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Analysis of Transition Dynamics caused by Technological Breakthroughs

-Cause of productivity slowdown and drop in existing firms' stock prices-

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-Cause of productivity slowdown and drop in existing firms' stock prices-[*]

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Abstract

This paper examines the transition dynamics caused by technological breakthroughs. Our results show that technological breakthroughs cause a productivity slowdown and a drop in the stock prices of existing firms; these findings are consistent with observations in the 1970s. We explain how technological breakthroughs cause these phenomena. The emergence of a new technology creates new business opportunities, which reduces existing firms' profits, thus causing their stock prices to drop. This decline in existing firms' profits discourages R&D-intensive firms from entering the sector; this decreases aggregate R&D activity, and thus the growth rate of productivity declines.

keyword: R&D, technological breakthrough, productivity slowdown, stock price, general purpose technology

JEL classification: O11, O31, O41

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1 Introduction

Since the beginning of the Industrial Revolution in Britain (1760–1850), we have witnessed a number of dramatic technological breakthroughs, which have had enormous and long-term impacts on economies. Schumpeter refers to these as Kondratieff cycles, caused by technological factors. For example, electrical technologies had a major influence between 1890 and 1930, and the impact of information technology (IT) during the second half of the 20th century was also important. These technologies created new markets that opened up new business opportunities, and induced a sequence of secondary incremental innovation, which Bresnahan and Trajtenberg (1995) characterized as general purpose technology (GPT). For example, household electrical appliance manufacturers and electric power companies both arose as a consequence of electricity, and now continue to improve the quality of their products to sustain consumer demand. Similarly, PC manufacturers, software companies and Internet service providers were set up following the emergence of IT. These companies also continually improve the quality of their products.

The purpose of this paper is to analyze the transition from the emergence of a new technology to its maturity. In particular, we consider the growth rate of productivity, the stock price of incumbents in the preexisting sector and aggregate stock price. Jovanovic and Rousseau (2005) present evidence that, following the emergence of a new and transforming technology, the growth rate of productivity declines for some decades, after which it begins to rise again. Indeed, the productivity slowdown is now recognized as a stylized fact. Our model describes these changes and shows that the slowdown is caused by declining R&D in existing technology. Greenwood and Jovanovic (1999) present evidence that, upon the arrival of a new technology, existing firms' stock prices fall, never to recover. They also show that several years after the emergence of a new technology, aggregate stock prices rise because of the rise of the prices of the stock of new firms exploiting the new technology. Our model describes these facts and shows that the decline in the stock prices of existing firms is caused by the decline in their profits, while the aggregate stock price rises with the new sector's maturity.

The model is the extension of Segerstrom (1998), which is a quality-ladder R&D model, freed from scale effects by assuming that the difficulty of R&D increases as R&D accumulates.[1] His model is a semi-endogenous growth model, where the long-run growth rate is never influenced by political variables or preferences. More recently, the short-run effects in semi-endogenous growth models have received more attention. Steger (2003) found that the speed of convergence in Segerstrom (1998) was only 0.019, which means that it takes about 38 years for half the initial gap to vanish (this is a very long time!). Therefore, analyzing the transition dynamics is very important. In this paper, we examine the technological cycle by analyzing the transition dynamics of an extended Segerstrom

[1] Aghion and Howitt (1992),Grossman and Helpman (1991) and Romer (1990) pioneered these R&D models.

(1998) model, which has very long transition dynamics.

In this paper, we expand Segerstrom (1998) by introducing heterogeneity across sectors in four ways: efficiency, difficulty and cost of R&D, and market size.[2] This extension enables the model to simultaneously analyze both the "old sector" and the "new sector", where the old sector has low R&D costs and a high difficulty of R&D , and the new sector has high R&D costs but a low difficulty of R&D. We examine the process caused by a technological breakthrough to analyze the transition dynamics from the existing old-sector equilibrium alone to the equilibrium of both the old and new sectors.

Consider the following situation. At stage 1, there exists an "old sector" with a continuum of products. At stage 2, an exogenous technological breakthrough occurs, leading to emergence of a "new sector" with a continuum of products. The initial cost of R&D in the new sector is high enough that no R&D is conducted in this new sector. Stage 3 is reached later, when another exogenous shock occurs, which drastically reduces the cost of R&D in the new sector, such that it is now conducted. This paper examines how economically important variables such as the growth rate of productivity and stock prices respond to these two enormous exogenous shocks.[3]

Some stage 2 results are consistent with empirical facts. First, the growth rate of productivity declines. New technology creates new business opportunities, thus forming the new sector, which captures market share from old-sector firms, thus causing their profits to decrease. This decreases the incentive to enter the old sector, reducing R&D in the old sector. Thus, the instantaneous probability of successful R&D in the old sector decreases. On the other hand, R&D is not being conducted in the new sector because of the very high cost of R&D. Therefore, aggregate R&D decreases, which leads to a decrease in the aggregate probability of successful R&D. Because the growth rate of productivity is calculated as the weighted sum of probabilities of successful R&D, this reduces productivity growth. Second, old-sector incumbents' stock prices decrease because their discounted stream of profits has decreased.

Stage 3 results are also consistent with empirical facts. First, the growth rate of productivity rises. Several years after a new technology is born, the cost of R&D falls. R&D activity then begins in the new sector, which dominates the effect of the decline of R&D activity conducted in the old sector. Therefore, aggregate R&D increases, which leads to an increase in the aggregate probability of successful R&D, and the growth rate of productivity rises. Second, old-sector incumbents' stock prices remain low. Because old-sector market share does not change between stages 2 and 3, the discounted profits of old-sector incumbents remain small and hence their stock prices remain

[2]Smulders and van de Klundert (1995), Startz (1998), Li (2000), Boucekkine and de la Croix (2003), Doi and Mino (2005) and Chu (2011) have investigated growth models with multiple sectors.

[3]By assuming that R&D cost decreases due to learning by doing, we can extend the model so that the time of initial new-sector R&D investment is determined endogenously; however, including these would complicate the model significantly and so is left for further research.

low. Third, the aggregate stock price rises when we include the new-sector firms, which were created in response to the new technology. As the new technology matures, the status of new-sector firms stabilizes, raising their discounted profits. Hence, new-sector firms' stock prices rise, dominating the decline of old-sector incumbents' stock prices. Therefore, the aggregate stock price rises.

The contributions of this paper are as follows. First, we explain the transition of economically important variables such as productivity and stock price that is observed empirically using a typical R&D growth model. Second, we construct a quality-ladder R&D model with ex ante heterogeneity (efficiency, difficulty and cost of R&D, and market share differences across sectors evaluated before R&D is conducted). The current literature examines quality-ladder R&D models with ex post heterogeneity (efficiency of product technology, degree of increment quality, shadow price and profits differ across firms evaluated after having conducted R&D).[4] Our model extends these models in another direction. Third, we carefully analyze the transition dynamics of a semi-endogenous growth model. As discussed in Steger (2003), the transition path of a semi-endogenous growth model is so long that analyzing transition dynamics is important for development of this model.

1.1 Related literature

Helpman and Trajtenberg (1998) and Aghion and Howitt (2008) analyze the relationship between the emergence of a new technology (GPT) and productivity slowdown. Helpman and Trajtenberg (1998) state that because labor is needed to introduce a new GPT, the amount of labor engaging in manufacturing declines, thus reducing output. Aghion and Howitt (2008) use a simple neoclassical model to consider a rate of capital obsolescence that is proportional to the innovation rate. They state that a new GPT raises both the innovation rate and the rate of capital obsolescence, which reduces the growth rate. On the other hand, this paper states that a new technology creates a new sector and diminishes the incentive to carry out R&D in the old sector, which reduces the growth rate of productivity.

Greenwood and Jovanovic (1999) and Hobijn and Jovanovic (2001) analyze the relationship between the emergence of IT and incumbents' stock prices. They use the Lucas tree model and assume that the emergence of IT causes permanently higher income several years later. The results show that the substitution effect causes current consumption to rise, and thus this reduces demand for assets (the right to future consumption), which reduces asset prices. On the other hand, we show that the initial fall in the stock price is due to the decline in old-sector incumbents' profits and that the subsequent rise is due to the maturation of the new technology sector.

[4] Dinopoulos and Unel (2011) and Minniti, Parello and Segerstrom (2012) examine R&D with ex post heterogeneity.

Cheng and Dinopoulos (1996) analyze the endogenous deterministic cycles caused by technological breakthroughs. However, they do not examine the effects of technological breakthroughs on the dynamics of productivity or stock price growth rates. Boucekkine and de la Croix (2003) study the effect on the growth rate of the IT revolution, which changes the productivity of the final goods sector and the efficiency of the R&D sector. They use a simulation model in which efficiency of R&D increases permanently and productivity of the final goods sector decreases temporarily to show that productivity slowdown occurs. They state that the productivity of labor initially decreases because of the negative shock to the final goods sector. However, the positive shock to the R&D sector stimulates R&D activity, which increases productivity. Thus, the productivity of labor eventually increases. This mechanism is clearly different from our model.

The rest of this paper is organized as follows. Section 2 describes the basic setup of the model. Section 3 describes the path of the economy and establishes the balanced-growth equilibrium. Section 4 analyzes the transition from the emergence of the new technology to its maturity. Section 5 concludes.

2 The Model

2.1 Environment

The economy consists of three agents: households, R&D firms and producing firms (incumbents). Households supply labor inelastically, save by investing in R&D firms as stockholders and consume products. R&D firms hire labor to engage in R&D, which improves products' quality. Producing firms hire labor to produce products, gain profits by selling the products and distribute those profits to households as dividends.

In the main analysis, we consider the following situation. Initially, there exists a sector with a continuum of products, the "old sector", and subsequently another sector with a continuum of products, the "new sector", emerges. The two sectors differ in four respects: efficiency, difficulty and cost of R&D, and market size. The old sector, which is denoted sector 0, is so mature that the cost of R&D is low; however, the probability of successful R&D is also low because a large amount of R&D has already been accumulated. This assumption follows Segerstrom's notion that the most obvious ideas are discovered first, making it harder to develop further ideas. The new sector, which is denoted sector 1, is so new that the cost of R&D is high; however, probability of successful R&D is high because little R&D has yet been accumulated.

In each sector, there are continuous industries indexed by $j \in [0, 1]$ producing differentiated goods. Each industry produces goods with different qualities, which are classified into countable quality generation $m = 0, 1, \ldots$.

The goods are perfectly substitutes for one another in each industry. Innovation in an industry results in a new generation of goods.

We designate as leaders (producing firms) (incumbents) those firms that use the latest R&D in each industry and are able to produce the state-of-the-art quality product in their industry. At the time of birth of each sector, we assume that the state-of-the-art quality product in each industry is m = 0 and that the size of quality increments induced by one successful R&D event is constant in any industry of any sector, which is denoted as $\lambda > 1$. Therefore, in industry j of sector i at time t, state-of-the-art quality is determined by the number of successful R&D events that have occurred in industry j of sector i from the birth of sector i until time t.

We designate as R&D firms those firms that attempt R&D investment in order to win a leadership position through a higher quality product. They may invest in any industry of any sector. If an R&D firm produces successful R&D in industry j of sector i, it can produce a state-of-the-art quality product in industry j of sector i, and thus becomes the leader in industry j of sector i. When the state-of-the-art quality in industry j of sector i is m – 1 generation, denoted as $q^i(j, m-1)$, the successful R&D firm can produce an m generation product, the quality of which is denoted by $q^i(j,m) = \lambda q^i(j,m-1)$. If we normalize the initial quality to unity (i.e., $q^i(j,0) = 1$), then an m generation of the state-of-the-art quality product in industry j of sector i is expressed as $q^i(j,m) = \lambda^m$.

2.2 Households

The economy is populated by a fixed number of identical households. Household members live forever and are endowed with one unit of labor, which is supplied inelastically. The number of members in each household grows exponentially at an exogenous rate $n > 0$. Assuming that the initial population is unity, then the population at time t is $L(t) = e^{nt}$. Each household maximizes its discounted utility:

$$U = \int_0^\infty \exp(-(\rho - n)t) \ln u(t) dt, \tag{1}$$

where $\rho > n$ is the subjective discount rate and $\ln u(t)$ is the instantaneous utility per person at time t, which is given by

$$\ln u(t) = \gamma^0 \int_0^1 \ln \left[\sum_m \lambda^m d^0(j,m,t) \right] dj + \gamma^1 \int_0^1 \ln \left[\sum_m \lambda^m d^1(j,m,t) \right] dj, \tag{2}$$

where γ^i is the market share of sector i, $d^i(j, m, t)$ is the quantity demanded of m generation product produced in industry j of sector i at time t.[5] The first term on the right-hand side (RHS) is the instantaneous utility from consuming the products of sector 0; the integration of this term is the instantaneous utility from consuming the products of industry j of sector 0, and the summation of this term is the instantaneous utility from consuming m generation products of industry j of sector 0. The second term on the RHS is the instantaneous utility from consuming products of sector 1, and so on. Because γ^i is the market share of sector i, $\gamma^0 + \gamma^1 = 1$ is always satisfied. If $\gamma^0 = 1$, then this instantaneous utility function is identical to that of Segerstrom (1998).

From (2), the demand function for the product with the lowest quality-adjusted price in industry j of sector i is given by

$$d^i(j,t) = \frac{\gamma^i c(t)}{p^i(j,t)}, \quad \text{for} \quad i = 0, 1 \ , \tag{3}$$

where $p^i(j, t)$ is the price of the product that has the lowest quality-adjusted price in industry j of sector i at time t, $d^i(j, t)$ is the quantity demanded of the product that has the lowest quality-adjusted price in industry j of sector i at time t, and $c(t)$ is households' expenditure at time t.[6] Given (3), intertemporal utility maximization yields

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho, \tag{4}$$

where $r(t)$ is the interest rate at time t.

2.3 Product markets

We assume the product market is monopolistically competitive. In each industry, firms compete in price. Labor is the only input in production. We assume that one unit of labor is required to produce one unit of output regardless of quality level, and that there is no fixed cost. We normalize the wage rate as numeraire. Thus, every firm has a constant marginal cost of production equal to unity.

In each industry, there is one leader and a number of followers. These followers were formerly able to produce the state-of-the-art quality product but cannot now. Because the quality increment from one successful R&D event is constant, leaders can produce a product of at least λ higher quality compared with the followers. Because the marginal cost of production is unity, followers never charge a price less than unity. Leaders can therefore

[5] As seen from (2), we assume that the intersector and interindustry elasticities of substitution are unity, and that the intra-industry elasticity of substitution is infinity.

[6] Because intra-industry elasticity of substitution is infinity, households demand only the product that has the lowest quality-adjusted price in each industry

monopolize demand for their industry's product by charging a price less than λ. Because interindustry elasticity of substitution is unity, leaders have no incentive to lower the price to less than λ.[7] Thus the price of the product is then determined by

$$p(t) = \lambda. \tag{5}$$

Because one unit of labor is required to produce one unit of output and there is no fixed cost, leaders in sector i earn the following profit flow:

$$\pi^i(t) = \gamma^i c(t) L(t)(1 - \frac{1}{\lambda}), \quad \text{for} \quad i = 0, 1, \tag{6}$$

where $\pi^i(t)$ is the leaders' profits in sector i at time t.[8]

2.4 R&D races

Labor is the only input to R&D. There is free entry into each R&D race. Any R&D firm that hires $H^i(t)l^i(j,k,t)dt$ units of labor in industry j of sector i at time interval [t,t+dt] is successful in discovering a state-of-the-art product with instantaneous probability $I^i(j,k,t) = l^i(j,k,t)A^i/X^i(t)dt$, where $H^i(t)$ is the cost of R&D, A^i is the efficiency of R&D and $X^i(t)$ is the difficulty of R&D in industry i. Assuming that the returns to engaging in the R&D race are independently distributed across firms, across industries and over time, the industry-wide instantaneous probability of successful R&D is given by

$$I^i(t)dt = \frac{L^i(t)A^i}{X^i(t)}dt, \quad \text{for} \quad i = 0, 1, \tag{7}$$

where $I^i(t)dt$ is industry-wide instantaneous probability of successful R&D in sector i, and $L^i(t)dt$ is the amount of labor engaging in R&D in sector i. Following Segerstrom (1998), the difficulty of R&D in sector i increases as R&D in sector i accumulates. Then we assume

$$\frac{\dot{X}^i(t)}{X^i(t)} = \mu I^i(t), \quad \text{for} \quad i = 0, 1, \tag{8}$$

[7] We assume that the consumer strictly prefers the higher-quality good when the quality-adjusted price is same.

[8] Note that all industries' profits in the same sector are identical because both the quantity demanded in a sector and their prices are identical.

where $0 < \mu < 1$ is exogenously given. We define s^i as the time of origin in sector i. We also assume that $X^i(s^i) = \bar{X}^i$, that is, the initial difficulty of R&D is identical for all industries in the same sector. Then, the difficulty of R&D in sector i at time t is $X^i(t) = \bar{X}^i \exp\left(\int_{s^i}^{t} \mu I_v^i d\nu\right)$.

Let $v^i(t)$ denote the leaders' stock prices in sector i at time t.[9] Each R&D firm attempts to maximize $v^i(t)\frac{A^i l^i(j,k,t)}{X^i(t)}dt - H^i(t)l^i(j,k,t)dt$. If $v^i(t)\frac{A^i}{X^i(t)} > H^i(t)$; then, $l^i = \infty$ is profit maximizing, which cannot be the equilibrium. If $v^i(t)\frac{A^i}{X^i(t)} < H^i(t)$, then $l^i = 0$ is profit maximizing. If $v^i(t)\frac{A^i}{X^i(t)} = H^i(t)$, then $l^i \in (0, +\infty)$ is profit maximizing. To sum up,

$$\begin{aligned} v^i(t) &= \frac{X^i(t)H^i(t)}{A^i} \quad when \quad I^i(t) > 0 \quad \text{for} \quad i = 0, 1 \\ v^i(t) &< \frac{X^i(t)H^i(t)}{A^i} \quad when \quad I^i(t) = 0 \quad \text{for} \quad i = 0, 1. \end{aligned} \tag{9}$$

2.5 No arbitrage condition

Over time interval dt, shareholders receive an income gain $\pi^i(t)dt$ and a capital gain (loss) $\dot{v}^i(t)dt$ in each industry of sector i. However, with instantaneous probability $I^i(t)dt$, a higher-quality product is discovered, the existing leader goes out of business and shareholders suffer a total capital loss $v^i(t)$. Efficient markets require that the expected return to holding the stock of the leader is equal to the risk-free interest rate $r(t)$.[10] Thus $\frac{\pi^i(t)+\dot{v}^i(t)-I^i(t)v^i(t)}{v^i(t)} = r(t)$. From (6), we obtain

$$v^i(t) = \frac{\gamma^i(\frac{\lambda-1}{\lambda})c(t)L(t)}{r(t) + I^i(t) - \frac{\dot{v}^i(t)}{v^i(t)}}, \quad \text{for} \quad i = 0, 1 \ \cdot \tag{10}$$

In further analysis, let us define indexes of stock prices, $V(t)$ and $\hat{V}(t)$, as follows:

$$V(t) = v^0(t) + v^1(t) \quad \forall I^0(t), \forall I^1(t) \ , \tag{11}$$

where $V(t)$ is defined as the sum of stock prices of leaders across sectors where the leaders are producing goods.

$$\begin{aligned} \hat{V}(t) &= v^0(t) \qquad\qquad when \quad I^0(t) > 0, I^1(t) = 0 \\ \hat{V}(t) &= v^0(t) + v^1(t) \quad when \quad I^0(t) > 0, I^1(t) > 0 \end{aligned} \ , \tag{12}$$

[9] As seen below, we interpret $v^i(t)$ as the expected discounted profit for winning an R&D race in sector i at time t. From (6), profits are identical for all industries in the same sector. Thus, the leader's stock price is the same in all industries in the same sector.

[10] Assuming that shareholders are risk neutral and can diversify risk completely.

where $\hat{V}(t)$ is defined as the sum of stock prices of leaders across sectors where the leaders are producing goods and R&D firms are carrying on R&D.[11]

2.6 The labor market

Because one unit of labor is required to produce one unit of output, $\gamma^i c(t)L(t)/\lambda$ workers are employed by the leader in sector i. $X^i(t)H^i(t)I^i(t)/A^i$ workers are employed by R&D firms in sector i. Because households supply one unit of labor inelastically, we obtain

$$\frac{X^0(t)H^0(t)I^0(t)}{A^0} + \frac{X^1(t)H^1(t)I^1(t)}{A^1} + \frac{c(t)L(t)}{\lambda} = L(t). \tag{13}$$

2.7 Growth rate of productivity

Because we assumed that the quality of 0 generation products for all industries of any sector is unity and that the size of an increment of one successful R&D event is λ, the quality of the product manufactured by the leader of industry j of sector i is determined by the number of times that successful R&D occurs in that industry. We define $Q(t)$ as the weighted sum of the quality of products across all industries of all sectors, and express $Q(t)$ as follows:

$$\ln Q(t) = \sum_{i=0}^{1} \gamma^i \int_0^1 \ln \lambda^{m^i(j,t)} dj = \sum_{i=0}^{1} \gamma^i \Phi(i,t) \ln \lambda. \tag{14}$$

where $m^i(j,t)$ is the generation of state-of-the-art quality in industry j of sector i at time t, and $\Phi(i,t) = \int_{s^i}^{t} I(i,\upsilon)d\upsilon$ is the expected value of successful R&D events that will occur in sector i in time interval $\left[s^i, t\right]$.

Define $g(t)$ as the growth rate of the weighted sum of product qualities. From (14), we obtain

$$g(t) \equiv \frac{\dot{Q}(t)}{Q(t)} = \gamma^0 I^0(t) \ln \lambda + \gamma^1 I^1(t) \ln \lambda. \tag{15}$$

Following Grossman and Helpman (1991), we interpret $g(t)$ as the growth rate of productivity.[12]

[11] We define two types of stock price index to deal with the problem of the timing of stock listing (the problem is when we should evaluate new-sector incumbents' stock prices.). Although the above definitions are not complete, they are sufficient for analyzing a rough motion of stock price.

[12] From (2) and (3), we obtain $\ln u(t) = \ln Q(t) + \gamma^0 \ln L^{p0}(t) + \gamma^1 \ln L^{p1}(t)$, where L^{pi} is labor engaging in manufacturing in sector i. Interpreting $u(t)$ as the final output, a higher $Q(t)$ means that more output can be produced with the same input. Therefore, we can take $Q(t)$ as Total Factor Productivity (TFP)and $g(t)$ as the growth rate of TFP.

3 Equilibrium

In this section, we derive the equilibrium path in two cases. In one case, R&D is done only in sector 0 (i.e., $I^0(t) > 0$ and $I^1(t) = 0$), and in the other case, R&D is done in both sectors (i.e., $I^0(t) > 0$ and $I^1(t) > 0$).

3.1 The case in which $I^0(t) > 0$ and $I^1(t) = 0$

First, derive the equilibrium path in the case where $I^0(t) > 0$ and $I^1(t) = 0$. Define the adjusted difficulty of R&D in sector i: $y^i(t) \equiv X^i(t)H^i(t)/A^iL(t)$ Differentiating this equation with respect to t yields $\dot{y}^i(t)/y^i(t) = I^i(t)\mu - n$ from (8). Because $I^1(t) = 0$, we obtain $y^0(t)I^0(t) + \frac{c(t)}{\lambda} = 1$ from (13). Then

$$\dot{y}^0(t) = \frac{(\lambda - c(t))}{\lambda}\mu - y^0(t)n. \tag{16}$$

Because $I^0(t) > 0$, we obtain $v^0(t) = y^0(t)L(t)$ from (9). We also obtain $v^0(t) = \frac{\gamma^0(\frac{\lambda-1}{\lambda})c(t)L(t)}{r(t)+(1-\mu)I^0(t)}$ from (8) and (10). Combining these two equations and using $y^0(t)I^0(t)+\frac{c(t)}{\lambda} = 1$, we obtain $r(t) = \frac{1}{\lambda y^0(t)}\left[\gamma^0(\lambda-1)c(t) - (1-\mu)(\lambda - c(t))\right]$. Substituting this equation into (4),

$$\dot{c}(t) = \frac{1}{\lambda y^0(t)}[(\gamma^0\lambda + \gamma^1 - \mu)(c(t))^2 - (1-\mu)\lambda c(t)] - \rho c(t). \tag{17}$$

This economy's dynamics are described by (16) and (17) in (y^0, c) space, as depicted in figure 1. The vertical axis represents consumption per capita, c, which is a jump variable, and the horizontal axis represents the adjusted difficulty of R&D in sector 0, y^0, which is a state variable. From (16), the $\dot{y}^0(t) = 0$ curve is downward-sloping. When the point (y^0, c) is to the left (right) of the $\dot{y}^0(t) = 0$ curve, then $\dot{y}^0(t) > 0$ ($\dot{y}^0(t) < 0$). From (17), the $\dot{c}(t) = 0$ curve is upward-sloping. When the point (y^0, c) is to the left (right) of the $\dot{c}(t) = 0$ curve, then $\dot{c}(t) > 0$ ($\dot{c}(t) < 0$).

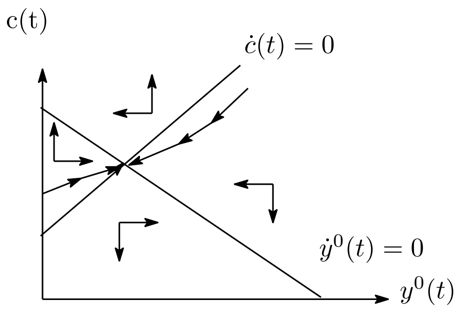

Figure 1. Phase diagram where $I^0(t) > 0$ and $I^1(t) = 0$.

We now derive the value of the balanced-growth equilibrium. Because the balanced-growth equilibrium is determined by the intersection of the $\dot{y}^0(t) = 0$ curve and the $\dot{c}(t) = 0$ curve, we obtain

$$y^{0*} = \frac{\gamma^0\mu(\lambda - 1)}{\mu\rho - \mu n + n(\gamma^0\lambda + 1 - \gamma^0)}, \tag{18}$$

and

$$c^* = \frac{\lambda(\mu\rho - \mu n + n)}{\mu\rho - \mu n + n(\gamma^0\lambda + 1 - \gamma^0)}, \tag{19}$$

from (16) and (17), where the steady-state equilibrium level is denoted by "*" in the present case (where $I^0(t) > 0$ and $I^1(t) = 0$).[13] Because c^* is constant in the balanced-growth equilibrium, we obtain $r^* = \rho$ from (4). Substituting (18) and (19) into (13) with $I^1(t) = 0$ and $y^0(t) \equiv X^0(t)H^0(t)/A^0L(t)$, we obtain

$$I^{0*} = \frac{n}{\mu}, \tag{20}$$

and from (15), we obtain

$$g^* = \gamma^0(\ln\lambda)\frac{n}{\mu}. \tag{21}$$

From $v^0(t) = y^0(t)L(t)$ and (18), we obtain

$$v^{0*}(t) = \frac{\mu\gamma^0(\lambda - 1)L(t)}{\mu\rho - \mu n + n(\gamma^0\lambda + 1 - \gamma^0)}, \tag{22}$$

and from (10) and $r^* = \rho$, we obtain

$$v^{1*}(t) = \frac{(\mu\rho - \mu n + n)}{(\mu\rho - \mu n)} \frac{\mu\gamma^1(\lambda - 1)L(t)}{\mu\rho - \mu n + n(\gamma^0\lambda + 1 - \gamma^0)}. \tag{23}$$

Substituting (22) and (23) into (11), we obtain

$$V^*(t) = \frac{(\mu\rho - \mu n + n\gamma^1)(\lambda - 1)L(t)}{(\rho - n)\left[\mu\rho - \mu n + n(\gamma^0\lambda + 1 - \gamma^0)\right]}. \tag{24}$$

[13] The equilibrium is saddle-path stable. Steger (2003) examines the stability conditions carefully.

From (12), we obtain

$$\hat{V}^*(t) = \frac{\mu\gamma^0(\lambda-1)L(t)}{\mu\rho - \mu n + n(\gamma^0\lambda + 1 - \gamma^0)}. \tag{25}$$

3.2 The case in which $I^0(t) > 0$ and $I^1(t) > 0$

In this subsection, we derive the equilibrium path in the case where $I^0(t) > 0$ and $I^1(t) > 0$. Because $I^0(t) > 0$ and $I^1(t) > 0$, we obtain $v^0(t) = y^0(t)L(t)$ and $v^1(t) = y^1(t)L(t)$ from (9). We also obtain $v^0(t) = \frac{\gamma^0(\frac{\lambda-1}{\lambda})c(t)L(t)}{r(t)+(1-\mu)I^0(t)}$ and $v^1(t) = \frac{\gamma^1(\frac{\lambda-1}{\lambda})c(t)L(t)}{r(t)+(1-\mu)I^1(t)}$ from (8) and (10). Substituting $v^0(t) = y^0(t)L(t)$ into $v^0(t) = \frac{\gamma^0(\frac{\lambda-1}{\lambda})c(t)L(t)}{r(t)+(1-\mu)I^0(t)}$, $v^1(t) = y^1(t)L(t)$ into $v^1(t) = \frac{\gamma^1(\frac{\lambda-1}{\lambda})c(t)L(t)}{r(t)+(1-\mu)I^1(t)}$, eliminating r(t) to combine these two equations, and using (13), we obtain

$$\begin{aligned} I^0(t) &= \tfrac{1}{\lambda(y^0(t)+y^1(t))}\Big[\big\{\gamma^0 y^1(t) - \gamma^1 y^0(t)\big\}\tfrac{(\lambda-1)c(t)}{(1-\mu)y^0(t)} + \lambda - c(t)\Big], \\ I^1(t) &= \tfrac{1}{\lambda(y^0(t)+y^1(t))}\Big[\big\{\gamma^1 y^0(t) - \gamma^0 y^1(t)\big\}\tfrac{(\lambda-1)c(t)}{(1-\mu)y^1(t)} + \lambda - c(t)\Big]. \end{aligned} \tag{26}$$

Substituting these equations into $\dot{y}^i(t)/y^i(t) = I^i(t)\mu - n$, we obtain

$$\begin{aligned} \tfrac{\dot{y}^0(t)}{y^0(t)} &= \tfrac{\mu}{\lambda(y^0(t)+y^1(t))}\Big[\big\{\gamma^0 y^1(t) - \gamma^1 y^0(t)\big\}\tfrac{(\lambda-1)c(t)}{(1-\mu)y^0(t)} + \lambda - c(t)\Big] - n, \\ \tfrac{\dot{y}^1(t)}{y^1(t)} &= \tfrac{\mu}{\lambda(y^0(t)+y^1(t))}\Big[\big\{\gamma^1 y^0(t) - \gamma^0 y^1(t)\big\}\tfrac{(\lambda-1)c(t)}{(1-\mu)y^1(t)} + \lambda - c(t)\Big] - n. \end{aligned} \tag{27}$$

We define $z(t)$ as the sum of adjusted difficulty of R&D across sectors where R&D is conducted:

$$\begin{aligned} z(t) &= y^0(t) + y^1(t) \quad \text{when} \quad I^0(t) > 0, I^1(t) > 0, \\ z(t) &= y^0(t) \qquad\qquad \text{when} \quad I^0(t) > 0, I^1(t) = 0; \end{aligned} \tag{28}$$

then, $\dot{z}(t) = \dot{y}^0(t) + \dot{y}^1(t)$ when R&D is conducted in both sectors.[14] Using (27), we obtain

$$\dot{z}(t) = \frac{(\lambda - c(t))\mu}{\lambda} - z(t)n. \tag{29}$$

[14]Note that $\dot{z}(t) = \dot{y}^0(t)$ when R&D is conducted only in sector 0. Therefore, we replace $y^0(t)$ with $z(t)$, and $\dot{y}^0(t)$ with $\dot{z}(t)$ in figure 1.

Substituting $v^i(t) = y^i(t)L(t)$ and (26) into (10), and solving for r(t), we obtain $r(t) = \frac{1}{\lambda z(t)}[(\lambda-\mu)c(t)-(1-\mu)\lambda]$. Substituting this equation into (4), we obtain

$$\dot{c}(t) = \frac{1}{\lambda z(t)}[(\lambda-\mu)(c(t))^2 - (1-\mu)\lambda c(t)] - \rho c(t). \tag{30}$$

This economy's dynamics are described by (29) and (30) in (z, c) space, seen in the upper part of figure 2. The vertical axis represents consumption per capita, c, which is a jump variable, and the horizontal axis represents the sum of adjusted difficulty of R&D across sectors, z, which is a state variable. The upper part of figure 2 is similar to figure 1, but there are two differences between the figures. First, the horizontal axis of figure 1 represents $y^0(t)$, but that of figure 2 represents z(t).[15] Second, the slope and the intercept of the $\dot{c}(t) = 0$ curve of figure 1 are greater than those of figure 2.[16]

From the upper part of figure 2, we can examine the macro variables, including consumption per capita and the sum of stock prices. However, we are also interested in each sector's variables, including the incumbents' stock prices in sector i and the industry-wide instantaneous probability of successful R&D in sector i. Define $Y(t) \equiv y^1(t)/y^0(t)$ as the ratio of adjusted difficulty of R&D in sector 1 to that of sector 0. From (27), we obtain

$$\dot{Y}(t) = \frac{c(t)\mu(\lambda-1)\gamma^0}{\lambda(1-\mu)y^0(t)}\left\{\frac{\gamma^1}{\gamma^0} - Y(t)\right\}. \tag{31}$$

The dynamics of $Y(t)$, which is a state variable, are illustrated in the lower part of figure 2. The vertical axis represents the adjusted difficulty of R&D in sector 1, y^1, and the horizontal axis represents the adjusted difficulty of R&D in sector 0, y^0. Because $Y(t)$ is the ratio of $y^1(t)$ to $y^0(t)$, the slope of the line connecting the origin and any point in $(y^0(t), y^1(t))$ space represents $Y(t)$. From (31), the $\dot{Y}(t) = 0$ curve is $Y(t) = \frac{\gamma^1}{\gamma^0}$. That is, the ratio of adjusted difficulty of R&D in sector 1 to that of sector 0 is equal to the ratio of market share of sector 1 to that of sector 0. When $Y(t)$ is smaller (greater) than $\frac{\gamma^1}{\gamma^0}$, $\dot{Y}(t) > 0$ ($\dot{Y}(t) < 0$). That is, when the ratio of adjusted difficulty of R&D is smaller (greater) than the ratio of market share at time t, the ratio of adjusted difficulty of R&D increases (decreases) at time t (i.e., industry-wide instantaneous probability of successful R&D in sector 1 is greater (smaller) than that for sector 0). This is because when $Y(t) < \frac{\gamma^1}{\gamma^0}$ ($Y(t) > \frac{\gamma^1}{\gamma^0}$), sector 1 is more (less) attractive than sector 0 because of the relatively high (low) probability of successful R&D or because of the relatively high (low) revenue from becoming the leader, and so more R&D firms are willing to conduct investigations in sector 1 (0). This process continues until the ratio of adjusted difficulty of R&D is equal to the

[15] However, $y^0(t)$ can be replaced with $z(t)$ and $\dot{y}^0(t)$ with $\dot{z}(t)$ in figure 1 from (28).
[16] Note that $\lambda > 1$.

ratio of market share.

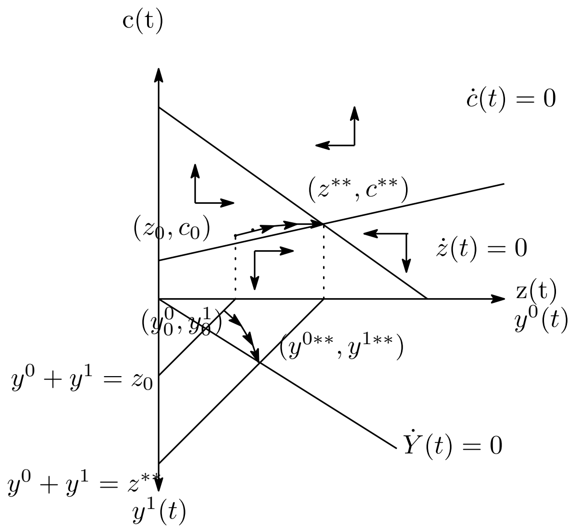

Figure 2. Phase diagram in the case where $I^0(t) > 0$ and $I^1(t) > 0$.

We give an example to promote better understanding of this dynamics. Suppose that the economy is initially at (z_0, c_0) in (z,c) space and at (y_0^0, y_0^1) in (y^0, y^1) space of figure 2. Because $z_0 = y_0^0 + y_0^1$ when $I^0(t) > 0$ and $I^1(t) > 0$, (y_0^0, y_0^1) is on the straight line that passes through the point $(z_0, 0)$ and has a slope of forty-five degrees. Because (z_0, c_0) is to the left of the $\dot{z}(t) = 0$ curve and the $\dot{c}(t) = 0$ curve, the economy moves northeast in (z,c) space. Because (y_0^0, y_0^1) is to the right of the $\dot{Y}(t) = 0$ curve, $Y(t)$ increases. Thus, the economy moves to the $\dot{Y}(t) = 0$ curve satisfying $y^0(t) + y^1(t) = z(t)$ in (y^0, y^1) space, as depicted in figure 2. In the long run, because $z(t)$ converges to z^{**}, $(y^0(t), y^1(t))$ converges to (y^{0**}, y^{1**}), which is the intersection of the $\dot{Y}(t) = 0$ curve and the $y^0(t) + y^1(t) = z^{**}$ curve. "**" denotes the steady-state equilibrium level in which $I^0(t) > 0$ and $I^1(t) > 0$.

Next we find the value of the balanced-growth equilibrium.[17] Because the balanced-growth equilibrium is determined by the intersection of the $\dot{z}(t) = 0$ curve and the $\dot{c}(t) = 0$ curve, from (29) and (30) we obtain

$$z^{**} = \frac{\mu(\lambda - 1)}{\mu\rho - \mu n + n\lambda} \tag{32}$$

and

$$c^{**} = \frac{\lambda(\mu\rho - \mu n + n)}{\mu\rho - \mu n + n\lambda}. \tag{33}$$

From (18) and (32), we confirm that $z^{**} > y^{0*} = z^*$, and from (19) and (33), we confirm that $c^{**} < c^*$.[18] That is, the proportion of workers in R&D (manufacturing) is greater (smaller) when $I^0(t) > 0, I^1(t) > 0$ than

[17] The equilibrium is saddle-path stable.
[18] Because $\lambda > 1$, $\gamma^0\lambda + 1 - \gamma^0 < \lambda$.

when $I^0(t) > 0, I^1(t) = 0$ from (13). Because c^{**} is also constant in the balanced-growth equilibrium under $I^0(t) > 0, I^1(t) > 0$, we obtain $r^{**} = \rho$ from (4). From (31), we obtain

$$Y^{**} = \frac{y^{1**}}{y^{0**}} = \frac{\gamma^1}{\gamma^0}. \tag{34}$$

Substituting (34) into (32), we obtain

$$y^{i**} = \frac{\gamma^i \mu(\lambda - 1)}{\mu\rho - \mu n + \lambda n} \quad \text{for} \quad i = 0, 1. \tag{35}$$

From (18) and (35), we confirm $\mathrm{y}^{0**} \leq \mathrm{y}^{0*}$. Substituting (33) and (35) into (26), we obtain

$$I^{i**} = \frac{n}{\mu} \quad \text{for} \quad i = 0, 1, \tag{36}$$

and from (15), we obtain

$$g^{**} = (\ln \lambda)\frac{n}{\mu}. \tag{37}$$

From (21) and (37), we confirm $g^{**} > g^*$.[19] That is, the growth rate of productivity is greater when $I^0(t) > 0, I^1(t) > 0$ than when $I^0(t) > 0, I^1(t) = 0$, because the proportion of workers engaging in R&D is greater when $I^0(t) > 0, I^1(t) > 0$. Because $v^i(t) = y^i(t)L(t), i = 0, 1$ from (9), we obtain

$$v^{i**}(t) = \frac{\gamma^i \mu(\lambda - 1)L(t)}{\mu\rho - \mu n + \lambda n}. \tag{38}$$

From (22) and (38), we confirm $v^{0**}(t) < v^{0*}(t)$. From (23) and (38), we confirm $v^{1**}(t) < v^{1*}(t)$. That is, the equilibrium value of the incumbents' stock price in both sectors is greater when $I^0(t) > 0, I^1(t) = 0$ than when $I^0(t) > 0, I^1(t) > 0$. From (11), we obtain

$$V^{**}(t) = \hat{V}^{**}(t) = \frac{\mu(\lambda - 1)L(t)}{\mu\rho - \mu n + \lambda n}. \tag{39}$$

From (24) and (39), we obtain $V^{**}(t) < V^*(t)$. From (25) and (39), we obtain $\hat{V}^*(t) < \hat{V}^{**}(t)$.

[19] From (21) and (37), we confirm that the growth rate of productivity is never influenced by the efficiency of R&D, A^i. This result is different from that in Boucekkine and de la Croix (2003), who state that the change in the efficiency of R&D affects the long-run growth rate.

4 Comparative Dynamics

In this section, we analyze the transition dynamics caused by an exogenous drastic technological breakthrough, which leads to the emergence of the new sector. The process of the emergence of a new technology is classified into three stages. Stage 1 is prior to the emergence of the new technology. In stage 1, there is only the old sector, sector 0 (i.e., $\gamma^0 = 1$). In this stage, households can only consume products of sector 0 and both manufacturing and R&D are carried out only in sector 0. When the economy is in stage 1, an unexpected major technological breakthrough happens and a new sector, sector 1, emerges, which we define as the point at which the economy enters stage 2. Stage 2 follows the emergence of a new technology, but the new technology is so new that R&D firms are not yet willing to carry out R&D in the new sector. In stage 2, $I^0(t) > 0$ and $I^1(t) = 0$ (as discussed in section 3.1). In this stage, households consume the products of both sectors 0 and 1 and manufacturing is carried out in both sectors; however, R&D is conducted only in sector 0. When the economy is in stage 2, unexpected major technological progress happens (cost of R&D falls drastically), which we define as the point at which the economy enters stage 3. By stage 3, the new technology have prevailed. In stage 3, $I^0(t) > 0$ and $I^1(t) > 0$ (as discussed in section 3.2). In this stage, households consume products of both sectors and manufacturing and R&D are carried out in both sectors. This section analyzes each stage in more detail.

4.1 Stage 1: Before the emergence of new technology

Before the emergence of a new technology, the economy is in a balanced-growth equilibrium. [20] The economy of stage 1 can be characterized by the present model with $\gamma^0 = 1$, which is identical with Segerstrom's (1998) model. In stage 1, the main endogenous variables are $c(t) = \frac{\lambda(\mu\rho - \mu n + n)}{\mu\rho - \mu n + \lambda n}$, $y^0(t) = z(t) = \frac{\mu(\lambda-1)}{\mu\rho - \mu n + \lambda n}$, $I^0(t) = \frac{n}{\mu}$, $g(t) = (\ln\lambda)\frac{n}{\mu}$, and $V(t) = \hat{V}(t) = \frac{\mu(\lambda-1)L(t)}{\mu\rho - \mu n + \lambda n}$, from (33), (35), (36), (37), and (39). This is point A in $(z(t), c(t))$ space and point A' in $(y^0(t), y^1(t))$ space of figure 3.[21] Point A is the intersection of the $\dot{z}(t) = 0$ curve: $\frac{(\lambda - c(t))\mu}{\lambda} - z(t)n = 0$ and the $\dot{c}(t) = 0$ curve: $\frac{1}{\lambda z(t)}[(\lambda-\mu)(c(t))^2 - (1-\mu)\lambda c(t)] - \rho c(t) = 0$.[22] Point A' is the intersection of the horizontal axis and the $z(t) = \frac{\mu(\lambda-1)}{(\mu\rho - \mu n + \lambda n)}$ line.

[20] This assumption is based on the fact that major technological breakthroughs are rare.
[21] $y^1(t) = 0$ in stage 1 because sector 1 does not yet exist.
[22] Note that when $\gamma^0 = 1$, the $\dot{z}(t) = 0$ curve is identical to the $\dot{y}^0(t) = 0$ curve.

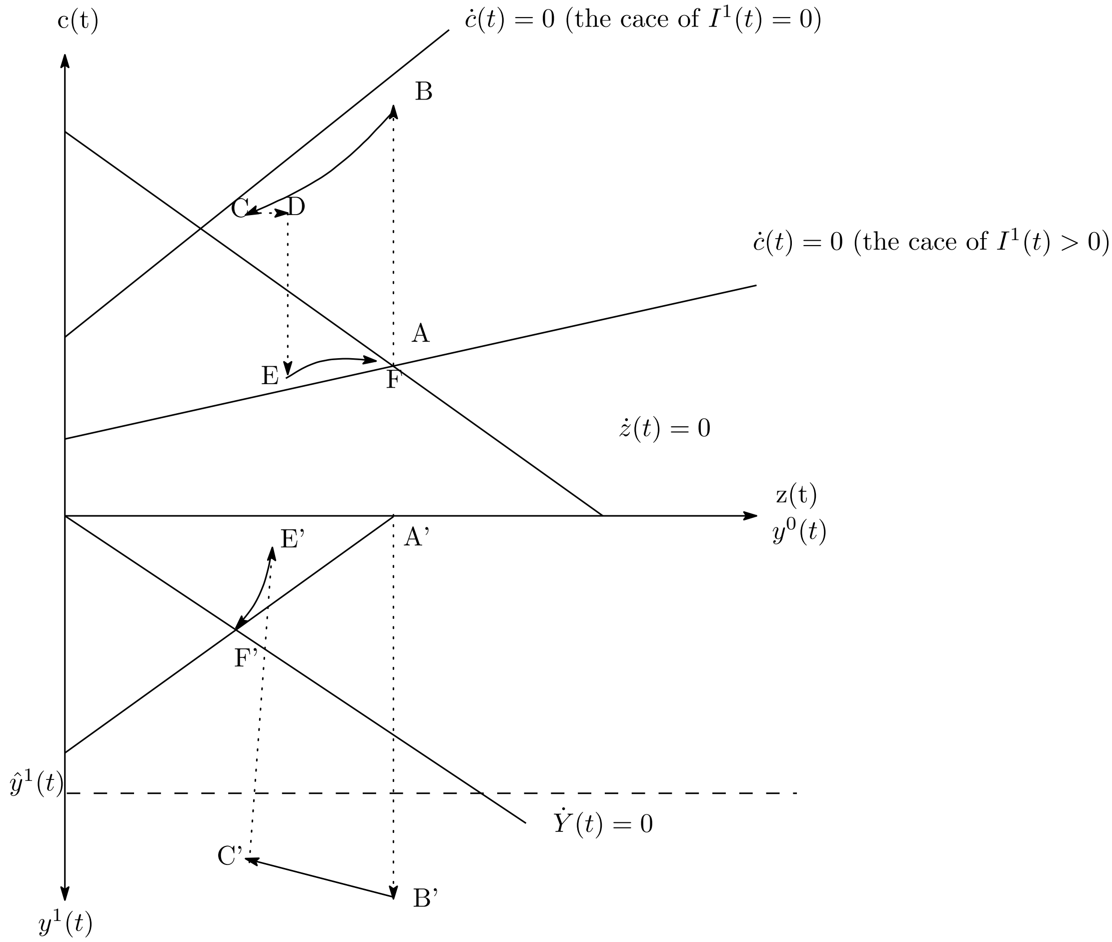

Figure 3. The movement of key variables in a phase diagram.

Figure 4 depicts the movements of the variables. The vertical axis represents the endogenous variables and the horizontal axis represents time, t. The panel on the left depicts the movements of key variables, the middle panel depicts the movements of variables related to the growth rate of productivity, and the panel on the right depicts the movements of variables related to the stock price.

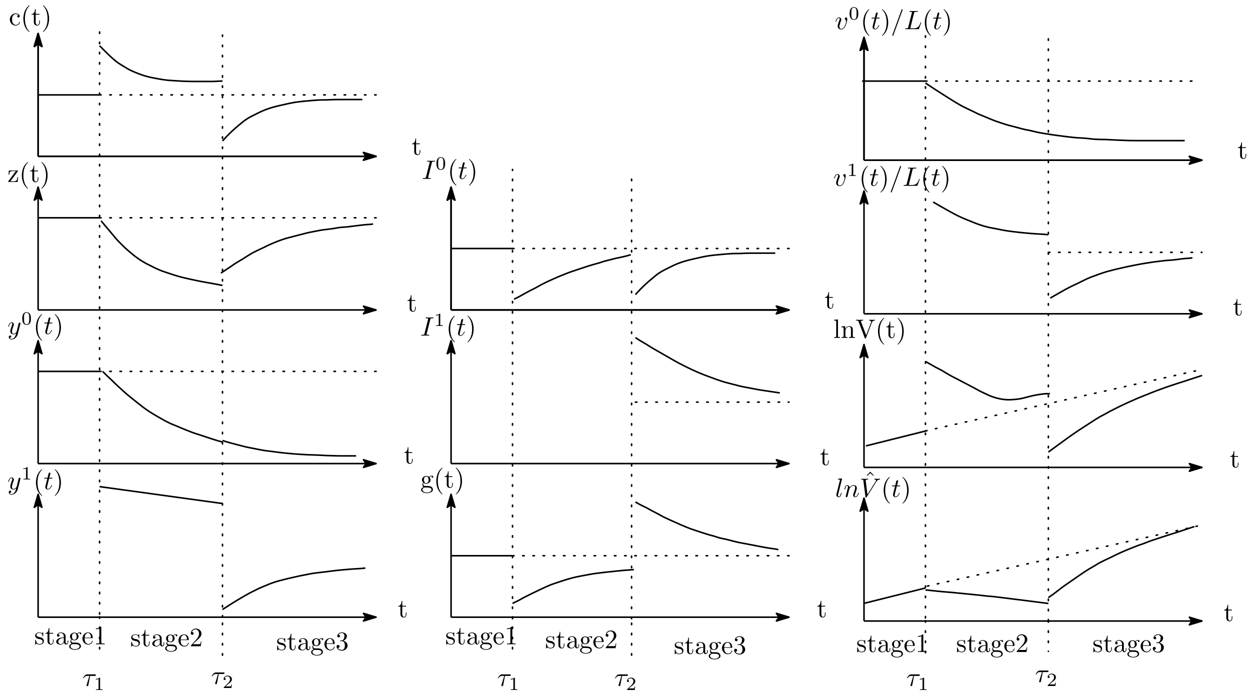

Figure 4. Summary of the movement of key and economically important variables.

4.2 Stage 2: Before the start of R&D in the new sector

Stage 2 covers the period between the time at which a new technology emerges, τ_1, and the beginning of R&D in the new sector. To guarantee the existence of this period, we assume that the cost of R&D in the new sector is too high for R&D firms for several periods after the new technology emerges, τ_2.[23] This assumption is based on the fact that the costs of innovative activity in technologies such as electricity and computing were very high for some time following the initial emergence of these new technologies, which discouraged people from using these new technologies; it is also based on the idea that workers engaging in R&D do not know enough about the new technology to be able to improve the quality of products based on the new technology for that interval of time after the breakthrough.[24] Here, this assumption is reflected in the high value of $H^1(t)$, which means a high value of adjusted difficulty of R&D in sector 1, $y^1(t)$. $\hat{y}^i(t)$ is defined as the critical value for the conduct of R&D sector i, for which $\hat{y}^i(t) = \frac{v^i(t)}{L(t)}$ from (9).[25][26] If $y^i(t) \leq \hat{y}^i(t)$, then R&D is conducted in sector i at time t; otherwise, R&D is not conducted in sector i. In stage 2, the inequality $y^1(t) > \hat{y}^1(t)$ is always satisfied because of the high value of $H^1(t)$. Stage 2 is where $I^0(t) > 0$ and $I^1(t) = 0$, as discussed in section 3.1. In stage 2, the $\dot{c}(t) = 0$ curve is expressed as (17), and the $\dot{z}(t) = 0$ curve is expressed as (16) with $z(t) = y^0(t)$.[27] Both the intercept and the

[23] If the cost of R&D is not too high, the economy goes directly from stage 1 to stage 3.

[24] See Jovanovic and Rousseau (2005).

[25] The critical value $\hat{y}^i(t)$ depends on time t because leaders' profits, interest rate and economy size vary over time.

[26] If R&D is conducted in sector i, the adjusted difficulty of R&D in sector i is equal to the critical value because of the free entry condition.

[27] Note that $I^1(t) = 0$ in stage 2 and (28).

slope of the $\dot{c}(t) = 0$ curve are greater in stage 2 than those in stage 1, whereas the $\dot{z}(t) = 0$ curve does not change between stages.

Figure 3 allows us to analyze the transition dynamics. At time τ_1, the economy is at point A in (z,c) space and at point A' in (y^0, y^1) space. As seen in the upper part of figure 3, at time τ_1, the $\dot{c}(t) = 0$ curve shifts counterclockwise, and then the intersection of the $\dot{c}(t) = 0$ curve and the $\dot{z}(t) = 0$ curve shifts to the left. Because point A is below the $\dot{c}(t) = 0$ curve in stage 2, point A is no longer the equilibrium. To remain on the saddle path, $c(\tau_1)$ has to shift upward, and then the economy should shift from point A to point B. Because point B is below the $\dot{c}(t) = 0$ curve and above the $\dot{z}(t) = 0$ curve in stage 2, $c(t)$ and $z(t)$ gradually fall to the new equilibrium, which is the intersection of $c(t) = -\frac{\lambda n}{\mu} z(t) + \lambda$ and $c(t) = \frac{\rho \lambda y^0(t) + (1-\mu)\lambda}{(\gamma^0 \lambda + \gamma^1 - \mu)}$.[28]

We turn now to the lower part of figure 3. When the new technology emerges, y^1 takes a value such that $y^1(\tau_1) > \hat{y}^1(\tau_1)$. The economy then shifts from point A' to point B'. Because $y^0(t) = z(t)$ throughout stage 2, $y^0(t)$ decreases together with $z(t)$. Because $I^1(t) = 0$ in stage 2, $\dot{y}^1(t)/y^1(t) = -n$. Then $y^0(t)$ and $y^1(t)$ decrease along arrow B'C'.[29] These stage 2 movements can be seen in the left-hand panel of figure 4.

Lemma 4.1. *Summary of movement of key variables in stage 2*

- *Consumption per capita, c, shifts upward at the emergence of the new technology, but decreases throughout stage 2 without ever falling below its level in stage 1.*
- *The adjusted difficulty of R&D in sector 0, $y^0 = z$, remains unchanged at the emergence of the new technology, but decreases throughout stage 2.*

In the above analysis, we confirm the movement of key variables in stage 2. Our next task is to analyze the movement of economically important variables such as the growth rate of productivity and stock price of old-sector incumbents in stage 2.

Because the equilibrium value of c in stage 2 is greater than that in stage 1 and c(t) is decreasing throughout stage 2, then consumption per capita in stage 2 is always strictly greater than that in stage 1. Therefore, the proportion of labor in R&D (manufacturing) in stage 2 is always strictly smaller (greater) than in stage 1.[30]

This result is related to the changes in the adjusted difficulty of R&D in sector 0, $y^0(t)$, in stage 2. Because $y^0(t)$ is decreasing and the size of the decrement of $y^0(t)$ decreases as it approaches equilibrium, $I^0(t) < \frac{n}{\mu}$ always holds in

[28] Because $\dot{c}(t) < 0$ and the size of the decrement of c(t) decreases as it approaches the stage 2 equilibrium, $r(t) < \rho$ is always satisfied and $r(t)$ is increasing throughout stage 2 from (4).

[29] In stage 2, $\frac{\dot{\hat{y}}^1(t)}{\hat{y}^1(t)} = \frac{\dot{v}^1(t)}{v^1(t)} - n$. $\frac{\dot{v}^1(t)}{v^1(t)} < 0$ is also satisfied, which is discussed later. Thus $y^1(t)$ is never less than $\hat{y}^1(t)$ in stage 2 as long as the cost of R&D, $H^1(t)$, remains constant.

[30] Dividing (13) by $L(t)$, we obtain $\frac{X^0(t)H^0(t)I^0(t)}{A^0 L(t)} + \frac{X^1(t)H^1(t)I^1(t)}{A^1 L(t)} + \frac{c(t)}{\lambda} = 1$. Thus, the proportion of labor in manufacturing and R&D is $\frac{c(t)}{\lambda}$ and $1 - \frac{c(t)}{\lambda}$ respectively.

stage 2 and $I^0(t)$ approaches $I^{0*} = \frac{n}{\mu}$ from below throughout stage 2. Then, on the only feasible path, $I^0(t)$ shifts downward at the emergence of the new technology and increases throughout stage 2. That is, the instantaneous probability of successful R&D in sector 0 in stage 2 is always strictly less than in stage 1, but increases throughout stage 2. We interpret this result intuitively as follows: at time τ_1, the profits of leaders in sector 0 fall because sector 1 takes market share from sector 0, which means that the rewards for successful R&D in sector 0 decrease. The number of R&D firms trying to enter sector 0 decreases, which means a decrease in the amount of labor engaged in R&D in sector 0. The industry-wide instantaneous probability of successful R&D in sector 0 falls, which is a downshift of $I^0(t)$ at time τ_1. However a lower value of $I^0(t)$ leads to a decrease in the adjusted difficulty of R&D in sector 0, which renders innovative activity more profitable in sector 0.[31] The number of R&D firms trying to enter sector 0 increases throughout stage 2, which means that the industry-wide instantaneous probability of successful R&D in sector 0 increases throughout stage 2.[32] On the other hand, R&D is not conducted in sector 1 in stage 2 because of the high cost of R&D. Thus, from (15), we obtain the important result that the growth rate of productivity is strictly less throughout stage 2 than in stage 1. This result is consistent with the stylized fact known as "the productivity slowdown," as depicted in the middle panel of figure 4.

In this model, the productivity slowdown occurs because of the following. The first cause is the decrease in the industry-wide instantaneous probability of successful R&D in sector 0. Sector 0 leaders' profits decrease because sector 1 takes market share from sector 0, which decreases the incentive to become the leader in sector 0. This decreases the amount of labor in R&D in sector 0, which in turn decreases the instantaneous probability of successful R&D in sector 0.[33] The second cause is the absence of R&D in sector 1, a consequence of the assumption that the cost of R&D is too high for several periods following the emergence of the new technology. From the evidence in Jovanovic and Rousseau (2005), this assumption seems plausible. However, it may be that the initial cost of R&D in the new sector is not high enough relative to its market share, which means that the economy goes directly from stage 1 to stage 3, with no stage 2. This possibility suggests that whether a technological breakthrough causes a productivity slowdown depends on the initial cost of R&D and the market share of the new sector.

Because $y^0(t)$ decreases over stage 2, $v^0(t)$ also decreases over stage 2 from (9). $v^1(t)$ also decreases over stage 2.[34] Thus, both $V(t)$ and $\hat{V}(t)$ decrease in stage 2 from (11) and (12). Because $y^0(t)$ is not changed by the

[31] Although the accumulation of R&D in sector 0 continues while $I^0(t) > 0$, the population grows faster than R&D accumulates in stage 2. Thus, the adjusted difficulty of R&D in sector 0 decreases.

[32] Note that because $y^0(t)$ decreases throughout stage 2, $I^0(t)$ never exceeds $\frac{n}{\mu}$.

[33] From (7) and (10), we see that $I^0(t)$ must decrease in order to satisfy (7) with equality.

[34] From (10) and $I^1(t) = 0$, we obtain $\frac{\dot{v}^1(t)}{v^1(t)} = r(t) - \gamma^1(\frac{\lambda-1}{\lambda})\frac{c(t)}{v^1(t)}L(t)$. This equation shows that $\dot{v}^1(t)$ increases with $v^1(t)$, $r(t)$ and decreases with $c(t)$. In stage 2, $r(t)$ increases and $c(t)$ decreases. Thus, $\frac{\dot{v}^1(t)}{v^1(t)}$ is increases throughout stage 2 as long as $v^1(t)$ does not decrease. Then, if $\frac{\dot{v}^1(t)}{v^1(t)} > 0$, $v^1(t)$ goes to infinity and never converges to the equilibrium. Therefore, $\frac{\dot{v}^1(t)}{v^1(t)} < 0$ must always hold in stage 2 if convergence is to occur. Therefore, the only feasible path of $v^1(t)$ is that $v^1(t)$ shifts upward at time τ_1 and decreases

emergence of a new technology, neither $v^0(t)$ nor $\hat{V}(t)$ is changed by that event. Assuming that sector 1 leaders' stocks are listed at the moment of the technology's emergence, $v^1(t)$ is evaluated at time τ_1. Then $V(t)$ shifts upward to the same degree as $v^1(\tau_1)$ at time τ_1. These movements in stage 2 are depicted in the right-hand panel of figure 4.

What happens to $v^0(t)$ in stage 2? At time τ_1, the new sector emerges, and the old sector loses market share to the new sector, which causes $\pi^0(t)$ to decline. From (10), $v^i(t)$ is an increasing function of $\pi(t)^i$ and a decreasing function of $I^i(t)$. Thus the stock price of incumbents in the old sector decreases unless the replacement risk, $I^0(t)$, changes. However, R&D activity, $I^0(t)$, also decreases to satisfy the free entry condition, then $v^0(t)$ does not shift downward at τ_1.[35] As seen above, $I^0(t) < \frac{n}{\mu}$ always holds in stage 2, in which case $\dot{y}(t)^0 < 0$ also holds.[36] This means that the adjusted difficulty of R&D in sector 0 decreases throughout stage 2, which leads to more R&D activity. Then $I^0(t)$ increases throughout stage 2. Because $\pi^0(t)$ remains small in stage 2, $v^0(t)$ decreases because of an increase in the replacement risk, $I^0(t)$. We note also that because $c(t)$ decreases and $r(t)$ increases throughout stage 2, the decline in $v^0(t)$ and $v^1(t)$ will be exacerbated.[37] As shown in the last paragraph, both $V(t)$ and $\hat{V}(t)$ decrease throughout stage 2, which is consistent with the empirical data. This shows that the aggregate stock price decreases regardless of the timing of new firms' stock listing. Thus the decline of $v^0(t)$, which is caused by the decline of the market share of sector 0, plays an important role in the decline of the aggregate stock price.

Proposition 4.1. *Summary of the movements of economically important variables in stage 2, in which R&D is not conducted in sector 1 after the emergence of a new technology.*

- *The growth rate of productivity, g, shifts downward at the moment of the emergence of the new technology, and increases throughout stage 2; however, it never goes over the level of stage 1.*
- *The stock price of incumbents in the old sector, v^0, decreases throughout stage 2.*
- *The sum of the stock price indexes, V and $\hat{V}$, decrease throughout stage 2.*

4.3 Stage 3: After R&D in the new sector begins

Third, we analyze stage 3, which is the stage after R&D begins in the new sector. We refer to the moment when R&D firms in new sector begin to carry out R&D as τ_2. When the economy is in stage 2, unexpected drastic technological progress occurs (cost of R&D falls drastically), which forces the economy to move from stage 2 to

throughout stage 2.

[35]From (9), $v^i(t)$ is a function of the state variable, $X(t)^i$, and exogenous variables, $H^i(t)$ and A^i, when $I^i(t) > 0$. Thus $v^i(t)$ does not shift unless the exogenous variables change.

[36]Note that a differential equation of $y(t)^i$ is $\dot{y}^i(t)/y^i(t) = I^i(t)\mu - n$

[37]From equation (10), $v^i(t)$ is an increasing function of $c(t)$ and a decreasing function of $r(t)$.

stage 3. In our model, this phenomenon reflects an unexpected sufficient drop in $H^1(t)$, which causes an unexpected drop in $y^1(t)$ satisfying $y^1(t) \leq \hat{y}^1(t)$ and $y^1(t)/y^0(t) < \gamma^1/\gamma^0$. Although this assumption seems to be restrictive, we obtain similar results by extending this model to the model where the cost of R&D decreases due to learning by doing.[38] However, as this expansion is somewhat complex and the results do not change dramatically, for now we regard the cost of R&D as an exogenous variable.

Because of the unexpected exogenous shock, $y^1(t)$ does not become greater than $\hat{y}^1(t)$; hence, we express the economy of stage 3 as the case where $I^0(t) > 0$ and $I^1(t) > 0$, which is discussed in section 3.2. Then, in stage 3, the $\dot{c}(t) = 0$ curve is expressed as (30), and the $\dot{z}(t) = 0$ curve is expressed as (29). Both the intercept and the slope of $\dot{c}(t) = 0$ curve are smaller in stage 3 than those in stage 2, whereas the $\dot{z}(t) = 0$ curve is identical in both stages. On the other hand, the $\dot{c}(t) = 0$ curve and the $\dot{z}(t) = 0$ curve of stage 3 and those of stage 1 are completely identical.

We then analyze the economic movement by using figure 3 again. At time τ_2, we consider that the economy is at point C in (z,c) space and at point C' in (y^0, y^1) space (i.e., the unexpected shock occurs when economy is at point C and point C'). First, we examine the upper part of figure 3. At time τ_2, $\dot{c}(t) = 0$ curve shifts clockwise, and the intersection of the $\dot{c}(t) = 0$ curve and the $\dot{z}(t) = 0$ curve shifts to the right. Because $I^0(t) > 0$ and $I^1(t) > 0$ by the drastic technological progress, $z(t)$ should be evaluated as $z(t) = y^0(t) + y^1(t)$ (c.f., $z(t)$ is evaluated as $z(t) = y^0(t)$ in stage 2). Then $z(t)$ shifts to the right to the same degree as $y^1(\tau_2)$, and the economy shifts from point C to point D. Because point D is above the $\dot{c}(t) = 0$ curve and above the intersection of the $\dot{c}(t) = 0$ curve and the $\dot{z}(t) = 0$ curve of stage 3, then point D is no longer on the saddle path. To return to the saddle path, $c(\tau_2)$ has to shift downward, and then the economy should shift from point D to point E. Because point E is above the $\dot{c}(t) = 0$ curve and below the $\dot{z}(t) = 0$ curve, $c(t)$ and $z(t)$ increase gradually to the new equilibrium point F, which is the intersection of $c(t) = -\frac{\lambda n}{\mu} z(t) + \lambda$ and $c(t) = \frac{\rho\lambda z(t) + (1-\mu)\lambda}{\lambda - \mu}$.[39]

We next examine the lower part of figure 3. At the moment that the unexpected exogenous shock occurs, the adjusted difficulty of sector 1 drops dramatically, which satisfies $y^1(\tau_2) \leq \hat{y}^1(\tau_2)$ and $y^1(\tau_2)/y^0(\tau_2) < \gamma^1/\gamma^0$. The economy then shifts from point C' to point E'. In stage 3, the dynamics of y^0 and y^1 are determined by (31). Because we assumed that $y^1(t)/y^0(t) < \gamma^1/\gamma^0$, the ratio of adjusted difficulty of R&D of sector 1 to that of sector 0, $Y(t)$, increases in stage 3. Note that from (18) and (35), $y^{0**} < y^{0*}$ and y^0 is a state variable; therefore, y^0

[38] We define the differential equation of $H^1(t)$ as $\begin{array}{lll} \dot{H}^1(t) = -\nu^1 \frac{A^1}{X^1(t)} L^{p1} & when & H^1(t) > \bar{H}(t) \\ \dot{H}^1(t) = 0 & when & H^1(t) = \bar{H}(t) \end{array}$, where $\bar{H}(t)$ and ν^1 are parameters and L^{p1} is the amount of labor engaging in manufacturing in sector 1. Then, the cost of R&D, $H^1(t)$, becomes an endogenous variable and the regime switch from stage 2 to stage 3 is determined endogenously.

[39] Because $\dot{c}(t) > 0$ and the size of increment of c(t) decreases when approaching the equilibrium in stage 3, $r(t) > \rho$ is always satisfied and $r(t)$ decreases throughout stage 3 from (4).

decreases in stage 3. Because $z(t)$ increases and $y^0(t)$ decreases in stage 3, $y^1(t)$ must increase in stage 3. Then $y^0(t)$ and $y^1(t)$ move to F', which is on the $\dot{Y}(t) = 0$ curve satisfying $y^{0**} + y^{1**} = z^{**}$. These movements are depicted in stage 3 of the left-hand panel of figure 4.

Lemma 4.2. *Summary of movement of key variables in stage 3*

- *Consumption per capita, c, shifts downward the moment R&D in the new sector begins, but increases throughout stage 3. However, it is never above the level in stage 1.*
- *The sum of the adjusted difficulty of R&D across sectors, z, shifts upward the moment R&D in the new sector begins, and increases throughout stage 3. However, it is never above the level in stage 1.*
- *The adjusted difficulty of R&D in sector 0, y^0, remains unchanged the moment R&D in the new sector begins, but decreases throughout stage 3.*
- *The adjusted difficulty of R&D in sector 1, y^1, shifts downward the moment R&D in the new sector begins, but increases throughout stage 3.*

In the above analysis, we confirm the movement of key variables in stage 3. Next, we analyze the movement of economically important variables such as the growth rate of productivity, the value of old-sector leaders' stock and the sum of stock prices in stage 3.

Because the equilibrium value of consumption per capita in stage 3 is smaller than that in stage 2 and c(t) increases throughout stage 3, consumption per capita in stage 3 is always strictly smaller than that in stage 2. Therefore, the share of labor engaging in R&D (manufacturing) in stage 3 is always strictly greater (smaller) than in stage 2 from (13). Although the total share of labor engaging in R&D is greater than that in stage 2, because $\dot{y}^0(t) < 0$, the industry-wide instantaneous probability of successful R&D in sector 0 is always less than the equilibrium value, $I^{0**} = n/\mu$, but increases throughout stage 3 to $I^{0**} = n/\mu$. Although the impact of $I^0(t)$ at time τ_2 is ambiguous, if stage 2 is long enough (i.e., $I^{0*} \simeq n/\mu$), then $I^0(\tau_2)$ shifts downward at the moment that R&D begins in sector 1. On the other hand, because $\dot{y}^1(t) > 0$ and the size of the increment to $y^1(t) > 0$ decreases throughout stage 3, the industry-wide instantaneous probability of successful R&D in sector 1 is always greater than the equilibrium value, $I^{1**} = n/\mu$, and decreases throughout stage 3 to $I^{1**} = n/\mu$. Then the only feasible path, $I^1(t)$, shifts upward precisely at τ_2 and decreases throughout stage 3. Therefore, $I^0(t) \leq I^1(t)$ is always satisfied throughout stage 3, which is consistent with $\dot{Y}(t) > 0$ throughout stage 3. Because $\dot{z}(t) > 0$, $\dot{Y}(t) > 0$ and $I^0(t) < I^1(t)$ throughout stage 3, $g(t) = \gamma^0 I^0(t) + \gamma^1 I^1(t) > n/\mu$ throughout stage 3 as depicted in the middle panel of figure 4 (see Appendix). This result is consistent with the empirical fact that the growth rate of productivity returns to a high level after a lapse of some years from the emergence of new technology.

This result has a simple explanation. When the economy arrives at stage 3, the cost of R&D in sector 1 falls dramatically. R&D firms in the new sector can then carry out R&D with a high probability of success (i.e., a low value of $X^1(t)$) and with a low cost of R&D, (i.e. a low value of $H^1(t)$), which induces a large volume of R&D activity by R&D firms in sector 1. Thus, the number of workers engaging in R&D activity in the new sector increases, which leads to a high industry-wide instantaneous probability of successful R&D in the new sector. By contrast, the number of workers engaging in R&D in sector 0 decreases because R&D firms in sector 1 take workers from R&D firms in sector 0, which reduces the industry-wide instantaneous probability of successful R&D in sector 0. However, the analysis in our appendix shows that the former effect dominates latter, and thus the growth rate of productivity is high throughout stage 3.

Because $y^0(t)$ ($y^1(t)$) decreases (increases) in stage 3, $v^0(t)$ ($v^1(t)$) also decreases (increases) in stage 3 from (7). Furthermore, because $z(t)$ increases in stage 3, the stock price indexes, $V(t)$ and $\hat{V}(t)$, also increase in stage 3 from (11), (12) and (28). Because $y^0(t)$ does not change at τ_2, $v^0(t)$ does not change at that point either. $v^1(t)$ in stage 2 is always higher than in stage 3 because new-sector leaders face no threat of replacement. In stage 3, these leaders face replacement because R&D commences in sector 1. Thus, the stock price of sector 1 leaders will fall as soon as R&D begins in sector 1. Therefore, $V(t)$ shifts downward at time τ_2. Consider instead the index $\hat{V}(t)$; because sector 1 leaders' stocks are listed at R&D in sector 1 begins (i.e., $v^1(t)$ is evaluated at time τ_2), then $\hat{V}(t)$ shifts upward to the same degree as $v^1(\tau_2)$ at time τ_2. These movements are shown in the right-hand panel of figure 4.

The reason $v^1(t)$ remains small and never returns to its stage 1 level is very simple: it is that $\pi^1(t)$ also remains small and never returns to its stage 1 level because of the emergence of the new sector. Then the cause of the increase in $V(t)$ and $\hat{V}(t)$ throughout stage 3 is the increase in $v^1(t)$ in stage 3. This comes mainly from the decrease in $I^1(t)$ throughout stage 3. As the new sector matures, that is, as product quality in the new sector improves, subsequent new-sector R&D becomes more difficult. This reduces the probability of successful R&D in the new sector (decreases the risk of new-sector leaders being replaced), which increases $v^1(t)$ throughout stage 3. In addition, the value of the increment of $v^1(t)$ relies in part on the results that $c(t)$ increases and $r(t)$ decreases throughout stage 3. Because the effect of this increment dominates the effect of the decline in $v^0(t)$, $V(t)$ and $\hat{V}(t)$ increase throughout stage 3.

Proposition 4.2. *Summary of movements of economically important variables in stage 3, where R&D is conducted* *in sector 1*

- *The growth rate of productivity, g, shifts upward when R&D in sector 1 begins, and decreases throughout stage 3. However, it never falls below the level in stage 1.*
- *The stock price of old-sector incumbents, v^1, remains small and never returns to the level in stage 1.*
- *The sum of the stock prices, V and $\hat{V}$, increases throughout stage 3.*

5 Conclusion

Using a typical R&D growth model, we examined the transition dynamics of a technological breakthrough. We showed that a technological breakthrough causes a productivity slowdown and a fall in the stock prices of existing firms. Both these results are consistent with events of the 1970s. Our model provides an explanation for these effects of technological breakthroughs and shows that productivity slowdown occurs if and only if the initial cost of R&D activity is high enough.

In this model, market share is exogenous and constant throughout the transition. However, the market share of a new sector can be small initially and grows throughout the transition. This problem is caused by our utility function having a unit elasticity of substitution between sectors. A utility function having any constant elasticity of substitution between sectors will solve this problem. This is an important direction for future work.

6 Appendix

6.1 A1

We show that the growth rate of productivity in stage 3 is greater than the equilibrium value. Because $\dot{z}(t) > 0$ in stage 3, and note that $\dot{y}^i(t)/y^i(t) = I^i(t)\mu - n$, the following holds:

$$
\begin{aligned}
&\dot{z}(t) > 0 \\
&\Leftrightarrow I^0(t)y^0(t)\mu - ny^0(t) + I^1(t)y^1(t)\mu - ny^1(t) > 0 \\
&\Leftrightarrow I^0(t)\tfrac{y^0(t)}{y^0(t)+y^1(t)} + I^1(t)\tfrac{y^1(t)}{y^0(t)+y^1(t)} > \tfrac{n}{\mu} \\
&\Leftrightarrow I^0(t)\tfrac{1}{1+Y(t)} + I^1(t)\tfrac{Y(t)}{1+Y(t)} > \tfrac{n}{\mu}.
\end{aligned}
\tag{40}
$$

Because $I^0(t) < I^1(t)$, then $I^0(t)\frac{1}{1+Y(t)} + I^1(t)\frac{Y(t)}{1+Y(t)}$ is an increasing function of $Y(t)$. From $\dot{Y}(t) > 0$, we obtain $Y(t) < \frac{\gamma^1}{\gamma^0}$. Then the following holds:

$$\begin{aligned}
&I^0(t)\frac{1}{1+Y(t)} + I^1(t)\frac{Y(t)}{1+Y(t)} \\
&< I^0(t)\frac{1}{1+\frac{\gamma^1}{\gamma^0}} + I^1(t)\frac{\frac{\gamma^1}{\gamma^0}}{1+\frac{\gamma^1}{\gamma^0}} \\
&= I^0(t)\frac{1}{\frac{\gamma^0+\gamma^1}{\gamma^0}} + I^1(t)\frac{\frac{\gamma^1}{\gamma^0}}{\frac{\gamma^0+\gamma^1}{\gamma^0}} \\
&= I^0(t)\gamma^0 + I^1(t)\gamma^1.
\end{aligned} \tag{41}$$

From (40) and (41), we obtain $g(t) = \gamma^0 I^0(t) + \gamma^1 I^1(t) > n/\mu$

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