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Old-age Social Security vs. Forward Intergenerational Public Goods Provision*

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Abstract

This paper introduces an overlapping-generations model with earnings heterogeneity and borrowing constraints. The labor income tax and the allocation of tax revenue across social security and forward intergenerational public goods are determined in a bidimensional majoritarian voting game played by successive generations. The political equilibrium is characterized by an ends-against-the-middle equilibrium where low- and high-income individuals form a coalition in favor of a low tax rate and less social security while middle-income individuals favor a high tax rate and greater social security. Government spending then shifts from social security to public goods provision if higher wage inequality is associated with the borrowing constraint and a low interest-rate elasticity of consumption.

Keywords: Borrowing constraint; Old-age social security; Forward intergenerational public goods; Ends-against-the-middle equilibrium; Wage inequality

JEL Classification: H41; H55; D72

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1 Introduction

The unfunded old-age social security program benefits the retired old at the expense of the current working young and thus creates an intergenerational conflict over government expenditure. The voting models of Browning (1975) and Boadway and Wildasin (1989) have precisely this flavor. In addition to this intergenerational conflict, the unfunded social security program also creates intragenerational conflict when income inequality within a generation is taken into account. Some previous studies introduce this inequality to emphasize the intragenerational redistribution component built in many social security systems (for example, Casamatta, Cremer, and Pestieau, 2000; Tabellini, 2000; Conde-Ruiz and Galasso, 2005; Galasso and Profeta, 2007; Koethenbuerger, Poutvaara and Profeta, 2008). They show that the redistribution from the rich to the poor via social security plays a crucial role in the voting game because social security becomes appealing to the low-income young.

The studies on the political economy of social security have been expanded in various directions for the past decade (see the surveys by Galasso and Profeta, 2002; de Walque, 2005; Borck, 2007b). In particular, the following two directions are of concern in the present study. The first is the introduction of borrowing constraints into the analysis and the second is the voting over multidimensional choice spaces.

As for the first direction, some previous studies exclude the presence of the constraints by assuming no consumption in youth (for example, Conde-Ruiz and Galasso, 2003, 2005; Bethencourt and Galasso, 2008) or by implicitly assuming that saving does not hit the borrowing constraint (for example, Galasso and Profeta, 2007). Given no restriction on saving, lower-income agents prefer a larger size of social security, which coincides with the standard result in the literature of the political economy of redistribution (Romer, 1975; Roberts, 1977; Meltzer and Richard, 1981). The result implies that higher inequality associated with a reduction of the decisive voter's income results in a larger volume of social security.

This theoretical prediction, however, is not necessarily supported by the empirical evidence: OECD cross-country data show that the size of social security is negatively correlated with inequality (Gottschalk and Smeeding, 1997; Chen and Song, 2009). Some studies suggest that the presence of borrowing constraints resolves this issue (Casamatta, Cremer, and Pestieau, 2000; Bellettini and Berti Ceroni, 2007; Conde-Ruiz and Profeta, 2007; Cremer et al., 2007). In particular, borrowing-constrained agents, who cannot re-allocate resources freely across periods, attach a large weight to the utility loss of income reduction in youth, and thus prefer a lower tax burden for social security finance in response to a reduction of their income, provided their interest-rate elasticity of consumption is low (Arawatari and Ono, 2011).

The second direction, with which we are concerned here, is a two-dimensional voting including intergenerational as well as intragenerational conflicts. Under a balanced government budget constraint, the unfunded old-age social security benefits are provided at the expense of other government programs such as intragenerational redistribution (Conde-Ruiz and Galasso, 2005), public goods provision that benefits only the young (Levy, 2005) and medical services for the elderly (Bethencourt and Galasso, 2008). Bernasconi and Profeta (2011) focus on two competing programs, public education and redistribution, and examine an intergenerational consequence of these two programs in the political economy. Society must choose via voting both the size of government (i.e., a tax) and how to allocate its resources between two competing programs. [1]

In this paper, we particularly focus on public goods, such as pure science and environmental maintenance, called *forward intergenerational public goods* in this study, which take a long time to mature and thus benefit only the young. Levy (2005) analyzed this issue in a two-party system. She focused on lump-sum redistribution and public education that benefits only the young, and showed that an increase in income inequality may decrease the size of redistribution when the majority is the young, which fits the empirical evidence. However, the predictions about the share of redistribution and forward intergenerational public goods (i.e., public education) in government expenditure are abstracted in her analysis because no provision of forward intergenerational public goods arises in the young-majority equilibrium because of the specification of the model.

The aim of this study is to combine the abovementioned two strands of literature that have received attention in recent years. In particular, we analyze how income inequality affects the size of government as well as the shares of old-age social security and forward intergenerational public goods in government expenditure, which has not been fully investigated in the previous studies. Therefore, the current paper sheds light on the role of borrowing constraints in the two-dimensional voting over old-age social security and forward intergenerational public goods provision.

For this purpose, we introduce an overlapping-generations model with heterogeneous agents. In this economy, young workers are of three types of income: low, middle, and high. Because they are not permitted to borrow in youth, as a result of imperfect financial markets, lower-income individuals are more likely to be borrowing constrained. Young workers then pay a fixed proportion of their labor income to the government, and the tax

[1] Besides the abovementioned studies, there is an emerging literature on two-dimensional voting, particularly focusing on voting over payroll tax and the pension system. This literature is classified into two categories according to the concept of equilibrium. The first includes the studies based on sequential voting (for example, Casamatta, Cremer, and Pestieau, 2000, 2005; Cremer et al., 2007); and the second includes the studies based on Shepsle's (1979) notion of structure-induced Nash equilibrium voting (for example, Conde-Ruiz and Galasso, 2003; Conde-Ruiz and Profeta, 2007). As explained further below, the current paper falls into the second category.

revenue is divided into social security payments from which the old can benefit and public goods provision that the old cannot capture.

The tax rate and the allocation of tax revenue between social security and public goods provision are determined in a two-dimensional majoritarian voting game played by the young and the old. Voters cast ballots over the labor income tax, which finances social security and public goods provision, and over the allocation of tax revenue between social security and public goods provision. Under this type of voting game, the existence of a Condorcet winner of the majority voting game is not necessarily guaranteed because of the multidimensionality of the issue space. To deal with this problem, we utilize the concept of a structure-induced equilibrium (Shepsle, 1979) with the notion of once-and-for-all voting, which is applied to an overlapping-generations framework by Conde-Ruiz and Galasso (2003, 2005).

Based on the abovementioned concept of equilibrium, we consider the voting behavior of each type of individual. The preferences of the old are identical across all types of individuals because they have no tax burden, receive the same level of social security benefit, and cannot capture the benefit of forward intergenerational public goods. Instead, they prefer the tax rate that attains the top of the Laffer curve and full use of the revenue for social security. In contrast, the preferences of the young depend on their income type because the tax burden differs across the types of income. In particular, the key factors for their preferences are the borrowing constraint and the interest-rate elasticity of consumption.

To understand the role of these two factors, consider the case where only low-income individuals are faced with the borrowing constraint. They wish to consume more in their youth but cannot because of the borrowing constraint. In this situation, a higher tax rate produces two opposing effects: a negative effect that results in lower after-tax income, and thus the utility loss of taxation in youth, and a positive effect that produces greater social security benefit, and thus the utility gain of taxation in old age.

When the interest-rate elasticity is high, substitution across periods is easy: the utility loss of taxation in youth is compensated for by the utility gain of taxation in old age. Therefore, low-income individuals choose a higher tax rate than middle- and high-income individuals. However, when the interest-rate elasticity is low, compensation is not necessarily available for low-income individuals because substitution across periods is difficult. Low-income individuals, then, prefer a lower tax rate than middle-income individuals. This results in an ends-against-the-middle equilibrium where low- and high-income individuals form a coalition in favor of a low tax rate while middle-income individuals favor a high tax rate.

Given the characterization of political equilibrium, we investigate how the shares of social security and forward intergenerational public goods in government expenditure are altered in response to changes in wage inequality. We show that the mean-preserving reduction of the decisive voter's wage creates an inverse U-shaped relationship between the decisive voter's wage and the share of social security in government expenditure when the interest-rate elasticity is low. The positive correlation arises when the decisive voter's wage is high, and thus he/she is borrowing unconstrained, while the negative correlation arises when his/her wage is low, and thus he/she is borrowing constrained. The same relation also holds between wage inequality and the size of social security. Therefore, the interest-rate elasticity and the borrowing constraint are the key factors needed to demonstrate the negative correlation between wage inequality and the share (or size) of social security.

The organization of this paper is as follows. Section 2 introduces the model and characterizes the economic equilibrium. Section 3 develops the political system, introduces the equilibrium concept of the voting game and demonstrates the voting behavior of each individual. Section 4 characterizes the political equilibrium. Section 5 examines how wage inequality affects the tax rate and the allocation of tax revenue between social security and forward intergenerational public goods. Section 6 briefly undertakes the analysis under a generalized framework. Section 7 provides concluding remarks. Proofs of the propositions are provided in the appendix.

2 The Economic Environment

Consider a discrete time economy where time is denoted by $t = 0, 1, 2 \cdots$. The economy is made up of overlapping generations of individuals, each of whom lives two periods: youth and old age. The size of a generation born in period t, called generation t, is denoted by N_t. Population grows at a constant rate $n > 0 : N_{t+1} = (1 + n)N_t$ for all $t \geq 0$. Within each generation, there are three types of agents according to ability, low, middle and high ($j = L, M, H$), whose proportions are respectively ρ^L, ρ^M and ρ^H, where $\sum_j \rho^j = 1$ and ρ^j satisfies the following assumption.

Assumption 1. *$\rho^j > n/\{2(1 + n)\}$, $j = L, M, H$.*

Assumption 1 ensures that a young individual who prefers the highest tax rate among young individuals becomes the decisive voter. To understand the argument stemming from Assumption 1, consider first the preferences of the old. As we explain below, the old choose a higher tax rate than any young individual because they bear no tax burden but benefit from taxation via social security; the tax burden when young is viewed as a sunk cost for the old. In addition, the old have the same preferences over the policy because they benefit from the same social security.

Next, consider the preferences of the young. Suppose that a type-k ($k = L, M$ or H) prefers the highest tax rate. As explained below, all the old have the same preferences over policies and choose a higher tax rate than any young agent. When the young and the old participate in voting, the sum of the type-k young and the old is given by $N_t\rho^k + N_{t-1}$, which is greater than half of the population in period t, $(N_t + N_{t-1})/2$, under the assumption of $\rho^k > n/2(1+n)$. This implies that the decisive voter becomes the old or the type-k young. However, the old cannot become the decisive voter because the population size of the old is smaller than that of the young under the assumption of $n > 0$. Therefore, the type-k young individual becomes the decisive voter. Figure 1 provides an example of preferences over the tax rate.

[Figure 1 about here.]

2.1 Individuals

Each individual is assumed to receive utility from private consumption and publicly provided goods. The utility function of a type-j young individual in period t is specified by:

$$U_t^j = \frac{(c_t^{yj})^{1-\sigma} - 1}{1-\sigma} + \eta\frac{(g_t)^{1-\sigma} - 1}{1-\sigma} + \beta \cdot \left[c_{t+1}^{oj} + \eta\frac{(g_{t+1})^{1-\sigma} - 1}{1-\sigma}\right],$$

where c_t^{yj} is consumption in youth, c_{t+1}^{oj} is consumption in old age, g_t is per capita public goods in period t, $\eta(> 0)$ is the parameter representing the preference for public goods, $\beta \in (0, 1]$ is the discount factor, and $\sigma(> 0)$ is the inverse of the elasticity of young-age consumption with respect to the interest rate. A lower $1/\sigma$ implies a lower interest-rate elasticity of young-age consumption.

Following the literature (Conde-Ruiz and Galasso, 2005; Borck, 2007; Bethencourt and Galasso, 2008; Leroux, Pestieau and Racionero, 2011), we assume a quasi-linear utility function for analytical tractability. In Section 6, we briefly investigate the case where the utility of old-age consumption is given by $\beta\{(c_{t+1}^{oj})^{1-\sigma} - 1\}/(1-\sigma)$ and show that the main result is not qualitatively unchanged under this alternative utility function.

Each individual works in his/her youth and retires in old age. The wage income is related to working ability. The wage of a type-j individual is given by $w^j (j = H, M, L)$, where w^j is constant over time and $w^L < w^M < w^H$. The average of the wage is denoted by $\bar{w} \equiv \rho^L w^L + \rho^M w^M + \rho^H w^H$.

Type-j's individual budget constraints in youth and old age are given respectively by:

$$c_t^{yj} + s_t^j \le (1-\tau_t)w^j,$$
$$c_{t+1}^{oj} \le Rs_t^j + b_{t+1},$$

where s_t^j is saving, τ_t is the income tax rate in period t, R is the gross interest rate, and b_{t+1} is the per capita social security benefit in old age. We impose the restriction of nonnegative savings, that is:

$$s_t^j \geq 0.$$

This rules out the possibility of borrowing in youth against future social security benefits (Diamond and Hausman, 1984; Conde-Ruiz and Profeta, 2007).

We assume that the economy is dynamically efficient.

Assumption 2. $R \geq 1 + n$.

The assumption implies that the rate of return from social security is lower than the private rate of return from saving. Nevertheless, low- and middle-income individuals may have an incentive to support this inferior system of intertemporal resource reallocation. This is because the current social security system involves an intragenerational redistribution component that transfers resources from the high to the low and the middle.

We also assume that (i) the interest rate is exogenous, and (ii) each individual receives the same amount of old age social security benefits regardless of contributions in their youth. The first assumption abstracts away the general equilibrium effect via the interest rate investigated by, for example, Cooley and Soares (1999) and Boldrin and Rustichini (2000). However, this simplification enables us to demonstrate more simply the analytical solution of the model. The second assumption abstracts away from the choice of social security systems (for example, Bismarckian vs. Beveridgean) analyzed by, for example, Borck (2007), Conde-Ruiz and Profeta (2007) and Cremer et al. (2007). We adopt the second assumption to concentrate on the role of the borrowing constraint in the political determination of social security and public goods provision.

The representative type-j young individual maximizes his/her utility subject to the budget constraints and the restriction of nonnegative saving. When $s_t^j > 0$, the first-order condition for an interior solution is $(c_t^{yj})^{-\sigma} = \beta R$, and thus defines the optimal saving decision of a type-j individual given by $s_t^j = (1-\tau_t)w^j - (\beta R)^{-1/\sigma}$. By taking the borrowing constraint into account, the saving function of a type-j individual is:

$$s_t^j = \max\left\{0, (1-\tau_t)w^j - (\beta R)^{-1/\sigma}\right\}. \quad (1)$$

Eq. (1) indicates that the saving decision depends on the current tax rate τ_t, but is independent of the future tax rate τ_{t+1} and the proportion of tax revenues devoted to social security in old age, denoted by λ_{t+1}. This property comes from the assumption of a linear utility function of old-age consumption. Because of this property, we easily demonstrate the joint political determination of the tax rate τ and the proportion λ.

The saving function (1) implies that there is a critical rate of tax such that:

$$s_t^j > 0 \Leftrightarrow \tau_t < \hat{\tau}(w^j) \equiv 1 - \frac{1}{(\beta R)^{1/\sigma} w^j}. \qquad (2)$$

A type-j individual chooses positive saving when the tax is below the critical rate. However, when the tax is above the critical rate, a type-j individual faces a borrowing constraint and can save nothing in youth. The critical rate of tax is higher when the wage income is larger because, given a tax rate common to all types of individuals, a higher ability individual receives a higher level of disposable income.

2.2 The Government

In each period, the government collects tax revenue from the young by imposing an income tax. Following the convention in the literature, we present the efficiency loss of taxation by assuming convex costs of collecting taxes (for example, Casamatta, Cremer, and Pestieau, 2000; Bellettini and Berti Ceroni, 2007; Cremer et al., 2007). Therefore, the actual tax revenue is given by $(1-\tau_t)\tau_t(\rho^L w^L + \rho^M w^M + \rho^H w^H) = (1-\tau_t)\tau_t\bar{w}$, where the term $(1-\tau_t)$ is the distortionary factor. The assumption of distortionary taxation is solely to ensure an interior solution to preferred tax rates and otherwise plays no role.

The government uses the tax revenue for old-age social security payments along with forward intergenerational public goods such as environmental preservation and pure science. The proportion $\lambda_t \in [0,1]$ of tax revenue is devoted to old-age social security benefits and the remainder $(1-\lambda_t)$ is devoted to forward intergenerational public goods provision. The old-age social security is then an intergenerational transfer from the young to the old within a period. The budget constraint is $\lambda_t N_t(1-\tau_t)\tau_t\bar{w} = N_{t-1}b_t$. The per capita social security benefit in period t, b_t, is given by:

$$b_t = (1+n)\lambda_t(1-\tau_t)\tau_t\bar{w}.$$

The formation of public goods requires investment one period ahead of time. This assumption reflects the idea that education, pure science, and investment in the environment do not obtain immediate results. Importantly, the current young can enjoy the outcomes of any investment in the future, while the current old cannot enjoy it while they are still alive. The budget constraint is $(1-\lambda_t)N_t(1-\tau_t)\tau_t\bar{w} = (N_t + N_{t+1})g_{t+1}$. The per capita pubic goods provision in period $t+1$, g_{t+1}, is given by:

$$g_{t+1} = \frac{1}{2+n}(1-\lambda_t)(1-\tau_t)\tau_t\bar{w}.$$

2.3 The Economic Equilibrium

We define the economic equilibrium as follows.

Definition 1. For a given sequence of tax rates and social security shares in government expenditure, $\{\tau_t, \lambda_t\}_{t=0}^{\infty}$, an *economic equilibrium* is a sequence of allocations, $\{c_t^{yj}, c_t^{oj}, s_t^j\}_{j=L,M,H}^{t=0,\cdots,\infty}$ with the initial condition $s_0^j (j = L, M, H)$, such that (i) in every period, a type-j individual maximizes his/her utility subject to the budget constraints and the nonnegativity constraint of saving, (ii) the social security budget and the public goods budget are balanced in every period, and (iii) the goods market clears every period.

From (1) and the private and government budget constraints, the consumption functions of a type-j individual in youth and old age are given respectively by:

$$c_t^{yj} = \begin{cases} (\beta R)^{-1/\sigma} & \text{if } \tau_t < \hat{\tau}(w^j) \\ (1-\tau_t)w^j & \text{if } \tau_t \geq \hat{\tau}(w^j) \end{cases}$$

$$c_{t+1}^{oj} = \begin{cases} R\{(1-\tau_t)w^j - (\beta R)^{-1/\sigma}\} + (1+n)\lambda_{t+1}(1-\tau_{t+1})\tau_{t+1}\bar{w} & \text{if } \tau_t < \hat{\tau}(w^j) \\ (1+n)\lambda_{t+1}(1-\tau_{t+1})\tau_{t+1}\bar{w} & \text{if } \tau_t \geq \hat{\tau}(w^j). \end{cases}$$

Because of the assumption of a quasi-linear utility function, the consumption in youth is type-independent and constant over time when the tax is below the critical rate.

The utility level obtained by individuals in economic equilibrium is represented by their indirect utility functions. We use the abovementioned consumption functions to obtain an indirect utility function of a type-j young individual:

$$V_t^{yj} = \begin{cases} V_{t,s>0}^{yj} & \text{if } \tau_t < \hat{\tau}(w^j) \\ V_{t,s=0}^{yj} & \text{if } \tau_t \geq \hat{\tau}(w^j), \end{cases} \tag{3}$$

where:

$$V_{t,s>0}^{y,j} \equiv \frac{\left((\beta R)^{-1/\sigma}\right)^{1-\sigma} - 1}{1-\sigma} + \beta\left[R\left\{(1-\tau_t)w^j - (\beta R)^{-1/\sigma}\right\} + (1+n)\lambda_{t+1}(1-\tau_{t+1})\tau_{t+1}\bar{w}\right] + \eta\left\{\frac{(g_t)^{1-\sigma}-1}{1-\sigma} + \beta\frac{\left(\frac{1}{2+n}(1-\lambda_t)(1-\tau_t)\tau_t\bar{w}\right)^{1-\sigma}-1}{1-\sigma}\right\},$$

$$V_{t,s=0}^{y,j} \equiv \frac{\left((1-\tau_t)w^j\right)^{1-\sigma} - 1}{1-\sigma} + \beta(1+n)\lambda_{t+1}(1-\tau_{t+1})\tau_{t+1}\bar{w} + \eta\left\{\frac{(g_t)^{1-\sigma}-1}{1-\sigma} + \beta\frac{\left(\frac{1}{2+n}(1-\lambda_t)(1-\tau_t)\tau_t\bar{w}\right)^{1-\sigma}-1}{1-\sigma}\right\}.$$

$V^{y,j}_{t,s>0}$ denotes the indirect utility of a type-j young individual when he/she saves some portion of his/her income, and $V^{y,j}_{t,s=0}$ denotes the indirect utility when he/she is faced with a borrowing constraint and saves nothing. For each indirect utility function, the first term on the right-hand side shows the utility of consumption in youth, the second term shows the utility of consumption in old age and the third term shows the utility of public goods in old age. The public goods provision in period t, g_t, is predetermined in period $t-1$.

For a type-j old individual in period t, the indirect utility function is:

$$V^{o,j}_t \equiv (1+n)\lambda_t(1-\tau_t)\tau_t\bar{w} + \eta\frac{(g_t)^{1-\sigma}-1}{1-\sigma}, \tag{4}$$

where the first-term on the right-hand side shows the social security benefits. Old individuals have the same indirect utility function regardless of their type because their saving in youth is predetermined and the level of public goods they enjoy is predetermined one period in advance. Therefore, old individuals have the same preferences for the tax rate, τ, and the share of social security, λ.

3 The Political Institution and Voting

The tax rate τ and the proportion λ are determined by individuals through a political process of majoritarian voting. Elections take place every period and all individuals alive, both young and old, cast a ballot over τ, the income tax, and λ, the share of social security in government expenditure. Individual preferences over the two issues are represented by the indirect utility functions at Eqs. (3) and (4) for the young and the old, respectively. Every individual has zero mass and thus no individual vote can change the outcome of the election. Thus, we assume individuals vote sincerely.

This majoritarian voting game has two significant characteristics. First, the issue space is bidimensional (τ and λ), and thus the Nash equilibrium of a majoritarian voting game may fail to exist. To deal with this feature, we use the concept of issue-by-issue voting, or *structure-induced equilibrium*, as formalized by Shepsle (1979) and applied by Conde-Ruiz and Galasso (2003, 2005) for the framework of overlapping generations.

Second, the game is intrinsically dynamic because it describes the interaction among successive generations. To deal with the second feature, we assume full commitment, i.e., once-and-for-all voting. That is, voters determine the constant sequence of the parameters: $\tau_t = \tau_{t+1} = \tau$ and $\lambda_t = \lambda_{t+1} = \lambda$ for all t as in Casamatta, Cremer, and Pestieau (2000) and Conde-Ruiz and Profeta (2007). We can view the full commitment solution as the solution including intergenerational interaction because the full commitment solution can be supported as the subgame perfect equilibrium (see, for example, Conde-Ruiz and Galasso, 2003, 2005).

Given the stationary environment, the current model presents a static voting game. Therefore, the result in Shepsle (1979) can be applied to obtain the sufficient conditions for the existence of a structure-induced equilibrium. In particular, if preferences are single peaked along every dimension of the issue space, a sufficient condition for (τ^*, λ^*) to be an equilibrium of the voting game with full commitment is that τ^* represents the outcome of majority voting over the jurisdiction τ when the other dimension is fixed at its level λ^*, and vice versa.

Preferences of the old are immediately shown to be single peaked along every dimension because they are given by $V_t^{o,j} \equiv (1+n)\lambda_t(1-\tau_t)\tau_t\bar{w}$ and satisfy $\partial^2 V^{o,j}/\partial\tau^2 < 0$ and $\partial^2 V^{o,j}/\partial\lambda^2 = 0$. The preferences of the young are also single peaked along every dimension. However, the proof of this argument is not straightforward because the preferences of the young are kinked at the critical rate $\hat{\tau}(w^j)$. The formal proof is given in Appendix 8.1.

In what follows, we demonstrate preferences of the old and the young over policy.

3.1 Preferences of the Old Over Policy

The old choose τ to maximize $V^{o,j}$ in (4) given λ, and λ to maximize $V^{o,j}$ in (4) given τ. Their preferred rate of tax and the share of social security are respectively given by:

$$\tau^{oj} = \frac{1}{2} \text{ and } \lambda^{oj} = 1 \text{ for all } j.$$

Maximization is realized when the tax rate is set to attain the top of the Laffer curve, $(1-\tau)\tau$. The maximized tax revenue is used exclusively for social security because the old cannot benefit from the investment in public goods that take a one-period lag in formation.

3.2 Preferences of the Young Over the Tax Rate

Consider first the preferences of the young over τ. A type-j young individual chooses τ to maximize $V_{s>0}^{y,j}$ when he/she is borrowing unconstrained; he/she chooses τ to maximize $V_{s=0}^{y,j}$ when he/she is borrowing constrained. Thus, the tax rate τ chosen by the type-j

young individual satisfies the following first-order condition:

$$\beta(1+n)\lambda(1-2\tau)\bar{w}+\beta\eta\left\{\frac{1}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right\}^{-\sigma}\frac{1}{2+n}(1-\lambda)(1-2\tau)\bar{w} \tag{5}$$
$$=\begin{cases}\beta R w^j & \text{if} \quad \tau<\hat{\tau}(w^j)\\ (1-\tau)^{-\sigma}(w^j)^{1-\sigma} & \text{if} \quad \tau\geq\hat{\tau}(w^j)\end{cases}$$

The first term on the left-hand side is the marginal benefit of social security, the second term is the marginal benefit of public goods provision, and the right-hand side shows the marginal cost of taxation. The marginal cost is given by βRw^j when the type-j young individual is borrowing unconstrained; and it is given by $(1-\tau)^{-\sigma}(w^j)^{1-\sigma}$ when he/she is borrowing constrained. The type-j young individual chooses the tax rate to equate marginal benefits and costs of taxation from the viewpoint of utility maximization.

Preferences of the young over τ are affected by the wage income. In order to understand the effect, we first assume that a type-j young individual is borrowing unconstrained: $s^j>0$. Given a tax rate τ, a reduction of w^j decreases the after-tax income in youth, but has no effect on the consumption in youth because $c^{yj}=(\beta R)^{-1/\sigma}$ if $s^j>0$. The negative income effect is then absorbed by saving because of the assumption of a quasi-linear utility function. In other words, a reduction of w^j only has an effect on consumption in old age. A young individual then wishes to offset the loss of saving by increasing the social security benefits. Therefore, a reduction of the wage gives a young individual an incentive to choose a higher tax rate when he/she is borrowing unconstrained.

Next, suppose that a type-j young individual is borrowing constrained: $s^j=0$. The equilibrium tax rate satisfies $y(\tau;\bar{w},n)=(w^j)^{1-\sigma}/(1-\tau)^{\sigma}$, where the right-hand side is the marginal utility loss of taxation when the type-j young individual is borrowing constrained (see Eq. (5)). A marginal reduction of w^j affects the marginal utility loss of taxation such that $\partial(w^j)^{1-\sigma}/(1-\tau)^{\sigma}/\partial w^j=(1-\sigma)(w^j)^{-\sigma}/(1-\tau)^{\sigma}\gtreqless 0\Leftrightarrow 1/\sigma\gtreqless 1$. When $1/\sigma>1$, a reduction of w^j leads to a decrease in the marginal utility loss, thereby, giving the decisive voter an incentive to increase the tax rate. However, when $1/\sigma<1$ holds, a reduction of w^j leads to the opposite effect. Therefore, if $1/\sigma<1$, a reduction of the wage gives a young individual an incentive to choose a lower tax rate when he/she is borrowing constrained.

In order to understand intuitively the result for the case of a borrowing-constrained young individual, we consider the following two opposing effects of a reduction in wage on the preferred tax rate. First, given a tax rate, a decrease in the wage reduces the tax burden and thus raises a benefit-to-burden ratio. This gives a young individual an incentive to choose a higher tax rate: this is a positive effect on the preferred tax rate. Second, a decrease in the wage reduces the disposable income and thus the consumption level when young. This gives a young individual an incentive to choose a lower tax rate from the viewpoint of maintaining the utility of consumption: this is a negative effect on the preferred tax rate.

When a young individual is borrowing constrained, the positive effect does not necessarily overcome the negative one. The borrowing-constrained individual wants to consume more when young, but his/her demand is restricted by the borrowing constraint. Under this situation, the borrowing-constrained individual attaches a large weight to the utility loss of a reduction in his/her wage. This might lead to a situation where the negative effect overcomes the positive effect, which results in a lower preferred tax rate in response to a wage reduction.

Which effect outweighs the other depends on the degree of the interest-rate elasticity. A lower elasticity results in a smaller change of consumption in response to a change in the interest rate. In other words, a lower elasticity implies a stronger incentive for a young individual to smooth consumption across periods. Because of this incentive, the borrowing-constrained individual attaches a larger weight to the negative effect on young-age consumption as the interest-rate elasticity becomes lower. Therefore, the borrowing-constrained individual prefers a lower tax rate as his/her wage becomes lower when the interest-rate elasticity is low enough so that $1/\sigma < 1$.

3.3 Preferences of the Young Over the Share of Social Security

Next, consider the preferences of the young over λ. The first derivative of $V^{y,j}$ with respect to λ is independent of the status of saving: $\partial V^{y,j}_{s>0}/\partial\lambda = \partial V^{y,j}_{s=0}/\partial\lambda$. Direct calculation leads to:

$$\begin{aligned}\frac{\partial V^{y,j}_{s>0}}{\partial\lambda} = \frac{\partial V^{y,j}_{s=0}}{\partial\lambda}& \\ &= \beta(1+n)(1-\tau)\tau\bar{w} - \beta\eta\left\{\frac{1}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right\}^{-\sigma}\frac{1}{2+n}(1-\tau)\tau\bar{w},\end{aligned}$$

where the first term on the right-hand side shows the marginal increase in the benefit of social security given by an increase in the share of social security, and the second term is the marginal loss of utility of public goods given by a decrease in the share of public goods provision. The share of social security, λ, is chosen to balance the marginal benefit and loss in terms of utility.

A noteworthy feature of the current model is that the preferred share by the young is type-independent. This is because (i) all types of young individuals enjoy the same level of forward intergenerational public goods, (ii) the utility of forward intergenerational public goods is separable from the utility of private goods, and (iii) the utility of old-age consumption is specified by a linear utility function.

Another noteworthy feature is that the share λ could be zero. In order to understand the mechanism behind the result of "a corner solution", we first assume an interior solution of λ and set $\partial V^{y,j}_{s>0}/\partial\lambda = 0$. By rearranging the terms, we obtain:

$$(1+n)\bar{w} = \eta\left(\frac{\bar{w}}{2+n}\right)^{1-\sigma}\left(\frac{1}{(1-\lambda)(1-\tau)\tau}\right)^{\sigma},$$

where the left-hand side shows the marginal benefit of an increase in social security benefits, and the right-hand side shows the marginal utility loss of a decrease in public goods provision. The left-hand side of the above equation is independent of τ, while the right-hand side depends on τ.

The marginal loss of taxation, represented by the right-hand side, increases as the tax rate decreases. There then exists a critical rate of the tax, denoted by $\underline{\tau}\in(0,1/2)$, such that for $\tau\in[0,\underline{\tau}]$, the marginal loss is greater than the marginal benefit regardless of the choice of $\lambda\in[0,1]$. Choosing $\lambda=0$ is optimal for a type-j young individual from the viewpoint of utility maximization when τ is below $\underline{\tau}$. The corner solution is not peculiar to the model with a quasi-linear utility function. The qualitatively similar result also holds under a generalized utility function, which is demonstrated in Section 6.

Based on the above argument, the preferred share of the young becomes:

$$\lambda = \begin{cases} 0 & \text{if } \tau\in[0,\underline{\tau}] \\ 1-\frac{2+n}{(1-\tau)\tau\bar{w}}\left(\frac{\eta}{(2+n)(1+n)}\right)^{1/\sigma} & \text{if } \tau\in\left(\underline{\tau},\frac{1}{2}\right], \end{cases} \tag{6}$$

where:

$$\underline{\tau} \equiv \frac{1-\sqrt{1-\frac{4(2+n)}{\bar{w}}\left(\frac{\eta}{(2+n)(1+n)}\right)^{1/\sigma}}}{2}.$$

Figure 2 illustrates the graph of (6). The range of τ is limited to $(0,1/2)$ because the preferred tax rate by the old is equal to $1/2$ and that by the young is less than $1/2$. When the tax rate is below the critical rate $\underline{\tau}$, the tax revenue is too small, so that the marginal benefit of raising the share of social security is lower than the marginal cost of reducing the share of public goods provision for any $\lambda\in[0,1]$. Therefore, the young choose no expenditure to social security, $\lambda=0$, if $\tau\le\underline{\tau}$.

[Figure 2 about here.]

When the tax rate is above the critical rate $\underline{\tau}$, the tax revenue is sufficient, so that there is a share $\lambda\in(0,1)$ that balances the marginal benefits and costs. The optimal share for the young increases as the tax rate increases. The optimal share attains its highest value at $\tau=1/2$, where the tax revenue is maximized.

4 The Political Equilibrium

The previous section analyzed the voting behavior of each type of individual along the two dimensions of the issue space, τ and λ. Given that preferences are single peaked for each issue, we now apply Shepsle's (1979) result and characterize the structure-induced equilibrium of the game. To proceed with the analysis, we impose the following assumption.

Assumption 3.

$$\text{(i) } 1 > \frac{4(2+n)}{\bar{w}}\left(\frac{\eta}{(2+n)(1+n)}\right)^{1/\sigma};$$

$$\text{(ii) } w^L > \underline{w}^L \equiv \frac{1-\sqrt{1-\frac{4(2+n)}{\bar{w}}\left(\frac{\eta}{(2+n)(1+n)}\right)^{1/\sigma}}}{2(\beta R)^{1/\sigma}\frac{(2+n)}{\bar{w}}\left(\frac{\eta}{(2+n)(1+n)}\right)^{1/\sigma}}.$$

The first assumption implies that the preferred share in (6), which attains the highest value at $\tau = 1/2$, takes a positive value at $\tau = 1/2$. Therefore, the assumption ensures that a political equilibrium exists with $\lambda > 0$ for a range of $[\underline{\tau}, 1/2)$; otherwise, $\lambda = 0$ holds for any $\tau \in [0, 1/2)$, implying a trivial outcome of no provision of PAYG social security. The second assumption enables us to demonstrate cases of borrowing-unconstrained as well as borrowing-constrained type-L individuals in the presence of social security, $\lambda > 0$. Otherwise, the type-L young individual is always borrowing constrained when $\lambda > 0$.

The structure-induced equilibrium outcome is found as follows. First, we determine the decisive voter over λ and calculate his/her most preferred share, denoted by $\lambda^{dec}(\tau)$, as a function of the tax rate τ, where the superscript "*dec*" indicates the decisive voter. Second, we determine the decisive voter over τ and calculate his/her most preferred tax rate, denoted by $\tau^{dec}(\lambda)$, as a function of the share parameter λ. Finally, we find the point where these reaction functions $\lambda^{dec}(\tau)$ and $\tau^{dec}(\lambda)$ cross. This point corresponds to the structure-induced equilibrium outcome of the voting game.

Consider the political determination of λ. The decisive voter over λ is a young individual because (i) the population size of the young is larger than that of the old, and (ii) all young individuals have the same preferences for λ regardless of their type. Therefore, from (6), the decisive voter's reaction function $\lambda^{dec}(\tau)$ is given by:

$$\lambda^{dec}(\tau) = \begin{cases} 0 & \text{if } \tau \in [0, \underline{\tau}] \\ 1 - \frac{2+n}{(1-\tau)\tau\bar{w}}\left(\frac{\eta}{(2+n)(1+n)}\right)^{1/\sigma} & \text{if } \tau \in \left(\underline{\tau}, \frac{1}{2}\right]. \end{cases} \tag{7}$$

Next, consider the political determination of τ. The decisive voter over τ belongs to the young generation because (i) young individuals choose lower tax rates than the old, and (ii) the population size of the young is larger than that of the old. In particular, to determine the type of the decisive voter, we focus on the parameter σ representing the inverse of the interest-rate elasticity of young-age consumption and consider two cases separately: a high elasticity ($1/\sigma \geq 1$ in Subsection 4.1) and a low elasticity ($1/\sigma < 1$ in Subsection 4.2).

We adopt the above classification because the order of preferences for the tax rate critically depends on the degree of interest-rate elasticity. For the case of $1/\sigma \geq 1$, a lower-income young individual prefers a higher tax rate. However, for the case of $1/\sigma < 1$, a low-income young individual may prefer a lower tax rate than middle-income (or middle- and high-income) individuals. For each case, we show the existence and uniqueness of a structure-induced equilibrium of the voting game and explain the mechanism underlying the result.

4.1 The Case of a High Interest-rate Elasticity of Consumption ($1/\sigma \geq 1$)

To determine the type of the decisive voter over τ in the case of $1/\sigma \geq 1$, we consider the preferred tax rate of a type-j young individual given by (5). Figure 3 illustrates the condition (5) that determines the preferred tax rate by a type-j young ($j = L, M, H$) individual. The left-hand side of (5), denoted by LHS, is decreasing in τ and is independent of the type of young individual. In contrast, the right-hand side of (5), denoted by RHS^j, is nondecreasing in τ, and dependent on the type of young individual and featured by $RHS^H \geq RHS^M \geq RHS^L$, where an equality holds if and only if $\sigma = 1$. The kink point of $\tau = \hat{\tau}(w^j)$ implies that a type-j young individual can save part of his/her income if $\tau < \hat{\tau}(w^j)$ and nothing if $\tau \geq \hat{\tau}(w^j)$. It is immediately observed from Figure 3 that given λ, a lower-income young individual prefers a higher tax rate: $\tau^{yH} < \tau^{yM} < \tau^{yL}$ for all $\lambda \in [0, 1]$, where $\tau^{yj} (j = L, M, H)$ denotes the preferred tax rate of a type-j young individual.

[Figure 3 about here.]

Given the assumption of demographic structure (Assumption 1) and the fact that $\tau^{yH} < \tau^{yM} < \tau^{yL} < \tau^{oj}$, the decisive voter over τ is the one who prefers the highest tax rate among young individuals, that is, a type-L young individual. Therefore, the reaction function of $\tau, \tau^{dec}(\lambda)$, is implicitly given by (5) with $j = L$. To find the crossing point of the two reaction functions, $\lambda^{dec}(\tau)$ and $\tau^{dec}(\lambda)$, we substitute (7) into (5) with $j = L$ to obtain:

$$y(\tau;\bar{w},n) = z(\tau;w^L),$$

where:

$$y(\tau;\bar{w},n) = \begin{cases} \beta\eta\left\{\frac{1}{2+n}\bar{w}\right\}^{1-\sigma}\frac{1-2\tau}{((1-\tau)\tau)^{\sigma}} & \text{if } \tau \in [0,\underline{\tau}] \\ \beta(1+n)(1-2\tau)\bar{w} & \text{if } \tau \in (\underline{\tau},1/2] \end{cases}$$

$$z(\tau;w^L) = \begin{cases} \beta R w^L & \text{if } \tau < \hat{\tau}(w^L) \\ \frac{(w^L)^{1-\sigma}}{(1-\tau)^{\sigma}} & \text{if } \tau \geq \hat{\tau}(w^L). \end{cases}$$

Solving $y(\tau;\bar{w},n) = z(\tau;w^L)$ for τ leads to the tax rate in a structure-induced equilibrium of the voting game. The corresponding λ is obtained by substituting the equilibrium τ into the reaction function λ^{dec} in (7).

Proposition 1. *Suppose that $1/\sigma \geq 1$ holds. There exists a unique structure-induced equilibrium of the voting game such that the decisive voter over τ is a type-L young individual.*

Proof. See Appendix 8.2.

There are two possible cases of the equilibrium. The first is the case where the wage of type-L individuals is high such that they can save part of their income in youth for future consumption. In this case, the equilibrium tax rate represented by the crossing point of $y(\tau;\bar{w},n)$ and $z(\tau;w^L)$ is below the critical rate of $\hat{\tau}(w^L)$. The second is the case where the wage of type-L individuals is low such that they save nothing in their youth. The equilibrium tax rate is given above the critical rate of $\hat{\tau}(w^L)$.

4.2 The Case of a Low Interest-rate Elasticity of Consumption $(1/\sigma < 1)$

Next, consider the case of a low interest-rate elasticity such that $1/\sigma < 1$. The decisive voter over λ is equivalent to that in the previous case; the reaction function $\lambda^{dec}(\tau)$ is given by (7). However, the decisive voter over τ may differ from the previous case; the order of preferred tax rates may change depending on the value of λ.

To determine the decisive voter over τ, we recall the condition (5) that determines the tax rate preferred by a type-j young individual for a given λ. The graphs of (5) for the case of $1/\sigma < 1$ are illustrated in Figure 4. The main difference from the previous case is that RHS^i and $RHS^j (i \neq j)$ cross at some tax rate $\tau \in (0,1/2)$. This is because when a type-j individual is borrowing constrained, the slope of RHS^j becomes steeper as the elasticity $1/\sigma$ decreases. There are two critical values of τ, $\tilde{\tau}^{LM}$ and $\tilde{\tau}^{MH}$, such that RHS^L and RHS^H cross at $\tau = \tilde{\tau}^{LM}$ and RHS^M and RHS^H cross at $\tau = \tilde{\tau}^{MH}$. By direct calculation, we obtain:

$$\tilde{\tau}^{LM} \equiv 1 - \left(\frac{(w^L)^{1-\sigma}}{\beta R w^M} \right)^{1/\sigma} \text{ and } \tilde{\tau}^{MH} \equiv 1 - \left(\frac{(w^M)^{1-\sigma}}{\beta R w^H} \right)^{1/\sigma},$$

where $\hat{\tau}(w^L) < \tilde{\tau}^{LM} < \hat{\tau}(w^M) < \tilde{\tau}^{LM} < \hat{\tau}(w^H)$ (see Figure 4). The derivation of $\tilde{\tau}^{LM}$ and $\tilde{\tau}^{MH}$ is given in Appendix 8.3.

[Figure 4 about here.]

The tax rate preferred by a type-j young individual is determined by the crossing point of LHS and RHS of (5). RHS is independent of λ while LHS is strictly increasing in λ. The tax rate preferred by a type-j young individual depends on the size of λ. Overall, he/she prefers a higher tax rate when λ is higher.

The order of tax rates preferred by the three types of agents is changed by the size of λ, as illustrated in Figure 4. First, when λ is low such that LHS of (5) crosses RHS of (5) with $j = L$ within the range $(0, \tilde{\tau}^{LM}]$, the tax rates preferred by the young are ordered by $\tau^{yH} < \tau^{yM} < \tau^{yL}$, where $\tau^{yj} (j = L, M, H)$ denotes the preferred tax rate by type-j young: the type-L young individual becomes the decisive voter. Second, when λ attains a middle value such that LHS of (5) crosses RHS of (5) with $j = M$ within the range $(\tilde{\tau}^{LM}, \tilde{\tau}^{MH}]$, the tax rates preferred by the young are ordered by $\tau^{yH} < \tau^{yL} < \tau^{yM}$ or $\tau^{yL} \leq \tau^{yH} < \tau^{yM}$: the decisive voter in this case is the type-M young individual. Finally, when λ is high such that LHS of (5) crosses RHS of (5) with $j = H$ within the range $[\tilde{\tau}^{MH}, 1/2]$, the tax rates preferred by the young are ordered by $\tau^{yL} < \tau^{yM} < \tau^{yH}$: the decisive voter becomes the type-H young individual.

Given the abovementioned feature, the reaction function of τ, $\tau = \tau^{dec}(\lambda)$, is now implicitly given by:

$$\beta(1+n)\lambda(1-2\tau)\bar{w} + (1+\beta)\eta \left\{ \frac{1}{2+n}(1-\lambda)\bar{w} \right\}^{1-\sigma} \frac{1-2\tau}{((1-\tau)\tau)^{\sigma}} = \tilde{z}(\tau; w^L, w^M, w^H), \quad (8)$$

where:

$$\tilde{z}(\tau; w^L, w^M, w^H) \equiv \begin{cases} \beta R w^L & \text{if } \tau < \hat{\tau}(w^L) \\ \frac{(w^L)^{1-\sigma}}{(1-\tau)^{\sigma}} & \text{if } \hat{\tau}(w^L) \leq \tau \leq \tilde{\tau}^{LM} \\ \beta R w^M & \text{if } \tilde{\tau}^{LM} < \tau < \hat{\tau}(w^M) \\ \frac{(w^M)^{1-\sigma}}{(1-\tau)^{\sigma}} & \text{if } \hat{\tau}(w^M) \leq \tau \leq \tilde{\tau}^{MH} \\ \beta R w^H & \text{if } \tilde{\tau}^{MH} < \tau < \hat{\tau}(w^H) \\ \frac{(w^H)^{1-\sigma}}{(1-\tau)^{\sigma}} & \text{if } \hat{\tau}(w^H) \leq \tau. \end{cases}$$

The graph of the function $\tilde{z}$ is illustrated by the bold curve in Figure 4.

We substitute the reaction function of $\lambda^{dec}(\tau)$, given by (7), into the left-hand side of (8) to obtain the condition that determines the equilibrium tax rate:

$$y(\tau; \bar{w}, n) = \tilde{z}(\tau; w^L, w^M, w^H),$$

where $y(\cdot)$ has been already defined in the previous subsection. Figure 5 illustrates the graphs of $y(\tau; \bar{w}, n)$ and $\tilde{z}(\tau; w^L, w^M, w^H)$. Solving $y(\tau; \bar{w}, n) = \tilde{z}(\tau; w^L, w^M, w^H)$ for τ leads to the tax rate in a structure-induced equilibrium of the voting game. The corresponding λ is obtained by substituting the equilibrium τ into the reaction function λ^{dec} in (7).

[Figure 5 about here.]

Proposition 2. *Suppose that $1/\sigma < 1$ holds. There exists a unique structure-induced equilibrium of the voting game such that the decisive voter over τ is:*

(i) *a type-L individual if* $-1 + 2\left\{(w^L)^{1-\sigma}/\beta R w^M\right\}^{1/\sigma} \le R w^M/(1+n)\bar{w}$;

(ii) *a type-M individual otherwise.*

Proof. See Appendix 8.4.

A noteworthy feature of Proposition 2 is that under certain conditions, the middle-income individuals prefer a higher tax rate than the low-income individuals. In particular, if the condition in statement (ii) of Proposition 2 holds, there exists an equilibrium, like an ends-against-the-middle equilibrium, where the low- and high-income young individuals form a coalition in favor of a low tax rate and the middle-income individual favoring a high tax rate becomes the decisive voter (see Figure 5).

The key factors in Proposition 2 are the borrowing constraints and the interest-rate elasticity of consumption. To understand the roles of these two factors, consider the case where the low-income individuals are faced with a borrowing constraint. Here, they wish to consume more in their youth, but cannot because of the borrowing constraint. In this situation, a higher tax rate produces two opposing effects: a negative effect that results in lower after-tax income and thus the utility loss of taxation in youth, and a positive effect that produces higher social security benefit and thus the utility gain in old age.

The net impact of taxation depends on the interest-rate elasticity. When the elasticity is high such that $1/\sigma > 1$, the positive effect overcomes the negative effect. The low-income individual then chooses the highest tax rate among the young and thus becomes the decisive voter. In contrast, when the elasticity is low such that $1/\sigma < 1$, the negative effect may dominate the positive effect for low-income individuals. They choose a lower tax rate than the middle-income individuals, and this results in an equilibrium where the middle-income individual becomes the decisive voter.

Figure 6 illustrates the conditions that determine the decisive voter and his/her status of saving in a $w^L - w^M$ space. From the figure, we find that the decisive voter is a type-L individual provided the wage income levels of the two types of individuals are high such that the pair (w^L, w^M) is set within the area marked by $(j = L)$ in Figure 6. The order of preferred tax rates is the reverse of the wage rates. However, the order is changed when the wage income level of the type-L individual is sufficiently low such that (w^L, w^M) is set within the area marked by $(j = M)$ in Figure 6. The decisive voter becomes the type-M young individual. The equilibrium is featured by the situation that resembles the ends-against-the-middle equilibrium.

[Figure 6 about here.]

5 Effects of Inequality on Policy

Given the characterization of the political equilibrium in Section 4, we now investigate how the tax rate (τ) and the share of social security (λ) change in response to a change in inequality. In particular, we consider a mean-preserving reduction of the decisive voters' wage in order to compare two groups of countries with similar per capita income levels but different levels of income inequality. We focus on the nontrivial equilibrium with $\lambda > 0$ to observe the marginal effect on the share of social security in government expenditure.

Proposition 3. *Consider a political equilibrium with* $\lambda > 0$.

(i) *In an economy with* $1/\sigma \geq 1$ *where the decisive voter is a type-L young individual, the tax rate and the share of social security are nondecreasing in response to a mean-preserving reduction of* w^L.

(ii) *In an economy with* $1/\sigma < 1$ *where the decisive voter is a type-j* ($j = L$ or M) *young individual, a mean-preserving change in the decisive voter's wage* (w^j) *locally produces inverse U-shaped relationships between* w^j *and the tax rate* (τ) *and between* w^j *and the share of social security* (λ).

Proof. See Appendix 8.5.

Figure 7 illustrates the effects of a mean-preserving change in the decisive voter's wage on the equilibrium tax rate. Panel (a) is for the case of $1/\sigma \geq 1$; Panel (b) is for the case of $1/\sigma < 1$. Proposition 3 states that if the interest-rate elasticity is high such that $1/\sigma \geq 1$, there is, in general, a monotone relationship between the decisive voter's wage and his/her preferred tax rate: the decisive voter prefers a higher tax rate as he/she becomes poorer. However, when the elasticity is low such that $1/\sigma < 1$, such a monotone relationship no longer holds. Once the decisive voter's wage falls below the threshold level that changes his/her status from unconstrained to constrained, he/she prefers a lower tax rate as he/she becomes poorer, as demonstrated in Subsection 3.2. Thus, there is an inverse U-shaped relationship between the decisive voter's wage and the preferred tax rate around the threshold level of wage, as illustrated in panel (c) of Figure 7. Given a positive correlation between the tax and the share of social security, there is also an inverse U-shaped relationship between the decisive voter's wage and the share of social security in government expenditure.

[Figure 7 about here.]

We should note that, in the current framework, the negative correlation arises only in the equilibrium where the following two conditions hold: (i) the interest-rate elasticity of consumption is low, and (ii) the decisive voter is borrowing constrained. When one of the conditions fails to hold, the economy displays a positive correlation between inequality and the share of social security. Therefore, our analysis suggests that these factors are the key to demonstrate the abovementioned inverse U-shaped relationships. These relationships also hold even if the assumption of a quasi-linear utility function is dropped, as we demonstrate in the next section.

6 A Generalized Utility Function

To this point, we have conducted the analysis by assuming a quasi-linear utility function where the utility of old-age consumption is given by βc_{t+1}^{oj}. This specification enables us to illustrate the existence and uniqueness of the political equilibrium. However, the specification also results in (i) a saving decision unaffected by social security, and (ii) type-independent preferences over the share of social security. We introduce a nonlinear utility function of old-age consumption to resolve these problems.

The main result of this section is that most of the previous results still hold true under the alternative utility function. That is, under a certain condition, there exists an equilibrium, like an ends-against-the-middle equilibrium, when the interest-rate elasticity is low and the decisive voter is borrowing constrained. In this equilibrium, a mean-preserving spread of income inequality results in a lower equilibrium tax rate and a lower share of social security in government expenditure.

For the purpose of analysis, we assume the following utility function:

$$U_t^j = \frac{(c_t^{yj})^{1-\sigma} - 1}{1-\sigma} + \eta \frac{(g_t)^{1-\sigma} - 1}{1-\sigma} + \beta \left[\frac{(c_{t+1}^{oj})^{1-\sigma} - 1}{1-\sigma} + \eta \frac{(g_{t+1})^{1-\sigma} - 1}{1-\sigma} \right].$$

The main difference from the previous model is that the utility of old-age consumption is given by $\beta\{(c_{t+1}^{oj})^{1-\sigma} - 1\}/(1-\sigma)$ rather than βc_{t+1}^{oj}. The maximization of their lifetime utility under the budget constraints leads to the following saving function:

$$s_t^j = \max \left\{ 0, \frac{(\beta R)^{1/\sigma}}{(\beta R)^{1/\sigma} + R} \left[(1-\tau_t) w^j - \frac{b_{t+1}}{(\beta R)^{1/\sigma}} \right] \right\}.$$

Saving now depends on the social security benefit b_{t+1} that gives individuals a disincentive to save. Hereafter, we drop the time subscript because our focus is on the time-invariant policy.

We substitute the government budget constraint for social security $b = (1+n)\lambda\tau(1-\tau)\bar{w}$ into the above saving function to obtain the following condition that determines the saving behavior of a type-j individual:

$$s^j > 0 \Leftrightarrow \frac{w^j}{\bar{w}} \cdot \frac{(\beta R)^{1/\sigma}}{1+n} > \lambda\tau.$$

This inequality condition states that a type-j individual is borrowing unconstrained if his/her wage is high, the tax burden is low and/or the share of social security in government expenditure is also low.

With the saving function and the government budget constraints, we give the consumption functions of a type-j individual in youth and old age as:

$$c_t^{yj} = \begin{cases} \frac{R}{(\beta R)^{1/\sigma}+R} \left[(1-\tau) w^j + \frac{(1+n)\lambda(1-\tau)\tau\bar{w}}{R} \right] & \text{if } \frac{w^j}{\bar{w}} \cdot \frac{(\beta R)^{1/\sigma}}{1+n} > \lambda\tau \\ (1-\tau) w^j & \text{if } \frac{w^j}{\bar{w}} \cdot \frac{(\beta R)^{1/\sigma}}{1+n} \leq \lambda\tau \end{cases}$$

$$c_{t+1}^{oj} = \begin{cases} \frac{(\beta R)^{1/\sigma} R}{(\beta R)^{1/\sigma}+R} \left[(1-\tau) w^j + \frac{(1+n)\lambda(1-\tau)\tau\bar{w}}{R} \right] & \text{if } \frac{w^j}{\bar{w}} \cdot \frac{(\beta R)^{1/\sigma}}{1+n} > \lambda\tau \\ (1+n)\lambda(1-\tau)\tau\bar{w} & \text{if } \frac{w^j}{\bar{w}} \cdot \frac{(\beta R)^{1/\sigma}}{1+n} \leq \lambda\tau. \end{cases}$$

Unlike the previous case, the consumption in youth is now type-dependent and is linearly related to lifetime income when individuals are borrowing unconstrained.

After some calculation, we obtain the indirect utility functions of type-j young and old individuals as:

$$V^{yj} = \begin{cases} V^{y,j}_{s>0} & \text{if } \frac{w^j}{\bar{w}} \cdot \frac{(\beta R)^{1/\sigma}}{1+n} > \lambda\tau \\ V^{y,j}_{s=0} & \text{if } \frac{w^j}{\bar{w}} \cdot \frac{(\beta R)^{1/\sigma}}{1+n} \le \lambda\tau \end{cases}$$

$$V^{oj} = \begin{cases} V^{o,j}_{s>0} & \text{if } \frac{w^j}{\bar{w}} \cdot \frac{(\beta R)^{1/\sigma}}{1+n} > \lambda\tau \\ V^{o,j}_{s=0} & \text{if } \frac{w^j}{\bar{w}} \cdot \frac{(\beta R)^{1/\sigma}}{1+n} \le \lambda\tau, \end{cases}$$

where:

$$\begin{aligned} V^{y,j}_{s>0} &\equiv \frac{1}{1-\sigma}\left(\frac{R}{(\beta R)^{1/\sigma}+R}\right)^{-\sigma}\left[(1-\tau)w^j + \frac{(1+n)\lambda(1-\tau)\tau\bar{w}}{R}\right]^{1-\sigma} + \frac{\eta}{1-\sigma}(g)^{-\sigma} \\ &\quad + \frac{\beta\eta}{1-\sigma}\left[\frac{1}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right]^{1-\sigma} - \frac{(1+\beta)(1+\eta)}{1-\sigma}; \\ V^{y,j}_{s=0} &\equiv \frac{1}{1-\sigma}\left((1-\tau)w^j\right)^{1-\sigma} + \frac{\beta}{1-\sigma}\left[(1+n)\lambda(1-\tau)\tau\bar{w}\right]^{1-\sigma} + \frac{\eta}{1-\sigma}(g)^{-\sigma} \\ &\quad + \frac{\beta\eta}{1-\sigma}\left[\frac{1}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right]^{1-\sigma} - \frac{(1+\beta)(1+\eta)}{1-\sigma}; \\ V^{o,j}_{s>0} &\equiv \frac{1}{1-\sigma}\left[Rs^j_{-1} + (1+n)\lambda(1-\tau)\tau\bar{w}\right]^{1-\sigma} + \frac{\eta}{1-\sigma}(g)^{-\sigma} - \frac{(1+\eta)}{1-\sigma}; \\ V^{o,j}_{s=0} &\equiv \frac{\beta\eta}{1-\sigma}\left[\frac{1+n}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right]^{1-\sigma} + \frac{\eta}{1-\sigma}(g)^{-\sigma} - \frac{(1+\eta)}{1-\sigma}. \end{aligned}$$

The term $\eta(g)^{-\sigma}/(1-\sigma)$ in the functions of $V^{o,j}_{s>0}$ and $V^{o,j}_{s=0}$ is predetermined. We can show that these preferences satisfy single-peaked properties in the same manner as in the case of a quasi-linear utility function.

The policy preferences of the old are the same as for the quasi-linear utility. That is, regardless of type and saving behavior, the old wish to maximize the tax revenue from the young and use it exclusively for social security: $\tau^{oj} = 1/2$ and $\lambda^{oj} = 1$ hold for all j. Accordingly, generalization of the utility function does not affect the policy preferences of the old.

We next consider the policy preferences of the young. Given λ, the preferred tax rate of a type-j young individual satisfies the following first-order condition with respect to τ:

$$LHS^{y} = RHS^{yj} \equiv \begin{cases} RHS^{yj}_{s>0} & \text{if } \tau < \tau^*(w^j,\lambda) \equiv \frac{w^j}{\bar{w}} \cdot \frac{(\beta R)^{1/\sigma}}{1+n} \cdot \frac{1}{\lambda} \\ RHS^{yj}_{s=0} & \text{if } \tau \ge \tau^*(w^j,\lambda) \end{cases}, \quad (9)$$

where:

$$LHS^y \equiv \beta\eta \left[\frac{1}{2+n}(1-\lambda)\tau\bar{w}\right]^{-\sigma} \frac{1}{2+n}(1-\lambda)(1-2\tau)\bar{w},$$
$$RHS^{yj}_{s>0} \equiv \left(\frac{R}{(\beta R)^{1/\sigma}+R}\right)^{-\sigma} \left[w^j + \frac{(1+n)\lambda\tau\bar{w}}{R}\right]^{-\sigma} \left[w^j - \frac{(1+n)\lambda(1-2\tau)\bar{w}}{R}\right], \text{and}$$
$$RHS^{yj}_{s=0} \equiv \left(w^j\right)^{1-\sigma} - \beta\left[(1+n)\lambda\tau\bar{w}\right]^{-\sigma}(1+n)\lambda(1-2\tau)\bar{w}.$$

LHS^y represents the marginal benefit of taxation in terms of the utility of public goods. This benefit is common to the three types of young agents because of the nature of public goods. RHS^{yj} represents the marginal loss of taxation plus the marginal benefit of social security in terms of the utility of consumption. The sum of these losses and benefits differs across individuals. In particular, the following properties hold (see Appendix 8.6 for the proof):

$$\begin{cases} RHS^{yL} \leq RHS^{yM} \leq RHS^{yH} & \text{if } 1/\sigma \geq 1 \\ RHS^{yL}_{s=0} > RHS^{yM}_{s=0} > RHS^{yH}_{s=0} & \text{if } 1/\sigma < 1, \end{cases} \tag{10}$$

where an equality in the first line holds if and only if $1/\sigma = 1$ and $s = 0$. Similar to the previous model, the order of $RHS^{yj}_{s=0}(j = L, M, H)$ critically depends on the degree of the interest-rate elasticity $1/\sigma$.[2]

Panel (a) of Figure 8 illustrates the graph of (9) when $1/\sigma \geq 1$ holds. The crossing point of LHS^y and RHS^{yj} determines the tax rate preferred by a type-j young individual. The figure shows that a lower-income young individual prefers a higher tax rate. Under the demographic structure assumption given in Assumption 1, a type-L young individual becomes the decisive voter over τ. That is, the ends-against-the-middle equilibrium never arises when the interest-rate elasticity is high such that $1/\sigma \geq 1$.

[Figure 8 about here.]

Panel (b) of Figure 8 illustrates the graph of (9) when $1/\sigma < 1$ holds. A noteworthy feature is that lower-income young individuals prefer a lower tax rate when they are borrowing constrained. In particular, there may arise an equilibrium where the low- and the high-income young individuals form a coalition against the middle, as illustrated in Panel (b) of Figure 8. Therefore, the low interest-rate elasticity and the borrowing constraint remain the keys to the existence of the ends-against-the-middle equilibrium.

The determination of the share of social security λ is slightly different from that in

[2]If $1/\sigma < 1$, the order of $RHS^{yj}_{s>0}(j = L, M, H)$ is ambiguous.

the previous quasi-linear utility case. The preferred share of a type-j young is given by:

$$\lambda^{yj} = \begin{cases} 0 & \text{if} \quad \tau \le \tilde{\tau}(w^j) \\ \dfrac{\frac{1}{2+n} - \left(\frac{\beta\eta R}{(2+n)(1+n)}\right)^{1/\sigma} \cdot \frac{R}{(\beta R)^{1/\sigma}+R} \cdot \frac{w^j}{\tau \bar{w}}}{\frac{1}{2+n} + \left(\frac{\beta\eta R}{(2+n)(1+n)}\right)^{1/\sigma} \frac{1+n}{(\beta R)^{1/\sigma}+R}} & \text{if } \tilde{\tau}(w^j) < \tau < \tau^*(w^j) \\ \dfrac{\frac{1}{2+n}}{\frac{1}{2+n} + \left(\frac{\eta}{(2+n)(1+n)}\right)^{1/\sigma}(1+n)} & \text{if} \quad \tau^*(w^j) \le \tau, \end{cases} \tag{11}$$

where:

$$\tilde{\tau}(w^j) \equiv \left(\frac{\beta\eta R}{(2+n)(1+n)}\right)^{1/\sigma} \cdot \frac{R}{(\beta R)^{1/\sigma} + R} \cdot \frac{(2+n)w^j}{\bar{w}};$$

$$\tau^*(w^j) \equiv (\beta R)^{1/\sigma} \cdot \left\{ \frac{1}{2+n} + \left(\frac{\eta}{(2+n)(1+n)}\right)^{1/\sigma} (1+n) \right\} \frac{(2+n)w^j}{(1+n)\bar{w}}.$$

The first two lines of the right-hand side in (11) represent the choice of λ when a type-j young is borrowing unconstrained; the third line represents the choice of λ when he/she is borrowing constrained. The derivation of (11) is given in Appendix 8.5.

When the tax burden is low such that $\tau \le \tilde{\tau}(w^j)$, a type-$j$ young individual can save much for his/her old-age consumption and thus finds it unnecessary to use tax revenue for social security: $\lambda = 0$. However, when the tax is above $\tilde{\tau}(w^j)$, a type-j young individual finds it optimal to offset part of their tax-induced consumption loss with a social security benefit. In particular, a lower income agent prefers a higher share of social security.

A type-j young individual is borrowing constrained when the tax rate is high such that $\tau \ge \tau^*(w^j)$. Borrowing-constrained individuals choose the same share of social security regardless of their type. This is because they have the same level of old-age consumption that is equal to the lump-sum pension benefit. They then choose that share to equate the marginal utilities of old-age consumption and public goods, both of which are type-independent. This result is different from that under a quasi-linear utility function.

Panel (c) of Figure 9 illustrates the reaction function of λ for each type of individual. The figure shows that $\lambda^{yH}(\tau) \le \lambda^{yM}(\tau) \le \lambda^{yL}(\tau) < \lambda^o$ holds for any τ. Thus, under the demographic structure in Assumption 1, a type-L individual agent becomes the decisive voter. We can derive the political equilibrium tax rate by substituting $\lambda = \lambda^{yL}$ into the decisive voter's first-order condition with respect to τ.

Given a brief characterization of the political equilibrium, we now compare the income inequality effects between the current and former models. In particular, we focus on the situation where the decisive voters over τ and λ are borrowing constrained. Thus, the decisive voter's choice of λ, in the current framework, is given by:

$$\lambda^{dec} = \frac{\frac{1}{2+n}}{\frac{1}{2+n} + \left(\frac{\eta}{(2+n)(1+n)}\right)^{1/\sigma}(1+n)},$$

which is independent of τ. We substitute this into the first-order condition with respect to τ, (9), for the case of $\tau \geq \tau^{*}(w^{j}, \lambda)$ and obtain the following condition that determines the equilibrium tax rate when the decisive voter is borrowing constrained:

$$\begin{aligned}\left(w^{j}\right)^{1-\sigma} &= \beta\eta\left(\frac{\left(1-\lambda^{dec}\right)\bar{w}}{2+n}\right)^{1-\sigma}(\tau)^{-\sigma}(1-2\tau)\\ &+\beta\left((1+n)\left(1-\lambda^{dec}\right)\bar{w}\right)^{1-\sigma}\left(\lambda^{dec}\right)^{1-\sigma}(\tau)^{-\sigma}(1-2\tau),\end{aligned}$$

where the left-hand side shows the marginal cost of taxation, the first term on the right-hand side shows the marginal benefit of public goods, and the second term on the right-hand side shows the marginal benefit of social security. Given τ, the right-hand side is independent of w^{j} whereas the left-hand side is decreasing (increasing) in w^{j} if $1/\sigma < (>)1$. Thus, a mean-preserving reduction of the decisive voter's wage decreases (increases) the equilibrium tax rate if the interest-rate elasticity is low (high) such that $1/\sigma < (>)1$. This result is qualitatively equivalent to that in the quasi-linear utility function model.

7 Conclusion

How does wage inequality affect the allocation of tax revenue between social security and forward intergenerational public goods provision in the presence of borrowing constraints? This paper develops a political economy model that addresses this question. Two features are crucial to our analysis and results: the interest-rate elasticity of consumption and the borrowing constraint. These features derive an ends-against-the-middle equilibrium where low- and high-income individuals form a coalition in favor of a low tax rate and middle-income individuals favor a high tax rate. In addition, higher wage inequality results in a lower level of social security and a lower share of social security in government expenditure when the decisive voter is borrowing constrained and the interest-rate elasticity is low.

To obtain these results, we simplify the analysis by adopting a quasi-linear utility function. Because of this simplification, we can remove the link between saving and the allocation of tax revenue between social security and public goods provision. However, as shown in our analysis, the effect of the interest-rate elasticity of consumption appears in the political equilibrium in spite of this simplification. In addition, Section 6 demonstrates that the main results are qualitatively unchanged under a generalized utility function.

Thus, our analysis is almost robust to the assumption of a quasi-linear utility function.

8 Appendix

8.1 Single-peakedness of Preferences

8.1.1 Single-peakedness of preferences over τ

The proof proceeds as follows. First, we show that both $V^{y,j}_{s>0}$ and $V^{y,j}_{s=0}$ are single peaked over τ. Then, we demonstrate that $\partial V^{y,j}_{s>0}/\partial\tau = \partial V^{y,j}_{s=0}/\partial\tau$ and $V^{y,j}_{s>0} = V^{y,j}_{s=0}$ hold at $\tau = \hat{\tau}(w^j)$, implying that $V^{y,j}$ has a unique local maximum over the whole range of τ and thus that $V^{y,j}$ is single peaked over τ.

The first and the second derivatives of $V^{y,j}_{s>0}$ and $V^{y,j}_{s=0}$ with respect to τ are:

$$\begin{aligned}
\frac{\partial V^{y,j}_{s>0}}{\partial\tau} &= -\beta R w^j + \beta(1+n)\lambda(1-2\tau)\bar{w} \\
&\quad + \beta\eta\left(\frac{(1-\lambda)(1-\tau)\tau\bar{w}}{2+n}\right)^{-\sigma}\cdot\frac{(1-\lambda)(1-2\tau)\bar{w}}{2+n}; \\
\frac{\partial^2 V^{y,j}_{s>0}}{\partial\tau^2} &= (-2)\beta(1+n)\lambda\bar{w} + (-2)\beta\eta\left(\frac{(1-\lambda)(1-\tau)\tau\bar{w}}{2+n}\right)^{-\sigma}\frac{(1-\lambda)\bar{w}}{2+n} \\
&\quad + \beta\eta(-\sigma)\left(\frac{(1-\lambda)(1-\tau)\tau\bar{w}}{2+n}\right)^{-\sigma-1}\left\{\frac{(1-\lambda)(1-2\tau)\bar{w}}{2+n}\right\}^2 \\
&< 0; \\
\frac{\partial V^{y,j}_{s=0}}{\partial\tau} &= (-1)(1-\tau)^{-\sigma}(w^j)^{1-\sigma} + \beta(1+n)\lambda(1-2\tau)\bar{w} \\
&\quad + \beta\eta\left(\frac{(1-\lambda)(1-\tau)\tau\bar{w}}{2+n}\right)^{-\sigma}\cdot\frac{(1-\lambda)(1-2\tau)\bar{w}}{2+n}; \\
\frac{\partial^2 V^{y,j}_{s=0}}{\partial\tau^2} &= (-\sigma)(w^j)^{1-\sigma}(1-\tau)^{-\sigma-1} + (-2)\beta(1+n)\lambda\bar{w} \\
&\quad + (-2)\beta\eta\left(\frac{(1-\lambda)(1-\tau)\tau\bar{w}}{2+n}\right)^{-\sigma}\frac{(1-\lambda)\bar{w}}{2+n} \\
&\quad + \beta\eta(-\sigma)\left(\frac{(1-\lambda)(1-\tau)\tau\bar{w}}{2+n}\right)^{-\sigma-1}\left\{\frac{(1-\lambda)(1-2\tau)\bar{w}}{2+n}\right\}^2 \\
&< 0.
\end{aligned}$$

$V^{y,j}_{s>0}$ and $V^{y,j}_{s=0}$ are single peaked over τ because the second derivatives are negative.

Next, we show that $\partial V^{y,j}_{s>0}/\partial\tau = \partial V^{y,j}_{s=0}/\partial\tau$ at $\tau = \hat{\tau}(w^j)$. By direct calculation, we have:

$$\left.\frac{\partial V^{y,j}_{s>0}}{\partial\tau}\right|_{\tau=\hat{\tau}(w^j)} \gtreqless \left.\frac{\partial V^{y,j}_{s=0}}{\partial\tau}\right|_{\tau=\hat{\tau}(w^j)} \Leftrightarrow -\beta R w^j \gtreqless (-1)(w^j)^{1-\sigma}(1-\tau)^{-\sigma}.$$

At $\tau = \hat{\tau}(w^j) \equiv 1 - 1/(\beta R)^{1/\sigma} w^j$, the right-hand side of the above condition is rewritten as:

$$(-1)(w^j)^{1-\sigma}(1-\hat{\tau}(w^j))^{-\sigma} = -\beta R w^j,$$

implying that $\partial V_{s>0}^{y,j}/\partial\tau = \partial V_{s=0}^{y,j}/\partial\tau$ at $\tau = \hat{\tau}(w^j)$.

Finally, we show that $V_{s>0}^{y,j} = V_{s=0}^{y,j}$ hold at $\tau = \hat{\tau}(w^j)$. By direct calculation, we have:

$$V_{s>0}^{y,j}\Big|_{\tau=\hat{\tau}(w^j)} \gtreqless V_{s=0}^{y,j}\Big|_{\tau=\hat{\tau}(w^j)}$$
$$\iff \frac{\left((\beta R)^{-1/\sigma}\right)^{1-\sigma}-1}{1-\sigma} + \beta R\left\{(1-\tau)w^j - (\beta R)^{-1/\sigma}\right\} = \frac{((1-\tau)w^j)^{1-\sigma}-1}{1-\sigma}.$$

At $\tau = \hat{\tau}(w^j) \equiv 1 - 1/(\beta R)^{1/\sigma} w^j$, the left-hand and right-hand sides of the above condition are reduced to, respectively:

$$\text{LHS} = \text{RHS} = \frac{\left((\beta R)^{-1/\sigma}\right)^{1-\sigma}-1}{1-\sigma},$$

implying that $V_{s>0}^{y,j} = V_{s=0}^{y,j}$ hold at $\tau = \hat{\tau}(w^j)$.

8.1.2 Single-peakedness of preferences over λ

Before proceeding to the proof, we note that the status of saving is independent of λ because of the assumption of a quasi-linear utility function. Thus, it is sufficient to show that $\partial^2 V_{s>0}^{y,j}/\partial\lambda^2 < 0$ and $\partial^2 V_{s=0}^{y,j}/\partial\lambda^2 < 0$ for the proof.

The first and the second derivatives of $V_{s>0}^{y,j}$ and $V_{s=0}^{y,j}$ with respect to λ are:

$$\frac{\partial V_{s>0}^{y,j}}{\partial\lambda} = \frac{\partial V_{s=0}^{y,j}}{\partial\lambda} = \beta(1+n)(1-\tau)\tau\bar{w} + (-1)\beta\eta\left(\frac{(1-\lambda)(1-\tau)\tau\bar{w}}{2+n}\right)^{-\sigma}\cdot\frac{(1-\tau)\tau\bar{w}}{2+n};$$
$$\frac{\partial^2 V_{s>0}^{y,j}}{\partial\lambda^2} = \frac{\partial^2 V_{s=0}^{y,j}}{\partial\lambda^2} = (-\sigma)\left(\frac{(1-\lambda)(1-\tau)\tau\bar{w}}{2+n}\right)^{-\sigma-1}\cdot\left(\frac{(1-\tau)\tau\bar{w}}{2+n}\right)^2 < 0.$$

8.2 Proof of Proposition 1

As shown in the text, when $1/\sigma \geq 1$, the decisive voter is a type-L individual and his/her preferred tax rate satisfies $y(\tau;\bar{w},n) = z(\tau;w^L)$. The functions $y(\tau;\bar{w},n)$ and $z(\tau;w^L)$ have the following properties: $\partial y(\tau;\bar{w},n)/\partial\tau < 0, \lim_{\tau\to 0} y(\tau;\bar{w},n) = \infty, y(1/2;\bar{w},n) = 0, \partial z(\tau;w^L)/\partial\tau \geq 0, z(0;w^L) = \max\{\beta R w^L, (w^L)^{1-\sigma}\} < \infty$, and $z(1/2;w^L) \in (0,\infty)$. These properties indicate that there exists a unique $\tau \in (0,1/2)$ that satisfies $y(\tau;\bar{w},n) = z(\tau;w^L)$. ■

8.3 The Derivation of $\tilde{\tau}^{LM}$ and $\tilde{\tau}^{MH}$

The derivation of $\tilde{\tau}^{LM}$ is as follows. For the range of $(\hat{\tau}(w^L), \hat{\tau}(w^M))$, the right-hand side of (5), denoted by RHS^j, is given by:

$$RHS^j = \begin{cases} RHS^L = (w^L)^{1-\sigma}/(1-\tau)^\sigma & \text{for } j = L \\ RHS^M = \beta R w^M & \text{for } j = M. \end{cases}$$

$RHS^L < RHS^M$ holds at $\tau = \hat{\tau}(w^L)$; $RHS^L > RHS^M$ holds at $\tau = \hat{\tau}(w^M)$. Thus, there exists a unique τ, denoted by $\tilde{\tau}^{LM} \in (\hat{\tau}(w^L), \hat{\tau}(w^M))$, that satisfies $RHS^L = RHS^M$ because RHS^L is continuous and strictly increasing in τ whereas RHS^M is independent of τ. We can derive $\tilde{\tau}^{LM}$ by solving $(w^L)^{1-\sigma}/(1-\tau)^\sigma = \beta R w^M$ for τ.

Similarly, the tax rate that satisfies $RHS^M = RHS^H$ for the range of $(\hat{\tau}(w^M), \hat{\tau}(w^H))$ is derived by solving $(w^M)^{1-\sigma}/(1-\tau)^\sigma = \beta R w^H$ for τ. The solution is denoted by $\tilde{\tau}^{MH}$.

8.4 Proof of Proposition 2

8.4.1 Existence and uniqueness of the equilibrium

As shown in the text, when $1/\sigma < 1$, the decisive voter's preferred tax rate satisfies $y(\tau; \bar{w}, n) = \tilde{z}(\tau; w^L, w^M, w^H)$. The functions $y(\tau; \bar{w}, n)$ and $\tilde{z}(\tau; w^L, w^M, w^H)$ have the following properties:

$$\begin{aligned} \partial y(\tau; \bar{w}, n)/\partial \tau &< 0, \\ \lim_{\tau \to 0} y(\tau; \bar{w}, n) &= \infty, \\ y(1/2; \bar{w}, n) &= 0, \\ \partial \tilde{z}(\tau; w^L, w^M, w^H)/\partial \tau &\geq 0, \\ \tilde{z}(0; w^L, w^M, w^H) &= \max\{\beta R w^L, (w^L)^{1-\sigma}\} < \infty, \\ \tilde{z}(1/2; w^L, w^M, w^H) &\in (0, \infty). \end{aligned}$$

These properties indicate that there exists a unique $\tau \in (0, 1/2)$ satisfying $y(\tau; \bar{w}, n) = \tilde{z}(\tau; w^L, w^M, w^H)$.

8.4.2 The determination of the decisive voter

Suppose that the type-H young individual is the decisive voter. Then, from Figure 5, it must hold that $y(\tau; \bar{w}, n) > \tilde{z}(\tau; w^L, w^M, w^H)$ at $\tau = \tilde{\tau}^{MH}$, that is:

$$(1+n)(1 - 2\tilde{\tau}^{MH})\bar{w} > R w^H.$$

This condition never holds under the assumptions of $R \geq 1+n$ (Assumption 2) and $w^H > \bar{w}$. Therefore, the decisive voter is a type-L or type-M young individual. From Figure 5, the type-L young individual becomes the decisive voter if $\beta(1+n)(1-2\tilde{\tau}^{LM})\bar{w} \leq \beta R w^M$, that is, if:

$$-1+2\cdot\left(\frac{(w^L)^{1-\sigma}}{\beta R w^M}\right)^{1/\sigma} \leq \frac{R}{(1+n)\bar{w}}w^M.$$

Otherwise, the decisive voter is a type-M young individual. ■

8.5 Proof of Proposition 3

(i) For the case of $1/\sigma \geq 1$, the decisive voter is a type-L young individual and the equilibrium tax rate satisfies $y(\tau;\bar{w},n) = z(\tau;w^L)$, as shown in Subsection 4.1. When the mean-preserving change in w^L is considered, $y(\tau;\bar{w},n)$ is unchanged while $z(\tau;w^L)$ is nonincreasing with reductions of w^L. Therefore, the equilibrium tax rate satisfying $y(\tau;\bar{w},n) = z(\tau;w^L)$ is nondecreasing in response to a mean-preserving reduction of w^L. Given that $\lambda^{dec}(\tau)$ is increasing in τ for $\tau \in (0,1/2)$, λ is also nondecreasing in response to a mean-preserving decrease in w^L.

(ii) For the case of $1/\sigma < 1$, the decisive voter is a type-j ($j = L$ or M) individual depending on parameter values, as shown in Proposition 2. To simplify the presentation, suppose that a type-L individual is the decisive voter. Note that the following argument applies for the case where a type-M is the decisive voter.

Assume that the equilibrium tax rate is given by $\tau^{equil} = \hat{\tau}(w^L)$: a type-$L$ individual is indifferent between saving and not saving. Under this situation, the decisive voter's wage w^L satisfies $\beta(1+n)(1-2\hat{\tau}(w^L))\bar{w} = \beta R w^L$, or:

$$R(w^L)^2 + (1+n)\bar{w}w^L - 2(1+n)\frac{\bar{w}}{(\beta R)^{1/\sigma}} = 0.$$

Solving this equation for w^L, we obtain:

$$w^L = \hat{w}^L \equiv \frac{-(1+n)\bar{w} + \sqrt{\{(1+n)\bar{w}\}^2 + 8R(1+n)\bar{w}/(\beta R)^{1/\sigma}}}{2R}.$$

Therefore, the equilibrium tax rate is given by $\tau^{equil} = \hat{\tau}(w^L)$ when a type-L individual with $w^L = \hat{w}^L$ is the decisive voter.

We now consider a mean-preserving change of w^L around $\hat{w}^L$. As shown in Subsection 4.2, the equilibrium tax rate satisfies $y(\tau;\bar{w},n) = \tilde{z}(\tau;w^L,w^M,w^H)$ if $1/\sigma < 1$. In particular, around $w^L = \hat{w}^L$, there exists a positive real number ε such that the equilibrium tax rate satisfies the following condition:

$$\tilde{z}(\tau; w^L, w^M, w^H) = \begin{cases} \beta R w^L & \text{for } w^L \in (\hat{w}^L - \varepsilon, \hat{w}^L], \\ \frac{(w^L)^{1-\sigma}}{(1-\tau)^\sigma} & \text{for } w^L \in [\hat{w}^L, \hat{w}^L + \varepsilon). \end{cases}$$

We focus on the range $(\hat{w}^L - \varepsilon, \hat{w}^L + \varepsilon)$ and consider a mean-preserving change of w^L around $\hat{w}^L$. The right-hand side of the above equation is increasing in w^L within the range $(\hat{w}^L - \varepsilon, \hat{w}^L)$ and decreasing in w^L within the range $(\hat{w}^L, \hat{w}^L + \varepsilon)$. This property implies that the equilibrium tax rate attains the highest value at $w^L = \hat{w}^L$ within the range $(\hat{w}^L - \varepsilon, \hat{w}^L + \varepsilon)$. Therefore, there is an inverse U-shaped relationship between the decisive voter's wage and the equilibrium tax rate around $w^L = \hat{w}^L$. Given $\lambda^{dec}(\tau)$ is increasing in τ for $\tau \in (0, 1/2)$, there is also an inverse U-shaped relationship between the decisive voter's wage and the equilibrium share of social security around $w^L = \hat{w}^L$.

8.6 Supplementary Explanation for Section 6

8.6.1 Derivation of (10)

To establish that condition (10) holds, we first investigate the property of $RHS^{yj}_{s>0}$. The first derivative of $RHS^{yj}_{s>0}$ with respect to w^j leads to:

$$\begin{aligned}
&\left(\frac{R}{(\beta R)^{1/\sigma} + R}\right)^{-\sigma} \cdot \frac{\partial RHS^{yj}_{s>0}}{\partial w^j} \\
&= \left(w^j + \frac{(1+n)\lambda\tau\bar{w}}{R}\right)^{-\sigma-1} \cdot \left[(1-\sigma)w^j + \frac{(1+n)\lambda\bar{w}}{R}\{\sigma(1-2\tau)+\tau\}\right],
\end{aligned}$$

where the term $\{\sigma(1-2\tau)+\tau\}$ is positive provided that $\tau > 1/2$. Thus, $\partial RHS^{yj}_{s>0}/\partial w^j > 0$ holds if $\sigma \geq 1$: this implies that $RHS^{yL}_{s>0} < RHS^{yM}_{s>0} < RHS^{yH}_{s>0}$ if $\sigma \geq 1$.

Next, we investigate the property of $RHS^{yj}_{s=0}$. Direct calculation leads to:

$$\begin{aligned}
RHS^{yL}_{s=0} \gtreqless RHS^{yM}_{s=0} &\Leftrightarrow \left(w^L\right)^{1-\sigma} \gtreqless \left(w^M\right)^{1-\sigma}; \\
RHS^{yM}_{s=0} \gtreqless RHS^{yH}_{s=0} &\Leftrightarrow \left(w^M\right)^{1-\sigma} \gtreqless \left(w^H\right)^{1-\sigma}.
\end{aligned}$$

Therefore, we obtain:

$$RHS^{yL}_{s=0} \gtreqless RHS^{yM}_{s=0} \gtreqless RHS^{yM}_{s=0} \Longleftrightarrow 1/\sigma \lesseqgtr 1.$$

An equality holds if and only if $1/\sigma = 1$.

8.6.2 Derivation of (11)

Suppose first that the type-j young agent is borrowing unconstrained. The first-order condition for the maximization of $V_{s>0}^{y,j}$ with respect to λ is given by:

$$\frac{\partial V_{s>0}^{y,j}}{\partial \lambda} = 0 \Leftrightarrow \lambda = \lambda_{s>0}^{yj} \equiv \frac{\frac{1}{2+n} - \left(\frac{\beta \eta R}{(2+n)(1+n)}\right)^{1/\sigma} \cdot \frac{R}{(\beta R)^{1/\sigma}+R} \cdot \frac{w^j}{\tau \bar{w}}}{\frac{1}{2+n} + \left(\frac{\beta \eta R}{(2+n)(1+n)}\right)^{1/\sigma} \frac{1+n}{(\beta R)^{1/\sigma}+R}} (< 1).$$

Taking into account the corner solution $\tau = 0$, we obtain:

$$\lambda_{s>0}^{yj} \equiv \max \left\{ 0, \frac{\frac{1}{2+n} - \left(\frac{\beta \eta R}{(2+n)(1+n)}\right)^{1/\sigma} \cdot \frac{R}{(\beta R)^{1/\sigma}+R} \cdot \frac{w^j}{\tau \bar{w}}}{\frac{1}{2+n} + \left(\frac{\beta \eta R}{(2+n)(1+n)}\right)^{1/\sigma} \frac{1+n}{(\beta R)^{1/\sigma}+R}} \right\}.$$

The preferred share λ is increasing in τ and is positive if and only if $\tau = \tau^*(w^j)$:

$$\frac{\partial V_{s>0}^{y,j}}{\partial \lambda} = 0 \Leftrightarrow \lambda = \lambda_{s=0}^{yj} \equiv \frac{\frac{1}{2+n}}{\frac{1}{2+n} + \left(\frac{\eta}{(2+n)(1+n)}\right)^{1/\sigma} (1+n)} (< 1),$$

where $\lambda_{s=0}^{yj}$ is constant and independent of τ. The equality holds between $\lambda_{s>0}^{yj}$ and $\lambda_{s=0}^{yj}$ at $\tau = \tau^*(w^j)$. Therefore, the preferred share λ by a type-j young agent is given as (11).

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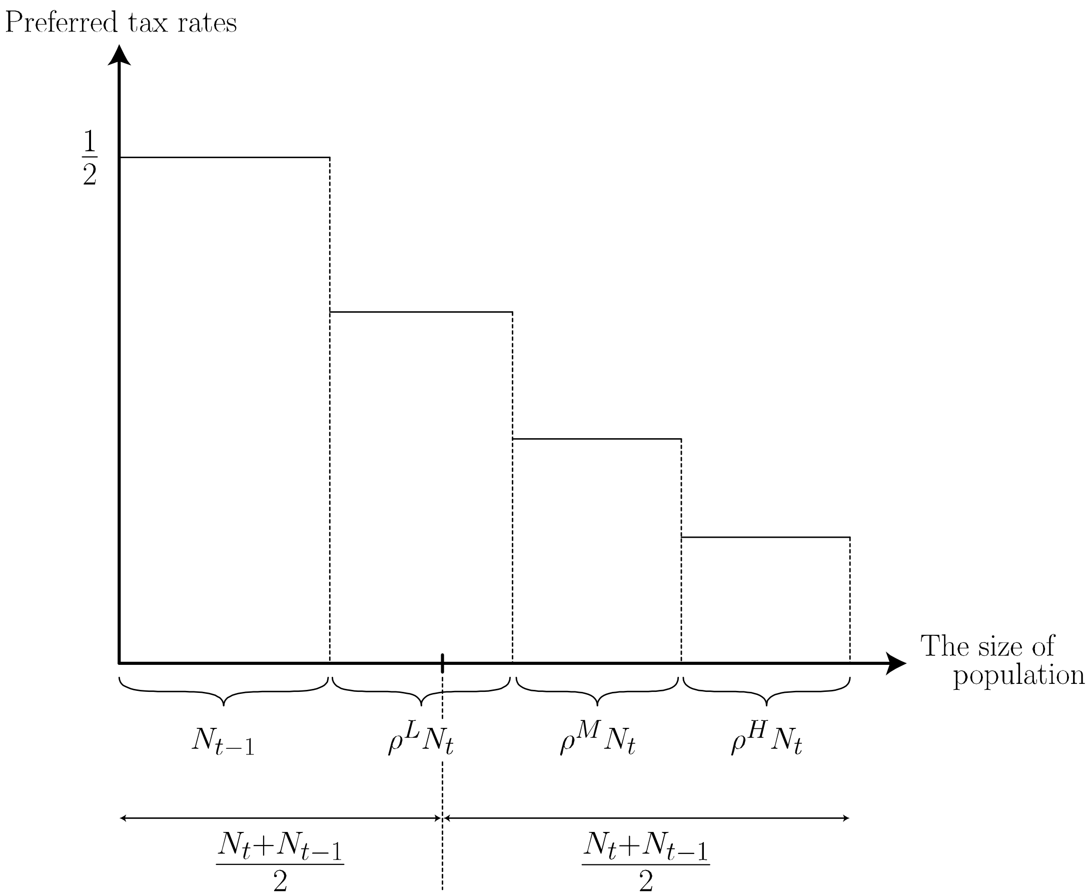

Figure 1: This figure illustrates an example of the tax rates preferred by the old and the young. In this example, a type-L young individual becomes a decisive voter.

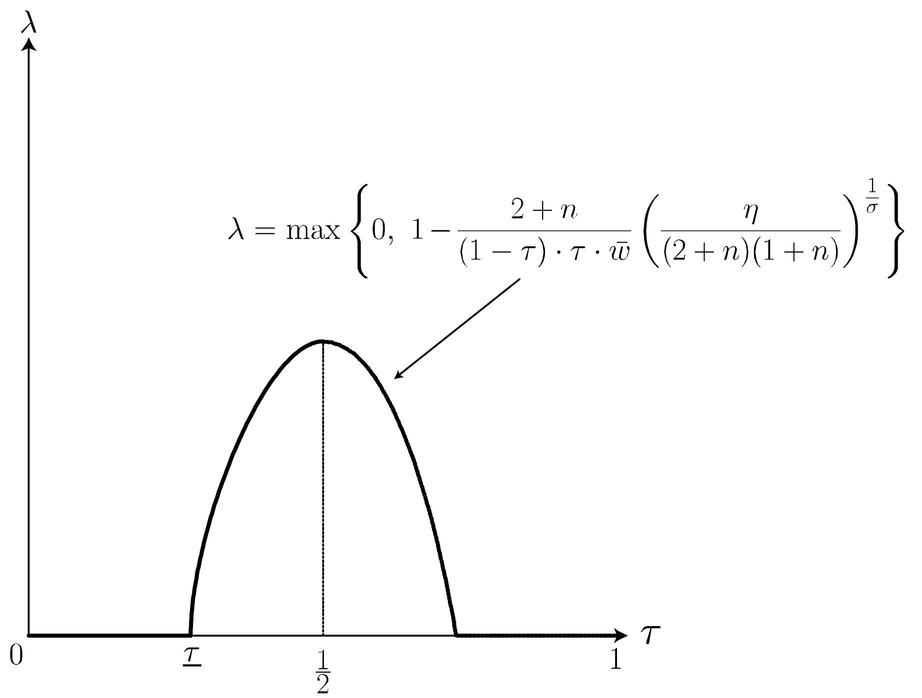

Figure 2: This figure illustrates the share of social security (λ) preferred by the young.

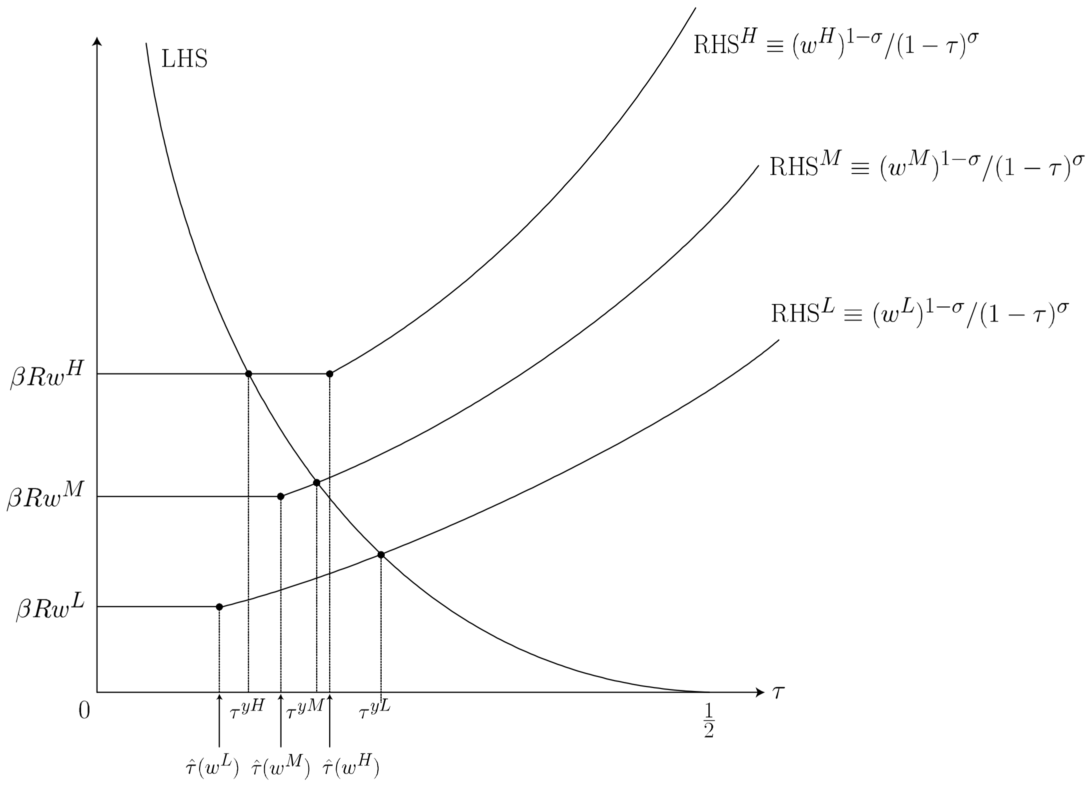

Figure 3: The tax rates preferred by the three types of young individuals in the case of $1/\sigma \geq 1$.

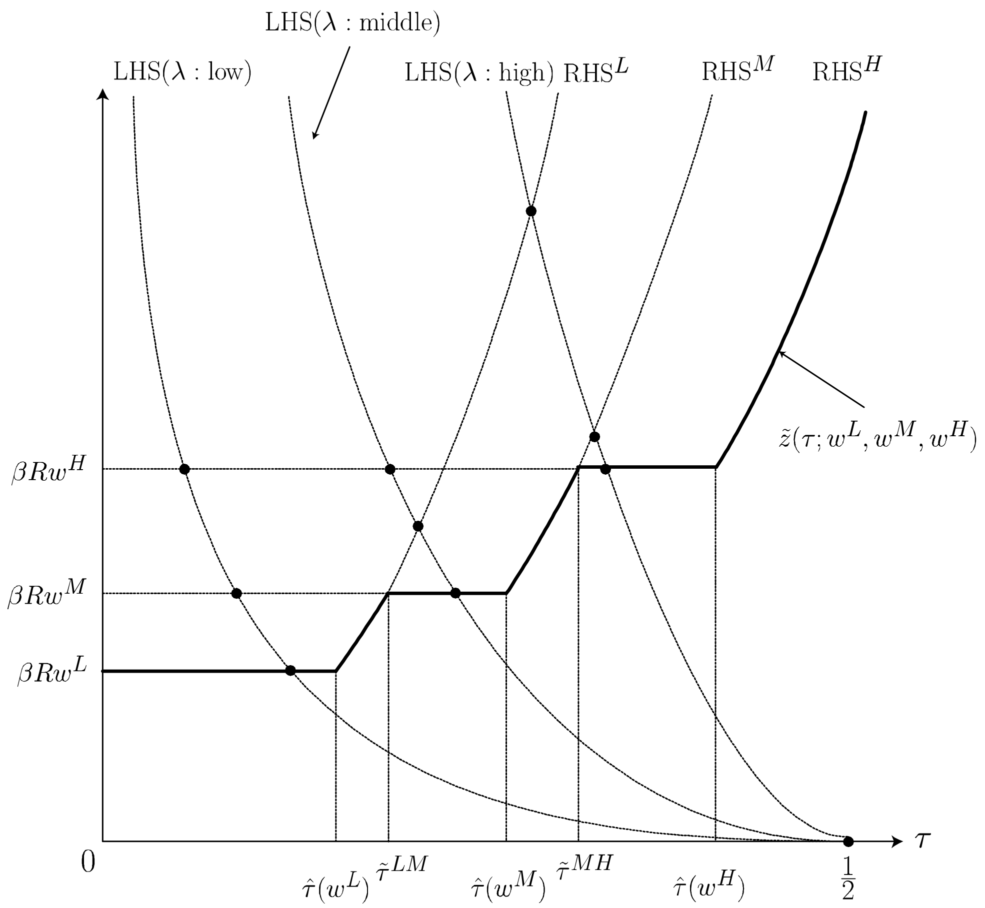

Figure 4: The tax rate preferred by a type-j young individual in the case of $1/\sigma < 1$. The bold curve illustrates the graph of $\widetilde{z}$ in (8).

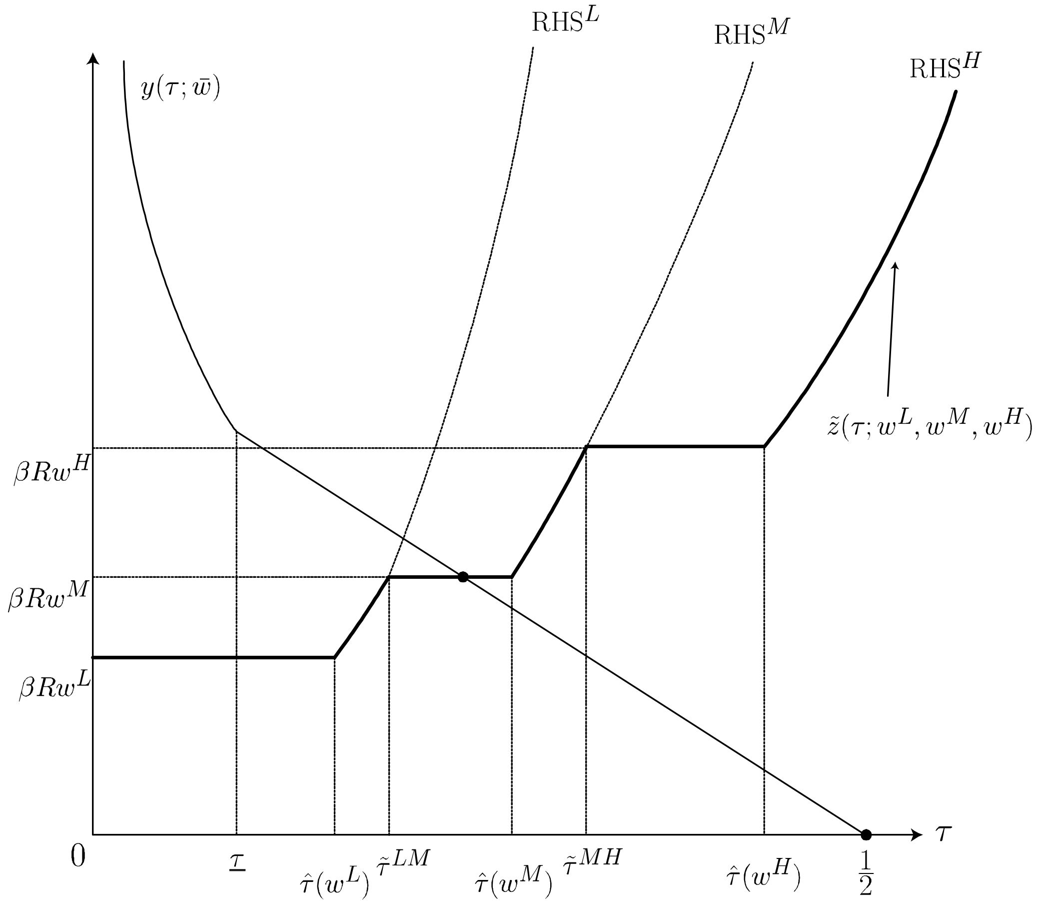

Figure 5: The determination of the tax rate in the case of $1/\sigma < 1$. The figure illustrates the case where the decisive voter is a type-M young individual.

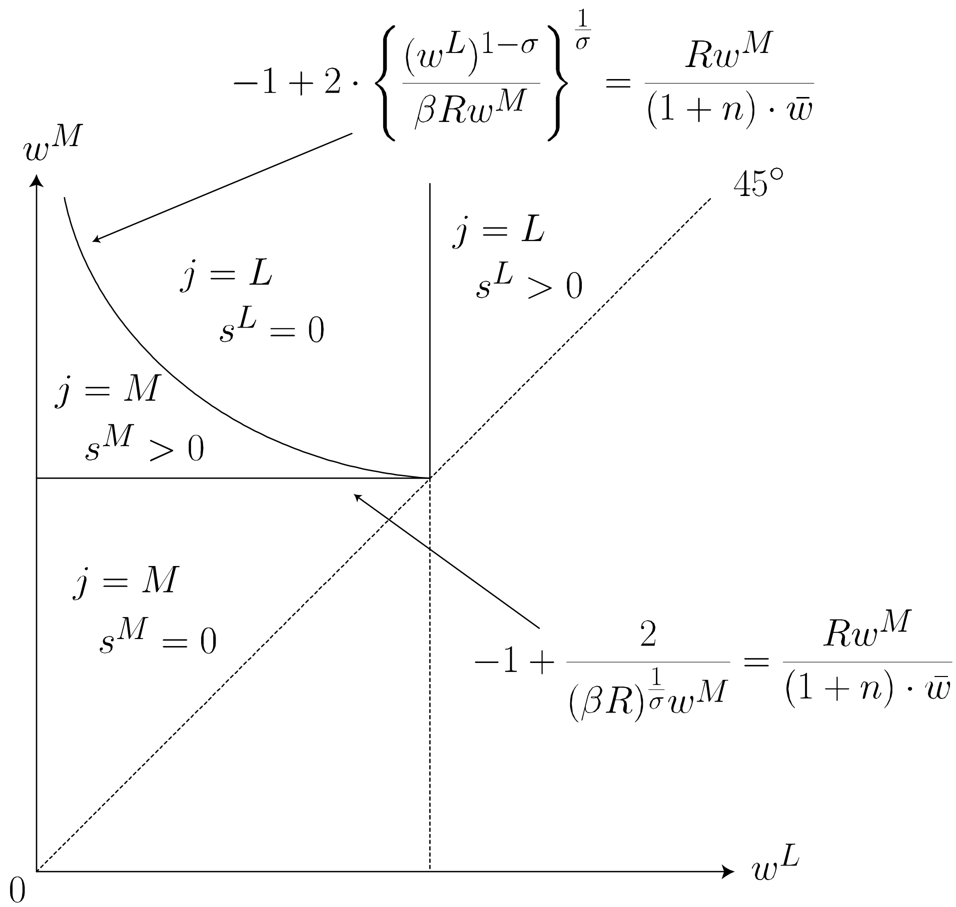

Figure 6: A decisive voter and his/her status of saving.

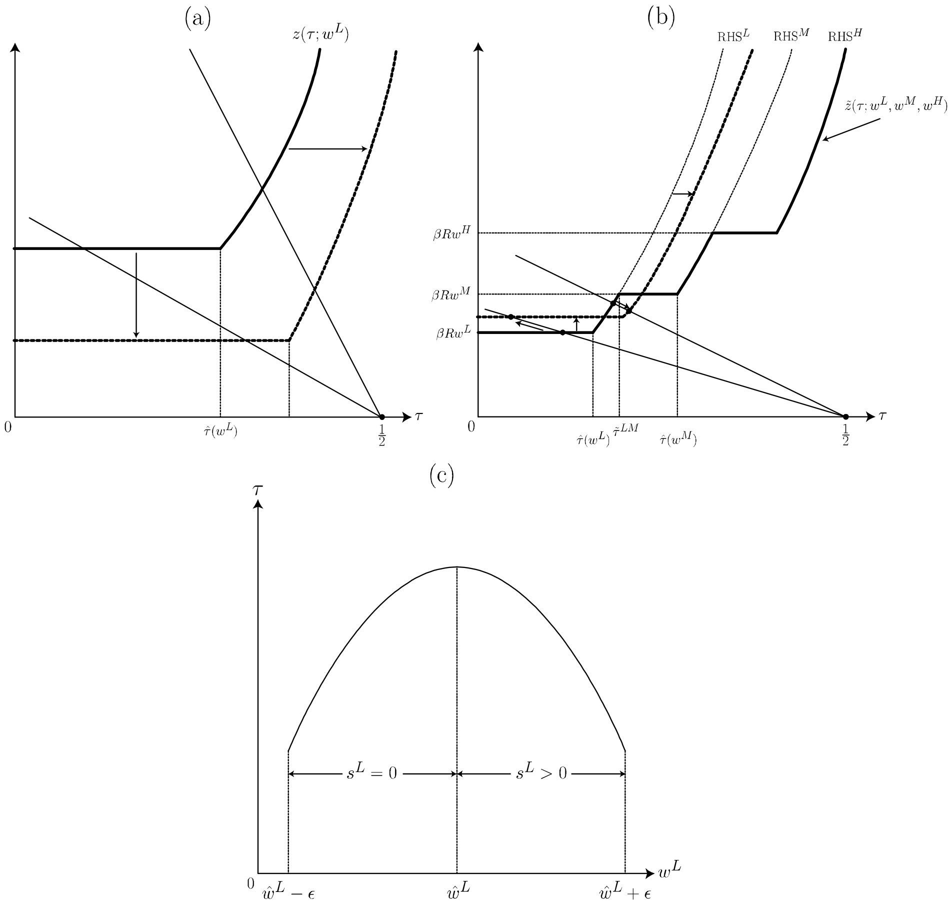

Figure 7: The effect of a mean-preserving reduction of the decisive voter's wage on the tax rate. Panel (a) illustrates the case of $1/\sigma \geq 1$. Panel (b) illustrates the case of $1/\sigma < 1$. Panel (c) illustrates the relation between the decisive voter's wage and the equilibrium tax rate in the case of $1/\sigma < 1$.

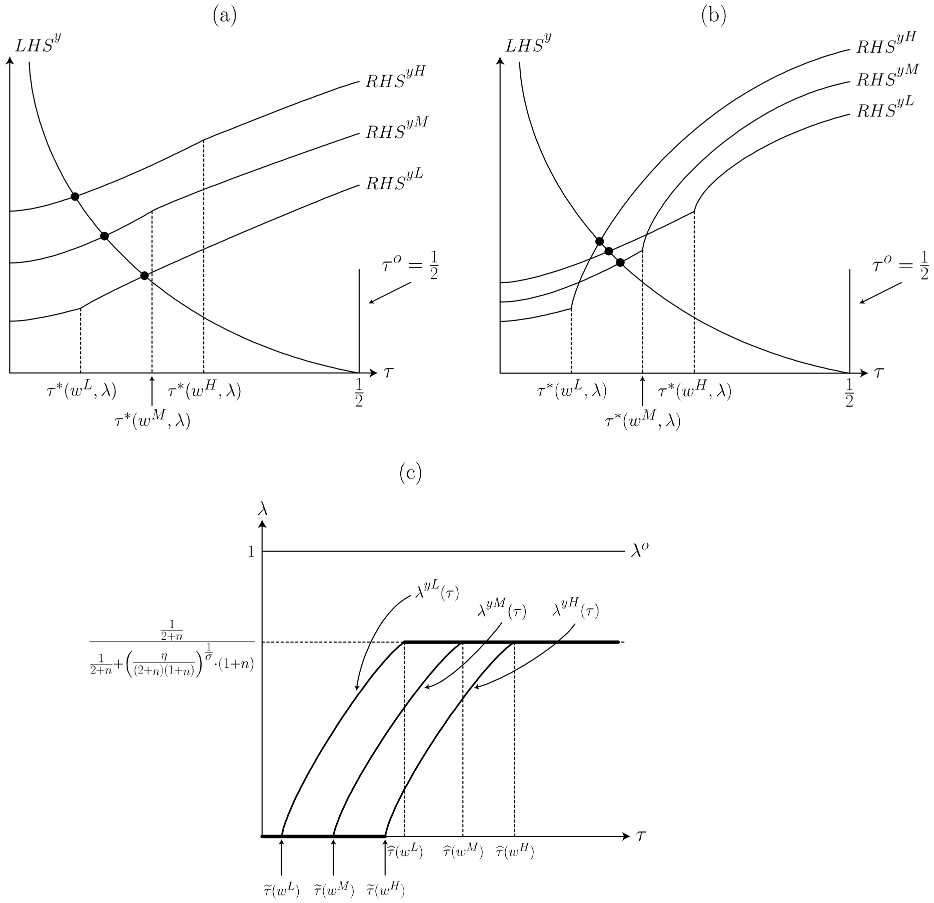

Figure 8: Panels (a) and (b) illustrate the preferred tax rates in cases where $1/\sigma \geq 1$ and $1/\sigma < 1$, respectively. Panel (c) illustrates the preferred shares of social security.