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Habit Formation, Interest-Rate Control and Equilibrium Determinacy

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Abstract

We examine macroeconomic stability of a monetary economy with habit formation in consumption. We assume that monetary authority controls the rate of nominal interest in response to inflation and output gap. We show that in the presence of habit persistence not only active but also passive monetary policy can generate equilibrium determinacy under empirically plausible values of the elasticity of intertemporal substitution in felicity.

Keywords and Phrases: equilibrium determinacy, habit formation, Taylor rule, endogenous labor.

JEL Classification Numbers: E21, E52, O42

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1 Introduction

This paper introduces habit persistence in consumption into a cash-in-advance (CIA) model with interest-rate control. The purpose of this paper is to investigate whether the presence of habit formation may alter the stabilization effects of monetary policy.

The presence of habit persistence in consumption has been confirmed in a number of empirical studies and the theoretical effects of habit formation have been discussed extensively. [1] Several authors have already examined monetary dynamic models with habit formation. For example, Auray et. al. (2002 and 2005) introduce money via the CIA constraint into macrodynamic models in which real money holdings bind consumption. They show that a high degree of habit persistence may yield multiple equilibria. In contrast, Auray et. al. (2004) reveal that in a money-in-the-utility-function model, the presence of habit does not affect macroeconomic stability.

The foregoing studies on monetary dynamics with habit formation usually assume the traditional monetary policy in the money and growth literature, that is, the monetary authority fixes the expansion rate of nominal money supply. It is now well understood that the exogenous money supply rule does not precisely describe the central banks' behaviors in many countries. Rather, as emphasized by Taylor (1993), they control the nominal interest rate by observing inflation and output gap. Considering this fact, a large number of authors have examined stabilization effect of interest-rate control rules: see, for example, Benhabib et. al. (2001), Meng (2002) and others. However, these authors ignore habit persistence in consumption. Our central concern is to reconsider the stabilizing effect of interest-rate control in the presence of habit formation.

It is to be noted that Graham (2008) constructs a savers-and-spenders model of monetary economy with habit formation and interest-rate control. He uses a

[1] For example, see Abel (1990), Carroll et. al. (1997), Carroll (2000), Constantinides (1990), Fuhrer (2000), Smith (2002), and Weder (2000).

stochastic New Keynesian model that includes imperfect competition, sticky price adjustment as well as capital accumulation, and his research concern focuses on numerical experiments rather than analytical implications. In contrast, we use a simple model of competitive, production economy in order to consider equilibrium determinacy analytically. Our model is based on Meng (2002) who studies stabilization effect of interest-rate control in a CIA economy with variable labor supply. We generalize Meng's (2002) analysis in two aspects.

First, as was emphasized, we introduce habit persistence in consumption into the base model. When the implicit cost of habit accumulation which represents a negative effect on the utility is taken into consideration under household's optimization, passive monetary policy which lowers the real rate of interest with a higher inflation may easily create determinate equilibrium under plausible values of the elasticity of intertemporal substitution. This is in contrast to the finding that in models without habit formation determinacy under passive interest-rate control can hardly emerge.

Second, we assume that the central bank adjusts the nominal interest rate in response not only to the rate of inflation but also to output gap, while Meng (2002) assumes that the nominal interest rate depends on inflation alone. [2] Given our policy rule, a rise in the rate of inflation may lower a real interest rate, even when monetary policy is active in which the real rate of interest rises with a higher inflation. For the effect on output via habit formation combining with this type of interest-rate control, it becomes more difficult to hold determinacy under active policy and high elasticity of intertemporal substitution.

The rest of this paper is organized as follows. Section 2 sets up the base model. Section 3 examines equilibrium determinacy/indeterminacy conditions for the base model. This section also considers the case with outward-looking habit persistence

[2]Whether it is significant for macroeconomic stability depends on a structure of the economy. For example, Meng and Yip (2004) show that it does not affect equilibrium determinacy in the Ramsey model, while it has an influence on stability in an AK growth economy as shown in Fujisaki and Mino (2007).

where habit formation in consumption is external to each household. Section 4 presents numerical examples for alternative forms of utility functions. Section 5 concludes.

2 The Model

The representative household's optimization problem is

$$\max \quad \int_0^\infty u(c - \theta h, l) e^{-\rho t} dt, \quad 0 < \theta < 1, \quad \rho > 0, \tag{1}$$

subject to

$$\dot{a} = (R - \pi)a - Rm + wz - c - \tau, \tag{2}$$

$$\alpha c \leq m, \quad \alpha > 0, \tag{3}$$

$$\dot{h} = \beta(c - h), \quad \beta > 0, \tag{4}$$

where $a \equiv b + m$ real financial assets, b bonds, m real money holdings, c is consumption, ρ the time discount rate, l leisure, $n \equiv 1 - l$ labor, R the nominal interest rate, π the rate of inflation, w the real wage, τ lump taxes, h the stock of consumption habits, θ habit persistence, and β denotes the speed of habit adjustment. The instantaneous felicity function, $u(c - \theta h, l)$, is monotonically increasing and strictly concave in $c - \theta h$ and l. Additionally, we assume that both leisure and (habit-adjusted) consumption are normal goods, that is,

$$\nu_1 \equiv u_{12}u_1 - u_2 u_{11} > 0, \quad \nu_2 \equiv u_{12}u_2 - u_1 u_{22} > 0, \text{ and } -(u_{12})^2 + u_{11}u_{22} > 0.$$

Equation (2) is the household's flow budget constraint. Condition (3) is the cash-in-advance constraint in which cash has to be held in advance of purchasing goods. The stock of habits equals weighted average of past consumption in such a way that $h_t = \beta e^{-\beta t} \int_{-\infty}^{t} e^{-\beta \tau} c_\tau d\tau$. Therefore, the dynamic behavior of h is represented as (4). The production function is

$$y = n = 1 - l, \tag{5}$$

and thus the real wage is normalized to hold $w = 1$.

We focus on the situation where the cash-in-advance constraint (3) is binding so that $\alpha c = m$ holds. Additionally, habit formation is assumed to be internal in the sense that the household takes into account the accumulation of habit stock (4) when it solves the optimization problem. The Hamiltonian function is

$$\mathcal{H} = u(c - \theta h, l) + q[(R - \pi)a + (1 - l) - (1 + \alpha R)c - \tau] + \lambda\beta(c - h), \tag{6}$$

where q and λ respectively denote the shadow value of assets, a, and habit stock, h. Since an increase in h lowers utility, the implicit value of h evaluated by utility, λ, has a negative value. The first-ordered conditions are as follows:

$$u_1(c - \theta h, l) = q(1 + \alpha R) - \lambda\beta, \tag{7}$$

$$u_2(c - \theta h, l) = q, \tag{8}$$

$$\dot{q} = [\rho + \pi - R]q, \tag{9}$$

$$\dot{\lambda} = (\rho + \beta)\lambda + \theta u_1(c - \theta h, l). \tag{10}$$

These conditions are standard except for the effect of habit persistence. Especially, (7) states that the marginal benefit of habit-adjusted consumption equals its marginal cost that equals the marginal (dis)utility of having an additional unit of real financial wealth plus that of an additional unit of habit stock. Additionally, the transversality conditions are $\lim_{t\to\infty} e^{-\rho t} q_t a_t = 0$ and $\lim_{t\to\infty} e^{-\rho t} \lambda_t h_t = 0$.

As Taylor (1993) originally suggests, we assume the monetary policy rule such that the central bank sets the rate of nominal interest according not only to the inflation rate but also to the output gap:

$$R(\pi, y) = \phi_\pi(\pi - \pi^*) + \phi_y(y - y^*) + R^*, \quad \phi_\pi > 0, \quad \phi_y \geq 0. \tag{11}$$

In (11), π^* and R^* respectively denote the target rates of inflation and nominal interest set by the monetary authority, which satisfy $\pi^* > -\rho$ and $\pi^* + \rho = R^* > 0$. We discuss determination of the steady-state value of output y^* in Section 3.2. The interest-rate control rule (11) is said to be active if $\phi_\pi > 1$. In this case, the monetary authority rises a real interest rate with a higher rate of inflation under a given level of output. Conversely, the rule is passive when $\phi_\pi < 1$.

The government's budget constraint with zero government purchases in real terms is given by the following: [3]

$$\dot{a} = (R - \pi)a - Rm - \tau. \tag{12}$$

Combining (12) with (2) gives the goods-market equilibrium condition

$$c = 1 - l. \tag{13}$$

3 Equilibrium Dynamics

3.1 Dynamic System

From (7) and (8), we obtain

$$\frac{u_2(c - \theta h, l)}{u_1(c - \theta h, l)} = \frac{1}{1 + \alpha R - \beta x}, \tag{14}$$

which means that the marginal rate of substitution between leisure l and the habit-adjusted consumption $c - \theta h$, equals the real wage in terms of the effective price including the opportunity cost of money holdings with internal habit, $1 + \alpha R - \beta x > 1$, where $x = \dfrac{\lambda}{q} < 0$. We can also interpret that the left-hand side in (14) represents a labor supply and the right-hand side is labor-demand.

Combining (14) with (13), we derive the demand function of leisure

$$l = l\left(h, \frac{1}{1 + \alpha R - \beta x}\right), \tag{15}$$

[3]Originally, the budget is expressed as

$$\dot{B} = RB - \dot{M} - P\tau,$$

where capital letters are nominal terms and P denotes the price level.

where

$$l_1 = -\frac{\theta \nu_1}{\nu_1 + \nu_2} \in (-1, 0) \text{ and } l_2 = -\frac{(u_1)^2}{\nu_1 + \nu_2} < 0.$$

Conversely, $\frac{\partial c}{\partial h} \in (0,1)$ and $\frac{\partial c}{\partial [1/(1+\alpha R - \beta x)]} > 0$ because of goods-market equilibrium (13). These properties are due to the assumption of normal goods. When habit increases under a given effective real wage $\frac{1}{1+\alpha R - \beta x}$, the gross consumption should be higher in order to keep the marginal rate of substitution $\frac{u_2}{u_1}$ constant. Since habit persistence satisfies $\theta \in (0,1)$, leisure decreases with habit stock less than one for one. When the effective real wage rises, the labor demand increases so that the household cuts leisure.

Using (11) and (15), we find that the equilibrium rate of inflation is expressed by

$$\pi = \pi(R, x, h) = \pi^* + \frac{R - R^*}{\phi_\pi} - \frac{\phi_y}{\phi_\pi}\left(1 - l\left(h, \frac{1}{1 + \alpha R - \beta x}\right) - y^*\right), \tag{16}$$

which satisfies

$$\pi_R = \frac{1}{\phi_\pi}\left(1 - \frac{\alpha \phi_y l_2}{(1 + \alpha R - \beta x)^2}\right) > 0, \pi_x = \frac{\beta \phi_y l_2}{\phi_\pi (1 + \alpha R - \beta x)^2} \leq 0, \pi_h = \frac{\phi_y}{\phi_\pi} l_1 \leq 0. \tag{17}$$

The equilibrium income becomes higher with a rise in habit stock h and a fall in the cost of accumulation of habit $-x$, and thus the equilibrium inflation rate falls when $\phi_y > 0$. Even though $\phi_\pi > 1$, π_R could be larger than one because an increase in a nominal interest rate lowers both the effective real wage and production. This means that an active interest-rate control may not generate a rise in the real interest rate with a higher inflation rate. When $\phi_y = 0$, these effects via the monetary-policy's response to output gap disappears so that $\pi_R = \frac{1}{\phi_\pi}$ and $\pi_h = \pi_x = 0$.

From (8) and (15), we obtain the following differential equations:

$$\frac{\dot{q}}{q} = -\frac{\theta u_{12}}{u_2} \dot{h} - \frac{u_{12} - u_{22}}{u_2} \dot{l}, \tag{18}$$

$$\dot{l} = l_1 \dot{h} - \frac{l_2}{(1 + \alpha R - \beta x)^2} (\alpha \dot{R} - \beta \dot{x}). \tag{19}$$

Combining these equations with (9), we see that R changes according to

$$\dot{R} = \frac{\beta}{\alpha}\dot{x} - E_1\{[\rho + \pi(R,x,h) - R] - E_2\dot{h}\}, \tag{20}$$

where

$$E_1 = \frac{\nu_1 + \nu_2}{\alpha u_2(u_{12} - u_{22})}; E_2 = \frac{\theta[-(u_{12})^2 + u_{11}u_{22}]}{\nu_1 + \nu_2} > 0.$$

Note that sign$[E_1]$ =sign$[u_{12} - u_{22}]$. Additionally, dynamic equations of x and h are respectively given by the following: [4]

$$\dot{x} = [\beta(1-\theta) - \pi(R,x,h) + R]x + \theta(1 + \alpha R), \tag{21}$$

$$\dot{h} = \beta\left[1 - l\left(h, \frac{1}{1 + \alpha R - \beta x}\right) - h\right]. \tag{22}$$

Consequently, we obtain a complete dynamic system consisting of (20)-(22) with respect to R, x and h.

3.2 Stability

In the following, asterisks "$*$" denote the steady-state values realized when it holds that $\dot{R} = \dot{h} = \dot{x} = 0$ in (20)-(22). In view of the interest-rate control rule (11), it also holds that $\pi = \pi^*$ and $y = y^*$ in the steady state. We can show that our model has a unique steady state. From (20), we derive

$$R^* = \rho + \pi^*, \tag{23}$$

which gives the steady-state rate of nominal interest. Combining this with (21), we obtain a unique level of x^* such that

$$x^* = -\frac{\theta(1 + \alpha R^*)}{\beta(1-\theta) + \rho} < 0. \tag{24}$$

To determinate the steady-state value of output y^*, note that (13), (22) and (24) yield

$$1 - l\left(h^*, \frac{\beta(1-\theta) + \rho}{(1 + \alpha R^*)(\beta + \rho)}\right) = h^*.$$

[4](21) is derived from $\frac{\dot{x}}{x} = \frac{\dot{\lambda}}{\lambda} - \frac{\dot{q}}{q}$, (7), (9), and (10). Substituting (15) into (4), we obtain (22) as the dynamic of h.

Since $-1 < l_1^* < 0$ for all $h^* > 0$, there is a unique level of h^* satisfying the above equation and thus $y^*(= c^* = h^* = 1 - l^*)$ is uniquely given as well.

Now, we examine the stability of this economy around the steady state. The coefficient matrix of the linearized system of the original one (20)-(22) around the steady state is

$$J = \begin{bmatrix} \dot{R}_R^* & \dot{R}_x^* & \dot{R}_h^* \\ \dot{x}_R^* & \dot{x}_x^* & \dot{x}_h^* \\ \dot{h}_R^* & \dot{h}_x^* & \dot{h}_h^* \end{bmatrix},$$

where

$$\dot{R}_R^* = \frac{\beta}{\alpha}\dot{x}_R^* + E_1^*\{(1 - \pi_R{}^*) + E_2^*\dot{h}_R^*\},$$

$$\dot{R}_x^* = \frac{\beta}{\alpha}\dot{x}_x^* - E_1^*(\pi_x{}^* - E_2^*\dot{h}_x^*), \quad \dot{R}_h^* = \frac{\beta}{\alpha}\dot{x}_h^* - E_1^*(\pi_h{}^* - E_2^*\dot{h}_h^*),$$

$$\dot{x}_R^* = x^* \cdot (1 - \pi_R{}^*) + \theta\alpha, \quad \dot{x}_x^* = \beta(1-\theta) + \rho - x^*\pi_x{}^*, \quad \dot{x}_h^* = -x^*\pi_h{}^* \leq 0,$$

$$\dot{h}_R^* = \alpha\beta l_2^*\left(\frac{u_2^*}{u_1^*}\right)^2 < 0, \quad \dot{h}_x^* = -\beta^2 l_2^*\left(\frac{u_2^*}{u_1^*}\right)^2 > 0,$$

$$\dot{h}_h^* = -\beta(l_1^* + 1) = -\beta\left[-\frac{\theta\nu_1^*}{\nu_1^* + \nu_2^*} + 1\right] < 0.$$

There are two jump variables, R and x, and one predetermined variable, h, in the dynamic system so that the steady state satisfies local determinacy if two eigenvalues of matrix J have positive real parts. When all eigenvalues have positive real parts, there is no equilibrium paths. We call this situation "non-stationary". Otherwise, equilibrium indeterminacy holds, that is, there exist multiple equilibrium paths.

Letting $\mu_i (i = 1, 2, 3)$ be the eigenvalues of J, we obtain the following: [5]

$$\det J = \mu_1\mu_2\mu_3 = -\frac{\beta[\beta(1-\theta) + \rho]E_1^*(1 + l_1{}^*)(\phi_\pi - 1 + \phi_y y_\pi^*)}{\phi_\pi}, \tag{25}$$

$$\begin{aligned} \text{trace}J = \mu_1 + \mu_2 + \mu_3 = \rho &+ \frac{\beta\theta u_{12}^*}{u_{12}^* - u_{22}^*} - \frac{\phi_y}{\phi_\pi}\frac{u_2^*}{u_{12}^* - u_{22}^*} \\ &+ \frac{\phi_\pi - 1}{\phi_\pi}\frac{1 + \alpha R^*}{\alpha u_1^*(u_{12}^* - u_{22}^*)}\left[\nu_1^* + \nu_2^* + u_2^*(u_{12}^* - u_{11}^*)\frac{\beta\theta}{\beta(1-\theta) + \rho}\right], \end{aligned} \tag{26}$$

[5] Since $1 = \frac{u_2}{u_1}(1 + \alpha R - \beta x)$, $u_{12} - u_{11} = \nu_1 - u_{11}\frac{u_2}{u_1}(\alpha R - \beta x) > 0$. On the other hand, $u_{12} - u_{22} = \frac{\nu_2 + u_{12}u_2(\alpha R - \beta x)}{u_2(1 + \alpha R - \beta x)}$ and thus the sign of $(u_{12} - u_{22})$ is ambiguous when $u_{12} < 0$.

where [6]

$$y_\pi^* \equiv \frac{\alpha l_2^*[\beta(1-\theta)+\rho]}{(1+l_1^*)(1+\alpha R^*)^2(\beta+\rho)} < 0. \tag{27}$$

From (25), we find that

$$\text{sign}[\det J] = -\text{sign}\left[\frac{\phi_\pi - 1 + \phi_y y_\pi^*}{u_{12}^* - u_{22}^*}\right]. \tag{28}$$

The key findings as for local determinacy are summarized in the following propositions and Table 1.

Proposition 1 *The necessary condition for determinate (resp. indeterminate) equilibrium is that* $\dfrac{\phi_\pi - 1 + \phi_y y_\pi^*}{u_{12}^* - u_{22}^*}$ *has a positive (resp. negative) value.* [7]

Proposition 2 *Suppose that* $u_{12}^* > 0$ *and* $\phi_\pi > 1$*. If monetary authority controls the nominal rate of interest in response to inflation alone (*$\phi_y = 0$*), equilibrium is locally determinate.* [8] *Otherwise, indeterminacy could be generated.*

These propositions suggest that the role of habit on equilibrium determinacy is emphasized by the response to output gap in the interest-rate control. When $\phi_y = 0$, the effect of habit on equilibrium determinacy depends only on the form of utility function. Hence, the result in Proposition 2 is similar to one obtained in the model without habit shown below.

3.3 Intuitive Implication

To obtain intuition behind the Propositions 1 and 2, first remember the main results obtained in the model without habit formation. If there is no habit persistence, the

[6] (27) is derived from $1 - l\left(h^*, \dfrac{\beta(1-\theta)+\rho}{(1+\alpha R^*)(\beta+\rho)}\right) = h^*$. We can derive y_π around the steady state in other cases analyzed in Section 3.3 by the same way.

[7] Even though $\det J < 0$, all eigenvalues may be negative when $\text{tr} J < 0$, which implies equilibrium indeterminacy. Non-stationary under $\det J > 0$ is because of not only habit stock but also endogenous labor. Since consumption is constant under the endowment economy, habit accumulation depresses if habit stock increases, but this may not hold when consumption is endogenously determined.

[8] This is because both $\det J > 0$ and $\text{tr} J > 0$ necessarily hold so that only one root is stable.

dynamic equation reduces to

$$\dot{R} = -E_1[\rho + \pi(R) - R], \tag{29}$$

where $E_1 = \dfrac{\nu_1 + \nu_2}{\alpha u_2(u_{12} - u_{22})}$. Inspecting (29) yields the following:

Proposition 3 *When there is no habit persistence, equilibrium determinacy around the steady state* [9] *holds if and only if* $\dfrac{\phi_\pi - 1 + \phi_y \bar{y}_\pi}{\bar{u}_{12} - \bar{u}_{22}} > 0$*, where*

$$\bar{y}_\pi \equiv \frac{\alpha \bar{l}_2}{(1+\alpha R^*)^2} < 0. \tag{30}$$

Otherwise, indeterminacy emerges.

The result is also classified in Table 2. Meng (2002) who assumes that $\phi_y = 0$ shows a special case of this setting.

We find that $\dfrac{\phi_\pi - 1 + \phi_y y_\pi}{u_{12} - u_{22}} > 0$ around the steady state in each economy analyzed in this and previous subsections is a necessary condition for equilibrium determinacy. To begin with, we investigate the implication of the condition without considering the effects of habit in consumption. If $\phi_\pi - 1 + \phi_y y_\pi$ is positive, a higher inflation rate rises the real rate of interest and thus consumption decreases, because it becomes more beneficial to accumulate financial assets. On the other hand, $u_{12} - u_{22}$ represents the effect on marginal utility of leisure when consumption increases one unit. When consumption decreases with the higher inflation rate, the marginal utility of leisure becomes smaller if $u_{12} > u_{22}$ and thus an agent tries to cut leisure, which contradicts to lower consumption under $\phi_\pi - 1 + \phi_y y_\pi > 0$. This is why determinacy can emerge when $\dfrac{\phi_\pi - 1 + \phi_y y_\pi}{u_{12} - u_{22}} > 0$.

If we consider the effects of internal habit, this process for equilibrium determinacy may be violated. Habit accumulation is decelerated by a decrease in gross consumption due to monetary policy such that $\phi_\pi - 1 + \phi_y y_\pi > 0$. If an agent

[9] When the target rate of inflation is π^*, the steady state of the variable z in this economy is represented as $\bar{z}$.

internalizes habit, he recognizes that this lowers the implicit cost of habit accumulation and thus raises consumption. Therefore, when there is habit persistence in consumption, indeterminacy could emerge even if $\phi_\pi - 1 + \phi_y y_\pi > 0$ and $u_{12} > u_{22}$.

In general, y_π is negative, because the opportunity cost of holding money increases with the rate of inflation and thus consumption falls, which equals to production and is constrained by real money balances. Since consumption rises with the stock of habit as in Section 3.1, y_π^* and $\bar{y}_\pi$ can be different. However, when $\phi_y = 0$, y_π becomes ineffective in the real interest rate and thus the criterion of monetary policy is sign$[\phi_\pi - 1]$ regardless of the existence of habit. Therefore, the difference between the case with internal habit and the one without habit comes only from the form of the utility function. Consequently, if the nominal interest rate does not respond to the output gap, stabilization effects of monetary policy rule in the model with habits are close to those established in the model without habits.

In order to clarify the magnitude of the effect from habit parameters, θ and β, on equilibrium determinacy, we have to examine the relation among the values of y_π and $u_{12} - u_{22}$ around the steady state in each economy, but we cannot analytically find this relation. We solve this problem by specifying the utility function in the next section.

3.4 External Habit Formation

In this subsection, we briefly consider the case where the habit stock represents the social average level so that habit formation is outward-looking. In other words, c in (4) is the average consumption in the economy at large so that an agent takes the motion of h as given when deciding his optimal consumption plans. [10] Then, the Hamiltonian function is now given by

$$\hat{\mathcal{H}} = u(c - \theta h, l) + q[(R - \pi)a + (1 - l) - (1 + \alpha R)c - \tau]. \tag{31}$$

[10]This assumption is often used in papers related with habit formation.

Using the optimization and equilibrium conditions, we find that the dynamic equations are as follows:

$$\dot{R} = -E_1\{[\rho + \pi(R,h) - R] - E_2\dot{h}\}, \tag{32}$$

$$\dot{h} = \beta\left[1 - l\left(h, \frac{1}{1+\alpha R}\right) - h\right]. \tag{33}$$

E_1 and E_2 has the same forms as in (20). Since the law of motion of habit is external, an agent does not consider the cost of habit accumulation $-\beta x$ so that x disappears from the reduced dynamic system. Given the target rate of inflation π^*, we add "**" to the steady state values of key variables in the case of external habits. We see that $c^* < c^{**}$ and $\bar{c} < c^{**}$, because the following results are satisfied:

$$\frac{u_2((1-\theta)c^*, 1-c^*)}{u_1((1-\theta)c^*, 1-c^*)} = \frac{\beta(1-\theta)+\rho}{(1+\alpha R^*)(\beta+\rho)} < \frac{u_2((1-\theta)c^{**}, 1-c^{**})}{u_1((1-\theta)c^{**}, 1-c^{**})} = \frac{1}{1+\alpha R^*},$$

$$\frac{u_2(\bar{c}, 1-\bar{c})}{u_1(\bar{c}, 1-\bar{c})} = \frac{u_2((1-\theta)c^{**}, 1-c^{**})}{u_1((1-\theta)c^{**}, 1-c^{**})} = \frac{1}{1+\alpha R^*}.$$

The coefficient matrix of the linearlized system of the original one (32)-(33) around the steady state is

$$\hat{J} = \begin{bmatrix} \dot{R}_R^{**} & \dot{R}_h^{**} \\ \dot{h}_R^{**} & \dot{h}_h^{**} \end{bmatrix},$$

where

$$\dot{R}_R^{**} = E_1^{**}\{(1-\pi_R^{**}) + E_2^{**}\dot{h}_R^{**}\}, \quad \dot{R}_h^{**} = -E_1^{**}\pi_h^{**} + E_1^{**}E_2^{**}\dot{h}_h^{**},$$

$$\dot{h}_R^{**} = \alpha\beta l_2^{**}\left(\frac{u_2^{**}}{u_1^{**}}\right)^2 < 0, \quad \dot{h}_h^{**} = -\beta(l_1^{**}+1) = -\beta\left[-\frac{\theta\nu_1^{**}}{\nu_1^{**}+\nu_2^{**}} + 1\right] < 0.$$

There is one jump variable, R, and one predetermined variable, h, in the dynamic system so that the steady state satisfies local determinacy, if one eigenvalues are positive. When all eigenvalues are positive, non-stationary holds. Otherwise, equilibrium is indeterminate. We find that

$$\det\hat{J} = \mu_1\mu_2 = -\frac{\beta E_1^{**}(1+l_1^{**})(\phi_\pi - 1 + \phi_y y_\pi^{**})}{\phi_\pi}, \tag{34}$$

$$\text{trace}\hat{J} = \mu_1 + \mu_2 = E_1^{**}\{(1-\pi_R^{**}) + E_2^{**}\dot{h}_R^{**}\} - \beta\left[-\frac{\theta\nu_1^{**}}{\nu_1^{**}+\nu_2^{**}} + 1\right], \tag{35}$$

where

$$y_\pi^{**} \equiv \frac{\alpha l_2^{**}}{(1+l_1^{**})(1+\alpha R^*)^2} < 0. \tag{36}$$

Table 3 and the following proposition represent the results concerning equilibrium determinacy:

Proposition 4 *In the case of external habit formation, local equilibrium determinacy holds if and only if* $\dfrac{\phi_\pi - 1 + \phi_y y_\pi^{**}}{u_{12}^{**} - u_{22}^{**}} > 0.$

Comparing propositions and tables, we again see that the determinacy condition is close to one for the model without habits.

4 Examples

4.1 Non-Separable Utility

We use an example of utility function such that

$$u(c-\theta h, l) = \frac{[(c-\theta h)^\eta l^{1-\eta}]^{1-\sigma} - 1}{1-\sigma}, \quad 0<\eta<1, \quad 0 \le \theta < 1, \quad \sigma > 0, \tag{37}$$

where σ is the inverse of the elasticity of intertemporal substitution. If we assume no habit formation ($\theta = 0$), then this specification corresponds to that in Meng (2002). As in (26) and (35), traces are complicated. In this subsection, we see only the values of $\dfrac{\phi_\pi - 1 + \phi_y y_\pi}{u_{12} - u_{22}}$ around the steady state, which is the necessary condition for equilibrium determinacy when it is positive.

Gross consumption function [11] and the corresponding steady-state values in each case are

$$c(R,x,h) = \frac{\theta h(1-\eta)(1+\alpha R - \beta x) + \eta}{(1-\eta)(1+\alpha R - \beta x) + \eta} : c^* = \frac{\eta}{(1-\eta)(1-\theta)(1+\alpha R^* - \beta x^*) + \eta},$$

$$c(R,h) = \frac{\theta h(1-\eta)(1+\alpha R) + \eta}{(1-\eta)(1+\alpha R) + \eta} : c^{**} = \frac{\eta}{(1-\eta)(1-\theta)(1+\alpha R^*) + \eta},$$

[11] The forms of functions are independent from σ, because we assume the Cobb-Douglas utility function as (37).

$$c(R) = \frac{\eta}{(1-\eta)(1+\alpha R)+\eta} : \bar{c} = \frac{\eta}{(1-\eta)(1+\alpha R^*)+\eta}.$$

Using these results, we obtain the absolute value of y_π:

$$|y_\pi^*| = \frac{\alpha c^*}{1+\alpha R^*}, \quad |y_\pi^{**}| = \frac{\alpha c^{**}}{1+\alpha R^*}, \quad |\bar{y}_\pi| = \frac{\alpha(1-\eta)\bar{c}}{(1-\eta)(1+\alpha R^*)+\eta}.$$

Additionally, there exists $\tilde{\sigma}$ such that $\text{sign}[\tilde{\sigma}-\sigma] = \text{sign}[u_{12}-u_{22}]$ around the steady state. More specifically, we obtain:

$$\tilde{\sigma}^* = 1 + \frac{\beta(1-\theta)+\rho}{(1-\eta)[\alpha R^*(\beta+\rho)+\beta\theta]} > 1, \quad \tilde{\sigma}^{**} = \bar{\tilde{\sigma}} = 1 + \frac{1}{(1-\eta)\alpha R^*} > 1.$$

We find that

$$\bar{c} < c^* < c^{**}, \quad |\bar{y}_\pi| < |y_\pi^*| < |y_\pi^{**}|, \quad \tilde{\sigma}^* < \tilde{\sigma}^{**} = \bar{\tilde{\sigma}},$$

and the results of comparative statics are shown in Table 4 and Figure 1. [12]

Circles in Figure 1 emphasize the intersecting point of $\tilde{\sigma}$ and $1+\phi_y|y_\pi|$ in each economy, because equilibrium determinacy can hold in the areas where satisfy $\frac{\phi_\pi - (1+\phi_y|y_\pi|)}{\tilde{\sigma}-\sigma} > 0$ around the steady state. However, we note that this is not a sufficient condition for determinacy when habit is internal. The range $1 \leq \phi_\pi \leq 1+\phi_y|y_\pi|$ becomes broader with a rise in θ and ϕ_y and a decrease in β and R^*, which implies higher steady-state consumption and an increase in policy response to output gap. Then, it becomes difficult to generate determinacy when $\sigma < \tilde{\sigma}$. On the other hand, $\sigma > \tilde{\sigma}$ is harder to hold when R^* is higher. Moreover, an increase in habit parameters θ and β lowers $\tilde{\sigma}$ when habit is internal, and thus determinacy under passive policy can emerge more easily.

We substitute a following numerical example into the critical values:

$$(\rho, \alpha, \eta) = (0.02, 1, 0.7).$$

Table 5 numerically examines the analytical results in Table 4. Totally, we find that the impact of habit persistence in consumption θ on the critical values is stronger

[12] Since $R^* = \rho + \pi^*$, a rise in R^* has the same effect as an increase in π^*. Thus, the negative impact of π^* on $\bar{\tilde{\sigma}}$ is equivalent to Meng (2002).

than that of the adjustment speed of habit β or the target rate of nominal interest R^*. We show more detailed results in the following.

If ϕ_y is enough high and consumption habit exists, the steady-state value of $-\phi_y y_\pi$ can be more than 0.5. This implies that real rate of interest does not increase with a higher inflation rate even when $\phi_\pi = 1.5$ which is the value empirically shown in Taylor (1993). Therefore, it becomes harder to emerge determinacy under $\sigma < \tilde{\sigma}$ and active monetary policy. As for this effect, whether habit is internal or external does not seem to be important.

However, unless habit is internal, the value of $\tilde{\sigma}$ is extremely high. This means that monetary authority can easily accomplish macroeconomic stability without habit by adopting active policy. In contrast, either active or passive interest-rate control can generate equilibrium determinacy under moderate values of the elasticity of intertemporal substitution, if habit is internal and habit persistence θ is high enough.

4.2 Separable Utility

In contrast to other cases, sign of trace is important for whether equilibrium path is uniquely determinate when habit is internal. In order to investigate this issue clearly, we focus on the additive separable utility (i.e., $\sigma = 1$ in (37)). In this subsection, we restrict our attention to the case of internal habits.

When $\sigma = 1$, we can rewirte (26) as

$$\text{trace}J = \rho - \frac{\phi_y}{\phi_\pi}(1 - c^*) + \frac{\phi_\pi - 1}{\phi_\pi}\frac{1 + \alpha R^*}{\alpha}\left(1 + \frac{1 - c^*}{(1-\theta)c^*}\frac{\beta + \rho}{\beta(1-\theta) + \rho}\right), \tag{38}$$

and the critical value of ϕ_π such that trace is zero is represented by

$$\begin{aligned} \phi_\pi(\text{trace}J = 0; \sigma = 1) &= \frac{\phi_y(1 - c^*) + \frac{1 + \alpha R^*}{\alpha}\left(1 + \frac{1 - c^*}{(1-\theta)c^*}\frac{\beta + \rho}{\beta(1-\theta) + \rho}\right)}{\rho + \frac{1 + \alpha R^*}{\alpha}\left(1 + \frac{1 - c^*}{(1-\theta)c^*}\frac{\beta + \rho}{\beta(1-\theta) + \rho}\right)} \\ &\equiv A_1\phi_y + A_2. \end{aligned}$$

Additionally, the critical line of $\det J = 0$ is

$$\phi_\pi(\det J = 0; \sigma = 1) = \frac{\alpha c^*}{1 + \alpha R^*}\phi_y + 1.$$

Since we have already derived

$$c^* = \frac{\eta}{(1-\eta)(1-\theta)(1+\alpha R^* - \beta x^*) + \eta} \text{ and } x^* = -\frac{\theta(1+\alpha R^*)}{\beta(1-\theta)+\rho},$$

we can see that these lines depend on β and θ. Using the lines and the fact that $A_2 < 1$ and $A_1 < \frac{\alpha c^*}{1+\alpha R^*}$, we draw Figure 2 which shows the relation between macroeconomic stability and monetary policy under $\sigma = 1$ in an economy with internal habit.

We find that monetary policy satisfying $\phi_\pi - (1 + \phi_y |y_\pi^*|) > 0$ necessarily makes equilibrium determinate. Additionally, the slope of $\phi_\pi(\det J = 0; \sigma = 1)$ becomes steeper when β falls and θ rises, which implies that it is harder to hold equilibrium determinacy. Using numerical example in the previous section and $R^* = 0.05$, we make Table 6 which shows the effects of β and θ on A_1, A_2, and $\frac{\alpha c^*}{1+\alpha R^*}$. When β and θ increase, the line $\phi_\pi(\text{trace}J = 0; \sigma = 1)$ moves clockwise because the slope (A_1) is flatter and the intercept (A_2) rises. That is, the area where either indeterminate or non-stationary can emerge widens if θ is higher. From Table 6, we can also find that $A_1 \in (0.02, 0.2)$, $A_2 \fallingdotseq 0.99$ and $\frac{\alpha c^*}{1+\alpha R^*} \fallingdotseq 0.7$. Therefore, equilibrium determinacy can easily hold when $(\phi_\pi, \phi_y) = (1.5, 0.5)$ in which Taylor (1993) empirically shows.

5 Concluding Remarks

We analyze the stabilization effects of the interest-rate control rule in the presence of habit formation in consumption. We assume the monetary policy under which the nominal rate of interest responds not only to inflation but also to output gap. Main results are as follows.

First, a necessary condition for determinate equilibrium is the combination of a higher (resp. lower) elasticity of intertemporal substitution and an interest-rate control such that the real rate of interest rises (resp. falls) with a higher inflation rate. However, in contrast to the model without habit, this is not sufficient when an agent takes the implicit cost of habit accumulation into consideration.

Second, we numerically show that the effect generated by the monetary policy's response to output gap is larger when habit exists, but whether habit is internal or external may not produce significant differences. Additionally, in the absence of internal habits, an extremely small elasticity of intertemporal substitution is necessary to hold that the marginal utility of leisure decreases with a rise in consumption. We have shown that determinacy under active policy and the high elasticity of intertemporal substitution becomes harder to emerge because of habit in consumption and of the monetary-policy's response to output gap: and that not only active but also passive interest-rate control can make equilibrium uniquely determinate under moderate values of the elasticity of intertemporal substitution if habit persistence is high enough and an agent internalizes habit.

We suggest some future themes for the study of stability in an economy with habit formation and the monetary policy rule of interest-rate control type. As Auray et. al. (2002, 2004 and 2005) show, the way of introduction of money can be important for macroeconomic stability. We have obtained a benchmark for comparing with the case of the MIUF. Since the timing of money holdings may play a critical role in the MIUF model, a model of Auray et. al. (2004) with interest-rate control, instead of the constant growth rate of nominal money supply, would be an interesting setting to be analyzed. Introducing material capital accumulation may be also an important issue.

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Table 1: Equilibrium Determinacy in the Economy with Internal Habit

	$u_{12}^* > u_{22}^*$	$u_{12}^* < u_{22}^*$
$\phi_\pi > 1 - \phi_y y_\pi^*$	D, I	I, NS
$1 \leq \phi_\pi \leq 1 - \phi_y y_\pi^*$	I, NS	D, I
$\phi_\pi < 1$	I, NS	D

D:determinate, I:indeterminate, NS:non-stationary

Table 2: Equilibrium Determinacy in the Economy without Habit

	$\bar{u}_{12} > \bar{u}_{22}$	$\bar{u}_{12} < \bar{u}_{22}$
$\phi_\pi > 1 - \phi_y \bar{y}_\pi$	D	I
$1 \leq \phi_\pi \leq 1 - \phi_y \bar{y}_\pi$	I	D
$\phi_\pi < 1$	I	D

D:determinate, I:indeterminate, NS:non-stationary

Table 3: Equilibrium Determinacy in the Economy with External Habit

	$u_{12}^{**} > u_{22}^{**}$	$u_{12}^{**} < u_{22}^{**}$
$\phi_\pi > 1 - \phi_y y_\pi^{**}$	D	I, NS
$1 \leq \phi_\pi \leq 1 - \phi_y y_\pi^{**}$	I, NS	D
$\phi_\pi < 1$	I	D

D:determinate, I:indeterminate, NS:non-stationary

Table 4: Comparative Statics

	$c^*, \lvert y_\pi^* \rvert$	$c^{**}, \lvert y_\pi^{**} \rvert$	$\bar{c}, \lvert \bar{y}_\pi \rvert$	$\tilde{\sigma}^*$	$\tilde{\sigma}^{**}$	$\bar{\tilde{\sigma}}$
θ	+	+	0	−	0	0
β	−	0	0	−	0	0
R^*	−	−	−	−	−	−

Table 5: Numerical Examples (Section 4.1)

θ	β	R^*	ϕ_y	c^*	c^{**}	$\bar{c}$	$-\phi_y y_\pi^*$	$-\phi_y y_\pi^{**}$	$-\phi_y \bar{y}_\pi$	$\tilde{\sigma}^*$	$\tilde{\sigma}^{**}$	$\bar{\tilde{\sigma}}$
0.2	0.2	0.03	0.5	0.6985	0.7290	0.6938	0.3391	0.3587	0.1031	13.876	112.111	112.111
0.2	0.2	0.03	1.5	0.6985	0.7290	0.6938	1.0172	1.0762	0.3094	13.876	112.111	112.111
0.2	0.2	0.07	0.5	0.6904	0.7316	0.6856	0.3326	0.3419	0.1007	11.830	48.619	48.619
0.2	0.2	0.07	1.5	0.6904	0.7316	0.6856	0.9679	1.0256	0.3002	11.830	48.619	48.619
0.2	0.8	0.03	0.5	0.6950	0.7290	0.6938	0.3374	0.3587	0.1031	12.918	112.111	112.111
0.2	0.8	0.03	1.5	0.6950	0.7290	0.6938	1.0122	1.0762	0.3094	12.918	112.111	112.111
0.2	0.8	0.07	0.5	0.6869	0.7316	0.6856	0.3210	0.3419	0.1007	11.120	48.619	48.619
0.2	0.8	0.07	1.5	0.6869	0.7316	0.6856	0.9630	1.0256	0.3002	11.120	48.619	48.619
0.5	0.2	0.03	0.5	0.7119	0.8192	0.6938	0.3456	0.3977	0.1031	4.752	112.111	112.111
0.5	0.2	0.03	1.5	0.7119	0.8192	0.6938	1.0368	1.1930	0.3094	4.752	112.111	112.111
0.5	0.2	0.07	0.5	0.7040	0.8135	0.6856	0.3290	0.3801	0.1007	4.466	48.619	48.619
0.5	0.2	0.07	1.5	0.7040	0.8135	0.6856	0.9870	1.1404	0.3022	4.466	48.619	48.619
0.5	0.8	0.03	0.5	0.6989	0.8192	0.6938	0.3392	0.3977	0.1031	4.297	112.111	112.111
0.5	0.8	0.03	1.5	0.6989	0.8192	0.6938	1.0177	1.1930	0.3094	4.297	112.111	112.111
0.5	0.8	0.07	0.5	0.6908	0.8135	0.6856	0.3228	0.3801	0.1007	4.061	48.619	48.619
0.5	0.8	0.07	1.5	0.6908	0.8135	0.6856	0.9684	1.1404	0.3094	4.061	48.619	48.619
0.8	0.2	0.03	0.5	0.7555	0.9189	0.6938	0.3667	0.4461	0.1031	2.200	112.111	112.111
0.8	0.2	0.03	1.5	0.7555	0.9189	0.6938	1.1002	1.3382	0.3094	2.200	112.111	112.111
0.8	0.2	0.07	0.5	0.7483	0.9160	0.6856	0.3497	0.4280	0.1007	2.140	48.619	48.619
0.8	0.2	0.07	1.5	0.7483	0.9160	0.6856	1.0491	1.2841	0.3022	2.140	48.619	48.619
0.8	0.8	0.03	0.5	0.7132	0.9189	0.6938	0.3462	0.4461	0.1031	1.903	112.111	112.111
0.8	0.8	0.03	1.5	0.7132	0.9189	0.6938	1.0386	1.3382	0.3094	1.903	112.111	112.111
0.8	0.8	0.07	0.5	0.7053	0.9160	0.6856	0.3296	0.4280	0.1007	1.860	48.619	48.619
0.8	0.8	0.07	1.5	0.7053	0.9160	0.6856	0.9888	1.2841	0.3022	1.860	48.619	48.619

Table 6: Numerical Examples (Section 4.2)

θ	β	A_1	A_2	$\frac{\alpha c^*}{1+\alpha R^*}$
0.2	0.2	0.17206	0.98874	0.6944
0.2	0.5	0.17182	0.98885	0.6917
0.2	0.8	0.17175	0.98888	0.6910
0.5	0.2	0.10987	0.99248	0.7080
0.5	0.5	0.10711	0.99291	0.6977
0.5	0.8	0.10631	0.99303	0.6948
0.8	0.2	0.03343	0.99731	0.7519
0.8	0.5	0.02822	0.99799	0.7194
0.8	0.8	0.02674	0.99816	0.7092

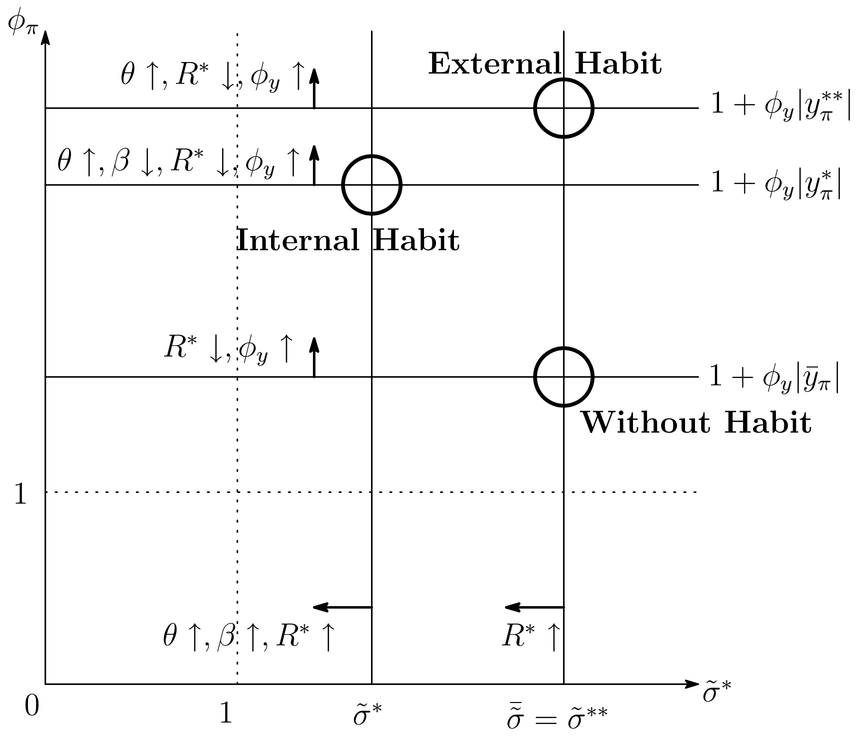

Figure 1: An Illustration of Comparative Statics

Circles emphasize the critical intersecting point of $\tilde{\sigma}$ and $1+\phi_y|y_\pi|$ in each case. Determinacy can hold only in the areas where satisfy $\dfrac{\phi_\pi-(1+\phi_y|y_\pi|)}{\tilde{\sigma}-\sigma}>0$ around the steady state.

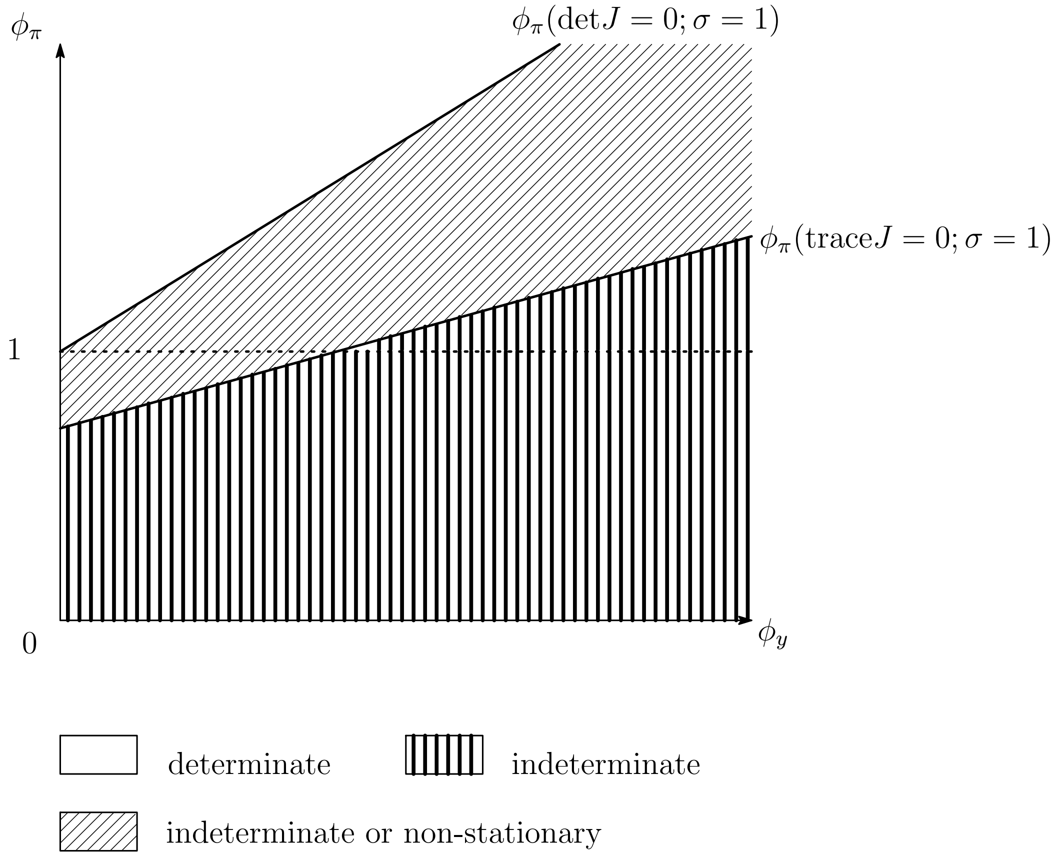

Figure 2: Equilibrium Determinacy under Internal Habit ($\sigma = 1$)