childcare assistance on choices during working adulthood.

Data availability

The replication package supplied for peer review contains the code, aggregate calibration inputs, numerical results, and instructions for reproducing the analysis. It includes code to reconstruct the childcare targets. The underlying China Health and Nutrition Survey records are available separately from the *Individual-Level Data* collection on UNC Dataverse (https://doi.org/10.15139/S3/9PQ5KV). Appendix B documents the calibration sources and data transformations.

Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

The author used ChatGPT and Codex (OpenAI) to assist with editing the manuscript, searching the literature, checking citations and mathematical derivations, and writing and debugging the Python code used for numerical analysis and plotting. After using these tools, the author reviewed and edited the manuscript and takes full responsibility for its content.

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CRediT authorship contribution statement

Tianye Zhou: Conceptualization, Methodology, Formal analysis, Software, Validation, Data curation, Visualization, Writing – original draft, Writing – review and editing.

Declaration of competing interests

The author has no competing interests to declare.

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References

Aísa, R., Larramona, G., Pueyo, F., 2015. Active aging, preventive health and dependency: Heterogeneous workers, differential behavior. Journal of Economic Behavior & Organization 117, 1–9. doi:10.1016/j.jebo.2015.06.001.

Baldi, M.M., Coppier, R., Michetti, E., 2025. Elderly labor supply, endogenous grandparental childcare, and fertility in an OLG model. Economic Modelling 149, 107102. doi:10.1016/j.econmod.2025.107102.

Battistin, E., De Nadai, M., Padula, M., 2014. Roadblocks on the road to grandma's house: Fertility consequences of delayed retirement. IZA Discussion Paper No. 8071. URL: https://www.iza.org/publications/dp/8071/roadblocks-on-the-road-to-grandmas-house-fertility-consequences-of-delayed-retirement.

Bertoni, M., Brunello, G., Mazzarella, G., 2018. Does postponing minimum retirement age improve healthy behaviors before retirement? evidence from middle-aged Italian workers. Journal of Health Economics 58, 215–227. doi:10.1016/j.jhealeco.2018.02.011.

Bratti, M., Frattini, T., Scervini, F., 2018. Grandparental availability for child care and maternal labor force participation: Pension reform evidence from Italy. Journal of Population Economics 31, 1239–1277. doi:10.1007/s00148-017-0683-7.

China Health and Nutrition Survey, n.d. Individual-level data, version 1.0. URL: https://doi.org/10.15139/S3/9PQ5KV, doi:10.15139/S3/9PQ5KV. uNC Dataverse. Waves 2004, 2006, 2009, and 2011; accessed November 4, 2026.

Cipriani, G.P., Fioroni, T., 2024. Grandparental childcare, family allowances and retirement policies. International Tax and Public Finance 31, 1669–1692. doi:10.1007/s10797-023-09822-9.

Cipriani, G.P., Fioroni, T., 2025. Healthy grands, fertility and pensions. The B.E. Journal of Macroeconomics 25, 121–145. doi:10.1515/bejm-2024-0112.

Dai, Y., Zhao, Z., Sui, J., Xu, J., 2025. The impact of delayed retirement on labor employment, fertility rate and economic growth in China. International Review of Economics & Finance 100, 104103. doi:10.1016/j.iref.2025.104103.

Du, F., Dong, X.Y., Zhang, Y., 2019. Grandparent-provided childcare and labor force participation of mothers with preschool children in urban China. China Population and Development Studies 2, 347–368. doi:10.1007/s42379-018-00020-3.

Etgeton, S., Fischer, B., Ye, H., 2023. The effect of increasing retirement age on households' savings and consumption expenditure. Journal of Public Economics 221, 104845. doi:10.1016/j.jpubeco.2023.104845.

Feng, J., Zhang, X., 2018. Retirement and grandchild care in urban China. Feminist Economics 24, 240–264. doi:10.1080/13545701.2017.1370120.

Frattola, E., 2025. Parental retirement and fertility decisions across family policy regimes. Review of Economics of the Household 23, 933–968. doi:10.1007/s11150-024-09757-y. appendix correction: doi:10.1007/s11150-025-09763-8.

Galama, T.J., Kapteyn, A., Fonseca, R., Michaud, P.C., 2013. A health production model with endogenous retirement. Health Economics 22, 883–902. doi:10.1002/hec.2865.

Geyer, J., Haan, P., Teschner-Hofmann, M., Weinhardt, F., 2026. Applying but not receiving: Longer working lives and institutionalized health investment. Working Paper 26/51. Institute for Fiscal Studies. doi:10.1920/wp.ifs.2026.5126.

Grossman, M., 1972. On the concept of health capital and the demand for health. Journal of Political Economy 80, 223–255. doi:10.1086/259880.

Huang, L., 2026. Postponing retirement under age discrimination and grandparenting. Journal of Asian Economics 103, 102111. doi:10.1016/j.asieco.2025.102111.

Ilciukas, J., 2023. Fertility and parental retirement. Journal of Public Economics 226, 104928. doi:10.1016/j.jpubeco.2023.104928.

Li, C., Cang, H., 2026. Health, incentives, and retirement choice: A life-cycle analysis from China. China Economic Review 96, 102667. doi:10.1016/j.chieco.2026.102667.

Luo, Y., LaPierre, T.A., Hughes, M.E., Waite, L.J., 2012. Grandparents providing care to grandchildren: A population-based study of continuity and change. Journal of Family Issues 33, 1143–1167. doi:10.1177/0192513X12438685.

Ma, J., Yang, Y., Ding, Y., 2026. Delayed retirement, grandparental support, and adult children's fertility intentions: Insights from a survey experiment in China. Social Policy and Society doi:10.1017/S1474746426101420. Advance online publication, pp. 1–19. First published online 27 March 2026.

Miyazawa, K., 2021. Elderly empowerment, fertility, and public pensions. International Tax and Public Finance 28, 941–964. doi:10.1007/s10797-020-09644-z.

National Bureau of Statistics, 2010. Population census: Health status by age and sex, cities, 2010. URL: https://www.stats.gov.cn/sj/pcsj/rkpc/6rp/html/B0802b.htm. table 8-2a. City women aged 60–84.

National Bureau of Statistics, 2011a. China statistical yearbook 2011. URL: https://www.stats.gov.cn/sj/ndsj/2011/html/J1008C.HTM. table 10-8, urban household consumption expenditure by income group, 2010.

National Bureau of Statistics, 2011b. Urban household income continued to grow and living standards improved [in Chinese]. URL: https://www.stats.gov.cn/zt_18555/ztfx/sywcj/202303/t20230301_1920369.html. march 7. Eleventh Five-Year Plan achievement report No. 9.

National Bureau of Statistics, 2020a. Population census: Age-specific fertility rates of women by region, cities, 2020. URL: https://www.stats.gov.cn/sj/pcsj/rkpc/7rp/zk/html/B0604a.jpg. table 6-4a. The national city total is 1118.44 births per 1000 women; the calibration uses the rounded TFR of 1.118.

National Bureau of Statistics, 2020b. Population census: Health status by age and sex, cities, 2020. URL: https://www.stats.gov.cn/sj/pcsj/rkpc/7rp/zk/html/B0802a.jpg. table 8-2a. City women aged 60–84.

National Bureau of Statistics, 2021. Household income and consumption expenditure in 2020 [in Chinese]. URL: https://www.stats.gov.cn/xxgk/sjfb/zxfb2020/202101/t20210118_1812464.html. january 18. Table 2, urban residents.

National Bureau of Statistics, 2023. Household survey: Frequently asked questions [in Chinese]. URL: https://www.stats.gov.cn/hd/cjwtjd/202302/t20230207_1902268.html. october 9. Question 3: wage income includes individual income tax and personal social-insurance contributions.

National Bureau of Statistics, 2025. Household income and consumption expenditure in 2024 [in Chinese]. URL: https://www.stats.gov.cn/sj/zxfb/202501/t20250117_1958325.html. statistical release, January 17.

National Bureau of Statistics, n.d. Population census: Deaths by age and sex, cities, 2000, 2010 and 2020. URL: https://www.stats.gov.cn/sj/pcsj/rkpc/6rp/html/A0604b.htm. table 6-4a in each census. Female population and death counts at ages 55–84. Tables: 5rp/html/t0604a.htm; 6rp/html/A0604b.htm; 7rp/zk/html/A0604a.xls.

National Bureau of Statistics, Department of Population and Employment Statistics, 2025. China population and employment statistical yearbook 2025. China Statistics Press. Table 2-45, Fertility of women of childbearing age in cities, by age and birth order, November 1, 2023-October 31, 2024. ISBN 9787523008638.

OECD, 2025. OECD global long-run economic scenarios: 2025 update. OECD Economic Policy Papers No. 36. doi:10.1787/00353678-en.

OECD, 2026. OECD economic outlook 117 long-term scenarios: China BAU1 potential GDP per capita, constant 2021 PPP US dollars. URL: https://data-explorer.oecd.org/vis?df[ds]=dsDisseminateFinalDMZ&df[id]=DSD_EO_LTB%40DF_EO_LTB&df[ag]=OECD.ECO.MAD&df[vs]=1.0&lc=en&dq=CHN.GDPVTRD_CAP.BAU1.A&pd=2024,2100&to[TIME_PERIOD]=false&vw=tb. official table. Retrieved September 21, 2026. Appendix B.2 documents calibration endpoints and extrapolation.

Penn World Table and FRED, 2026. Share of labour compensation in GDP at current national prices for China, series LABSHPCNA156NRUG. URL: https://fred.stlouisfed.org/data/LABSHPCNA156NRUG. 2023 observation: 0.56118. Annual data; updated February 4, 2026; accessed September 21, 2026.

Qiao, X., Wang, L., 2019. Fertility and old-age labor supply in aging China. China Economic Review 57, 101261. doi:10.1016/j.chieco.2018.12.004.

Standing Committee of the National People's Congress, 2024. Decision on implementing a gradual increase in statutory retirement ages, and State Council measures on gradually increasing statutory retirement ages [in Chinese]. URL: https://www.cqrd.gov.cn/site/article/1284174374269034496/web/content_1284174374269034496.html. adopted September 13, 2024; effective January 1, 2025. Published full text.

World Bank, 2026. World development indicators, GDP per capita (constant 2015 US$), series NY.GDP.PCAP.KD, China. URL: https://data.worldbank.org/indicator/NY.GDP.PCAP.KD?locations=CN. data vintage: July 13, 2026.

Yu, X., Shen, K., Angeline, P.T., Guan, Y., Wu, Y., 2026. Grandchild care and its links to disability and depressive symptoms among urban residents aged 60 and older. Healthcare 14, 2418. URL: https://www.mdpi.com/2227-9032/14/15/2418, doi:10.3390/healthcare14152418.

Zhang, J., Emery, T., 2023. Grandparental childcare and second births in China. PLOS ONE 18, e0286496. doi:10.1371/journal.pone.0286496.

Zhang, J., Zhang, J., Leung, M.C.M., 2006. Health investment, saving, and public policy. Canadian Journal of Economics 39, 68–93. doi:10.1111/j.0008-4085.2006.00339.x.

Zhang, Y., Luh, Y.H., 2018. Grandparents' health and family fertility choice: Evidence from Taiwan. China Economic Review 51, 294–308. doi:10.1016/j.chieco.2018.06.003.

Zhu, M., Shen, Y., Kang, Z., 2024. Delayed retirement intention, medical service utilization and health benefits. China Economic Quarterly 24, 84–100. doi:10.13821/j.cnki.ceq.2024.01.06. in Chinese.